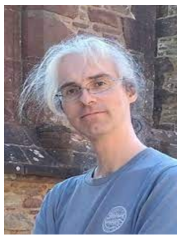
Polaron spectroscopy of many-body systems

Ivan Amelio

Université Libre de Bruxelles

Nathan Goldman's group






IA et al., Phys. Rev. B **107**, 155303 (2023)
IA, PRB **107** (10), 104519 (2024)
IA et N Goldman, SciPost Physics **16** (2), 056 (2024)
A Vashisht, IA et al., arXiv:2407.19093
IA, G Mazza, N Goldman, B **110** (23), 235302 (2024)
P Comaron,Goldman,Imamoglu, IA arXiv:2412.08546

Outline

- **Invitation to polarons**
- Fermi polaron & Chevy ansatz approach

A brief introduction

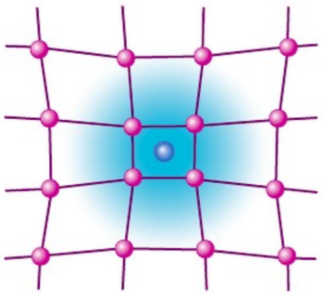
- Polarons in fermionic superfluids
- Polarons in insulators (CDW & Mott transition)
- Chiral polarons
- Repulsive impurity across BKT & BEC transitions

Overview of results

- Conclusions

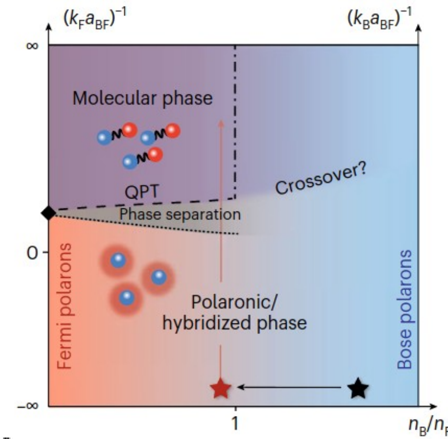
Polarons: motivation

polaron = Impurity dressed by excitations out of many-body background



Transport of electrons (mass renormalization by polar phonons)

Landau et Pekar (1965)

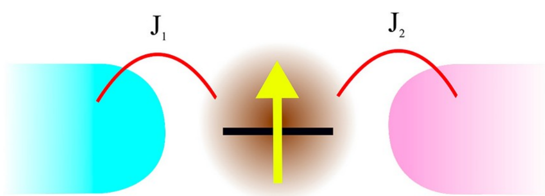


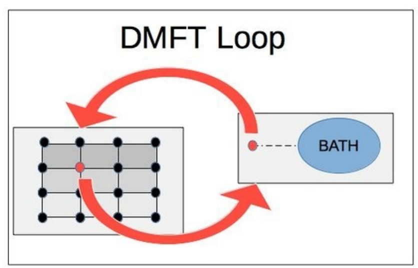
Phase diagram of imbalanced mixtures (cold atoms provide many possibilities!)

Chevy, PRA **74**, 063628 (2006)
Duda et al., Nat. Phys., **19**, 720–725 (2023)

The renormalization group: Critical phenomena and the Kondo problem*†

Kenneth G. Wilson



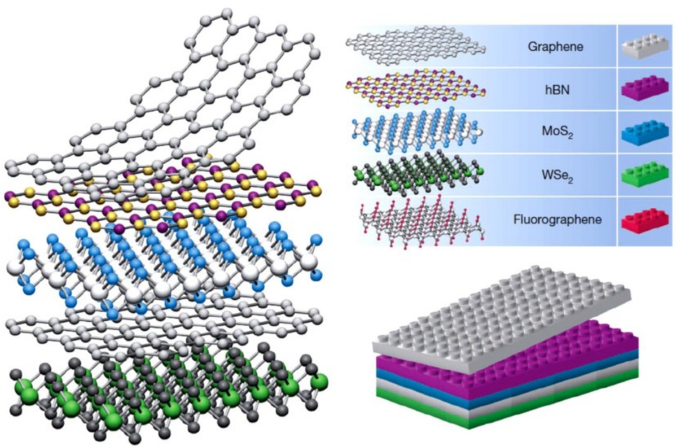


Impurity problems are the simplest nontrivial many-body problems: theoretical development of methods like numerical RG (Kondo), DMFT (Anderson impurity model), etc...

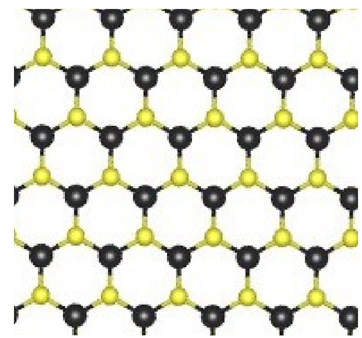
Here:
Polarons as sensors:
Polaron spectroscopy!

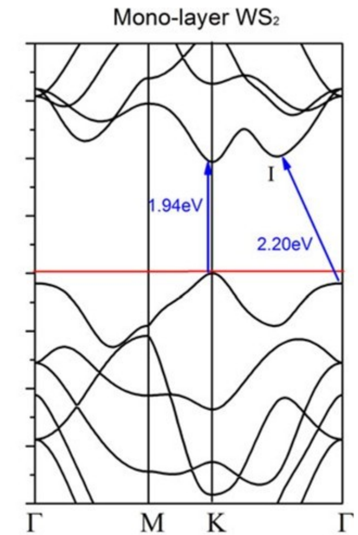
Polaron spectroscopy: TMDs

TMD = transition metal dichalchogenide
aka the <u>gapped</u> brothers of graphene
=> atomically thin semiconductors!



Nature 499, 419–425 (2013)





Spin-valley d.o.f

Exotic & correlated physics

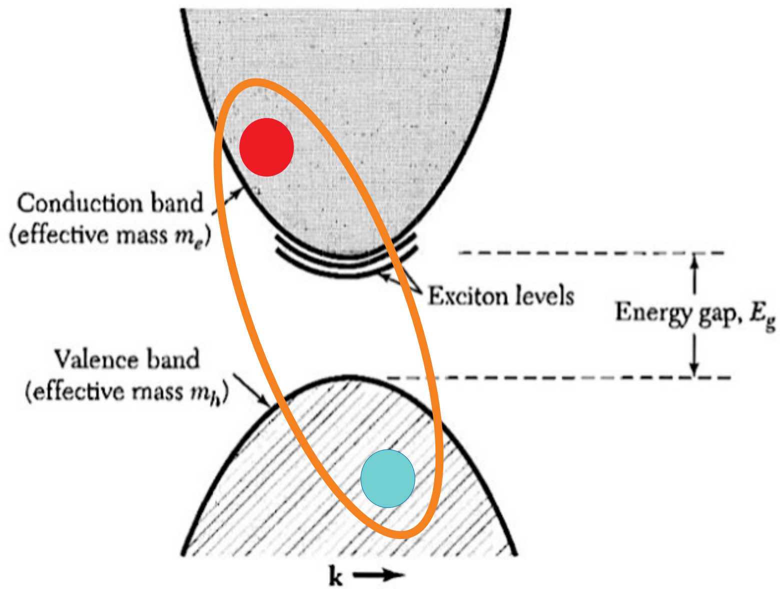
Experimentally observed:
Wigner crystals, Mott insulators, superconductivity, excitonic condensates, Chern & fractional Chern insulators,...

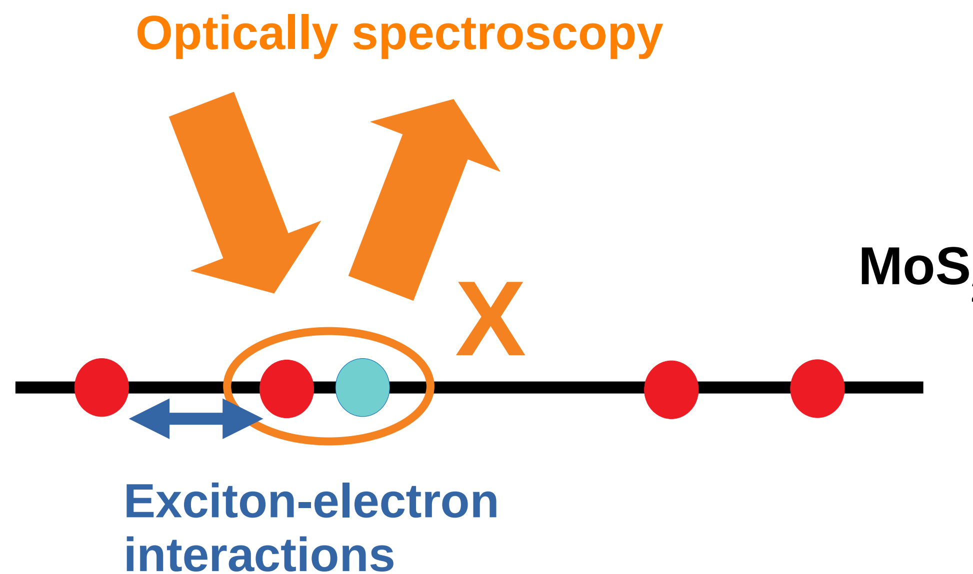
Possibility of stacking, twisting etc. => moire' potentials, flat bands

Reduced screening => strong interactions!

Polaron spectroscopy: TMDs

TMD = transition metal dichalchogenide…   aka the <u>gapped</u> brothers of graphene
However: reduced screening => tightly bound excitons (E_B ~ 200 meV)

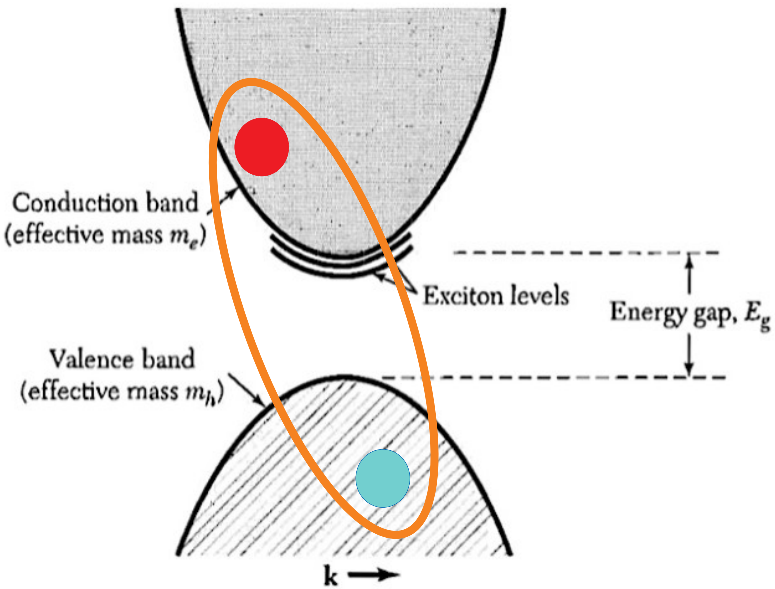


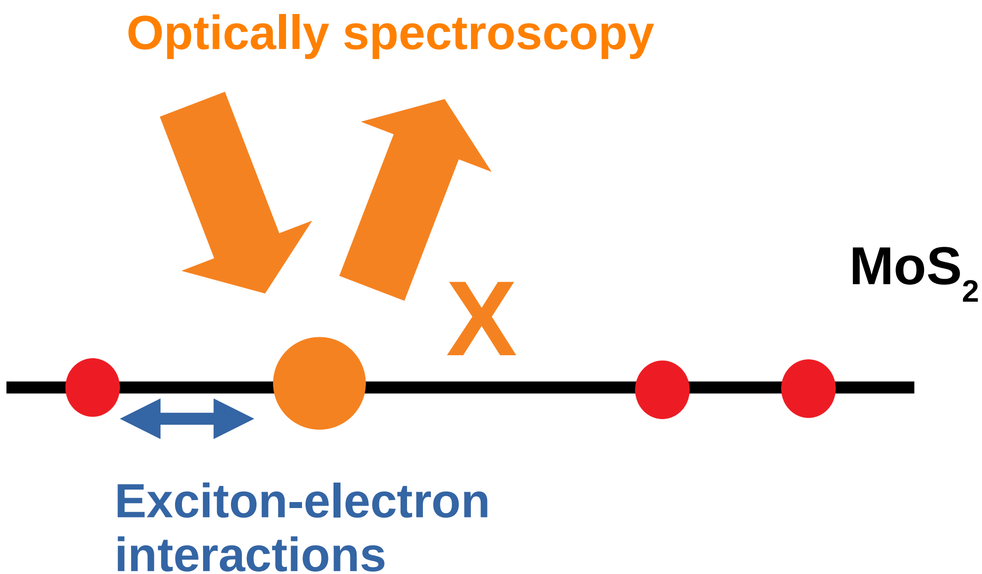


Momentum of light ~ 0 compared to electronic scales...

Polaron spectroscopy: TMDs

TMD = transition metal dichalchogenide… aka the <u>gapped</u> brothers of graphene
<u>However: reduced screening => tightly bound excitons (E_B ~ 200 meV)</u>



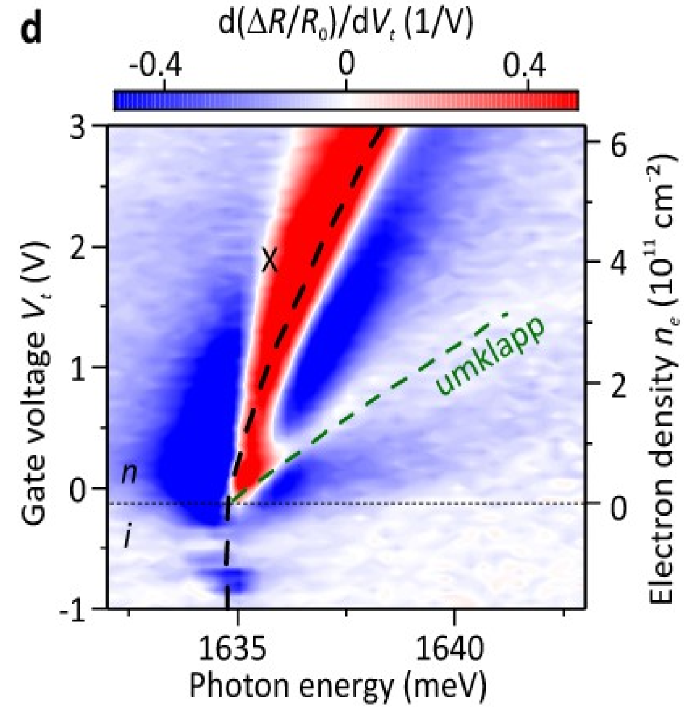


Momentum of light ~ 0 compared to electronic scales...

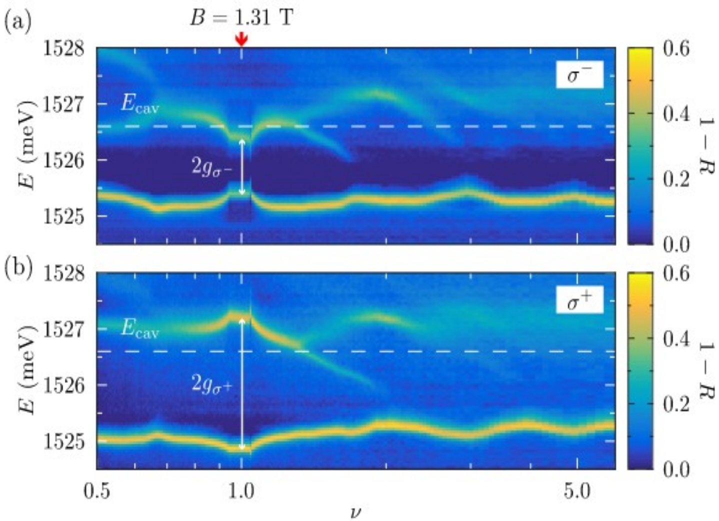
Polaron spectroscopy: TMDs

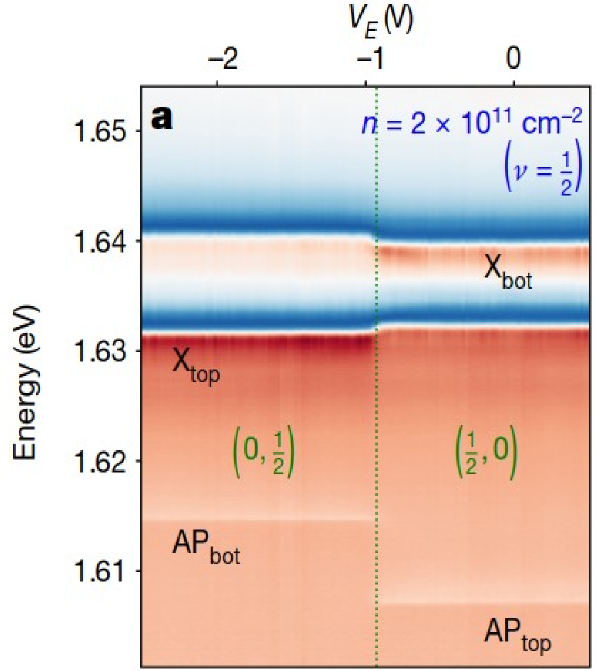
p.e. in Imamoglu's lab @ ETH….

Wigner crystal
Nature **595** (7865), 53-57 (2021)



Quantum Hall phases
PRL **120** (5) 057401 (2018)



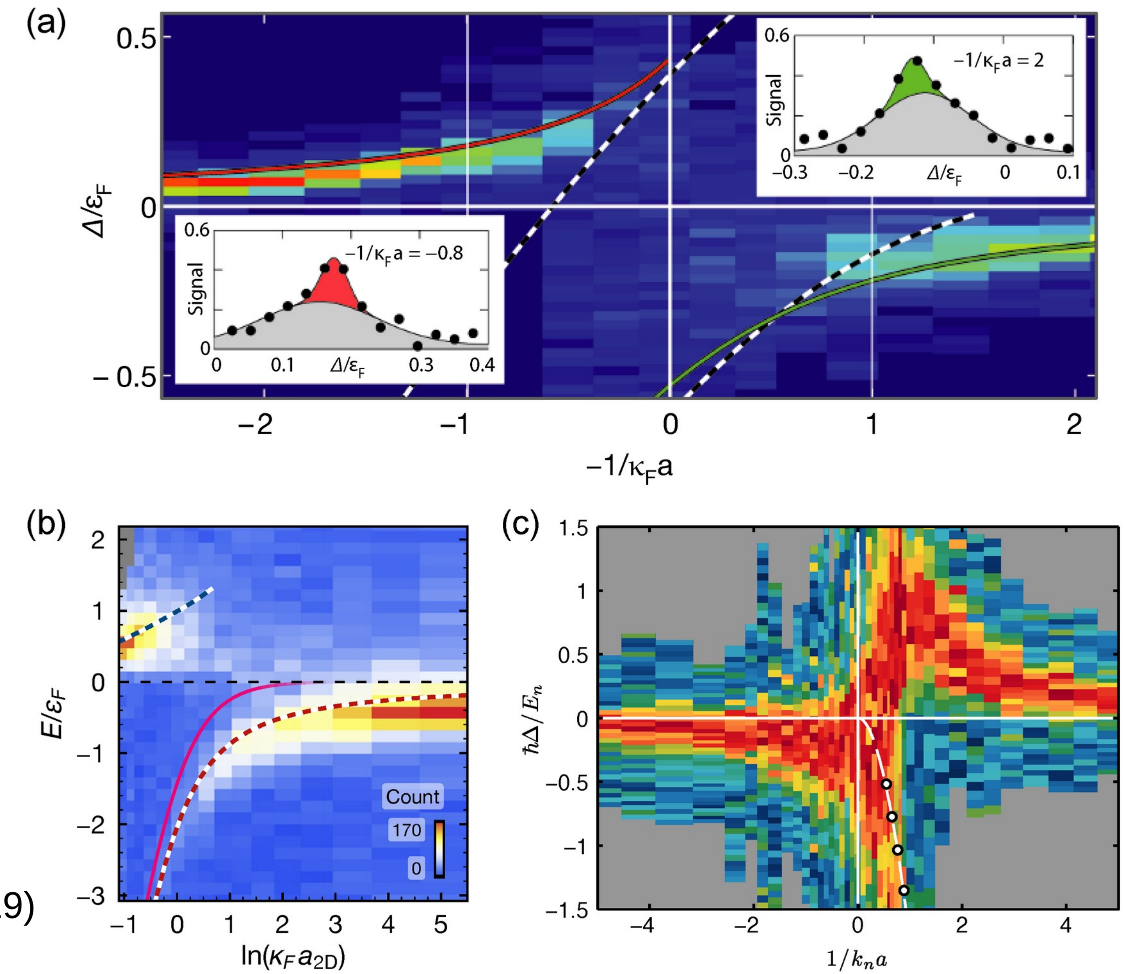


Correlated insulators
Nature **580** (7804), 472-477 (2020)

...but polarons also in cold atoms!

Injection spectroscopy:

Radio-frequency pulse flips hyperfine state so to be close to Feshbach resonance



C. Kohstall, et al, Nature 485, 615 (2012)
Jørgensen et al., PRL 117, 055302 (2016)
Darkwah Oppong et al., PRL 122, 193604 (2019)

Outline

- Invitation to polarons
- **Fermi polaron & Chevy ansatz approach**
- Polarons in fermionic superfluids
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Fermi polaron

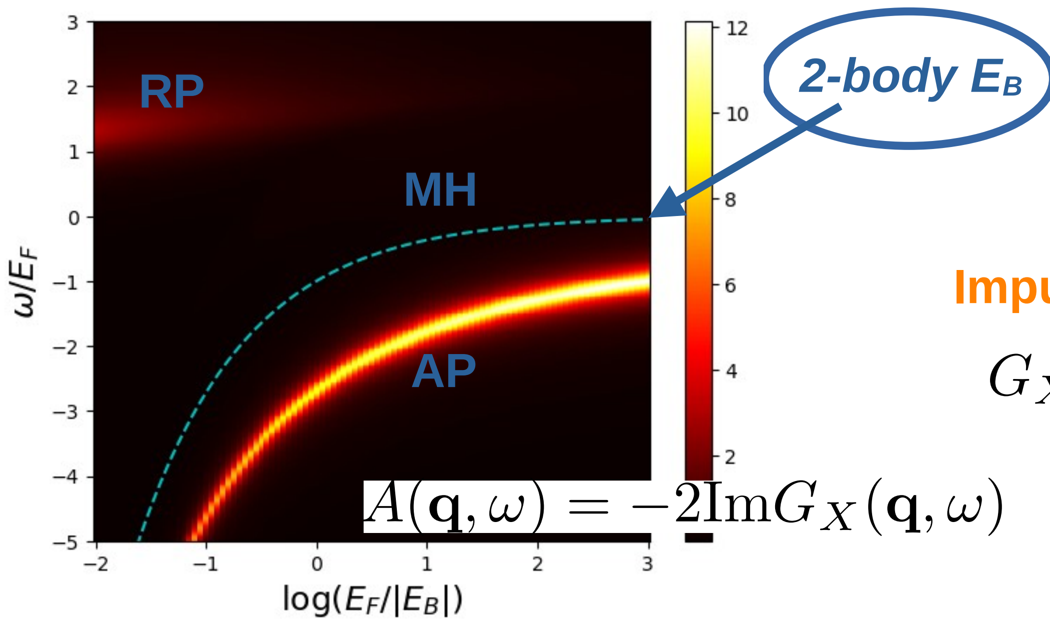
Impurity gets dressed by particle-hole excitations out of a Fermi sea

$$H = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + \sum_{\mathbf{q}} \epsilon^X_{\mathbf{q}} x^\dagger_{\mathbf{q}} x_{\mathbf{q}} + g \sum_{\mathbf{kpq}} x^\dagger_{\mathbf{q}-\mathbf{k}+\mathbf{p}} x_{\mathbf{q}} c^\dagger_{\mathbf{k}} c_{\mathbf{p}}$$

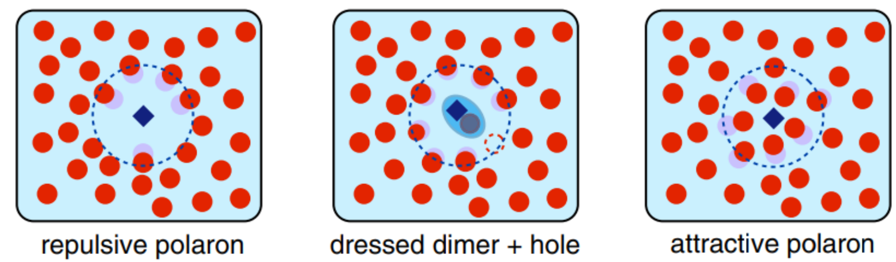
Attractive (AP) and repulsive (RP) polaron branch, molecule-hole continuum (MH).

Fermi dispersion $\xi_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m} - \mu$

Impurity dispersion $\epsilon^X_{\mathbf{q}} = \frac{\mathbf{q}^2}{2M}$



$$A(\mathbf{q},\omega) = -2\mathrm{Im}G_X(\mathbf{q},\omega)$$



Impurity Green's function:

$$G_X(\mathbf{q},\omega) = \langle FS| x_{\mathbf{q}} \frac{1}{\omega - H + i0^+} x^\dagger_{\mathbf{q}} |FS\rangle$$

Massignan et al. Rep. Prog. Phys. **77** 034401 (2014)
Schmidt et al. PRA **85**, 021602(R) (2012)
…

Chevy ansatz approach

Impurity in a Fermi bath: $$H = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + \sum_{\mathbf{q}} \epsilon^X_{\mathbf{q}} x^\dagger_{\mathbf{q}} x_{\mathbf{q}} + g \sum_{\mathbf{kpq}} x^\dagger_{\mathbf{q}-\mathbf{k}+\mathbf{p}} x_{\mathbf{q}} c^\dagger_{\mathbf{k}} c_{\mathbf{p}}$$

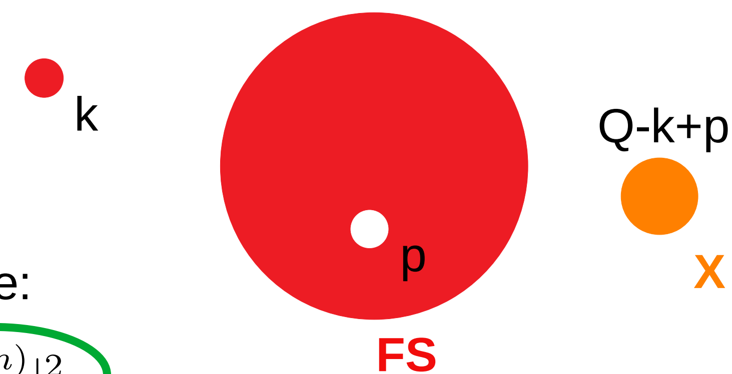
<u>Variational restriction of Hilbert space to 0 & 1 particle-hole excitations on top of Fermi sea</u>

$$|\Psi_{\mathbf{Q}}\rangle = \left\{ \psi_0 x^\dagger_{\mathbf{Q}} + \sum_{\mathbf{kp}} \psi_{\mathbf{kp}} x^\dagger_{\mathbf{Q}-\mathbf{k}+\mathbf{p}} c^\dagger_{\mathbf{k}} c_{\mathbf{p}} \right\} |FS\rangle + O(x^\dagger c^\dagger c^\dagger cc|FS\rangle) + \ldots$$

neglect states with multiple p-h excitations

Basis expansion of generic state.
Notice that total momentum **Q** is conserved.

F. Chevy, PRA **74**, 063628 (2006)



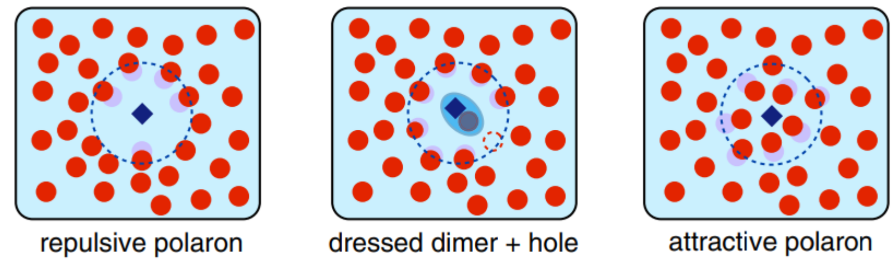
Hamiltonian can be numerically diagonalized in this subspace:

Impurity Green's function: $$G_X(\mathbf{q}, \omega) = \langle FS| x_{\mathbf{q}} \frac{1}{\omega - H + i0^+} x^\dagger_{\mathbf{q}} |FS\rangle = \sum_n \frac{|\psi_0^{(n)}|^2}{\omega - E^{(n)} + i0^+}$$

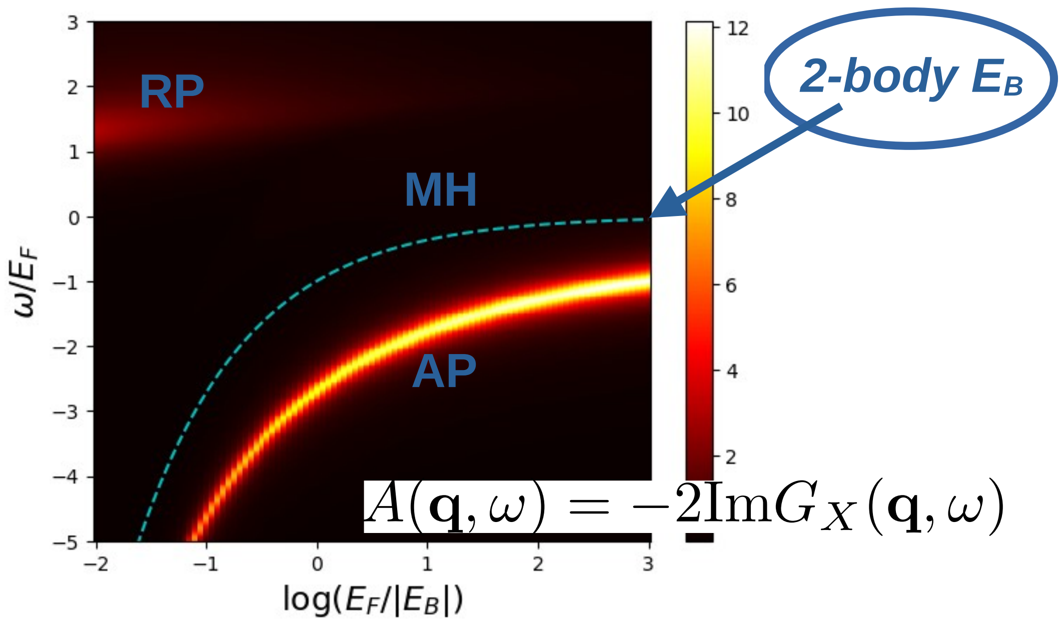
Oscillator strength of n-th eigenstate of energy $E^{(n)}$

Fermi polaron spectrum (Chevy)

$$|\Psi_{\mathbf{Q}}\rangle = \left\{ \psi_0 x^\dagger_{\mathbf{Q}} + \sum_{\mathbf{kp}} \psi_{\mathbf{kp}} x^\dagger_{\mathbf{Q}-\mathbf{k}+\mathbf{p}} c^\dagger_{\mathbf{k}} c_{\mathbf{p}} \right\} |FS\rangle$$



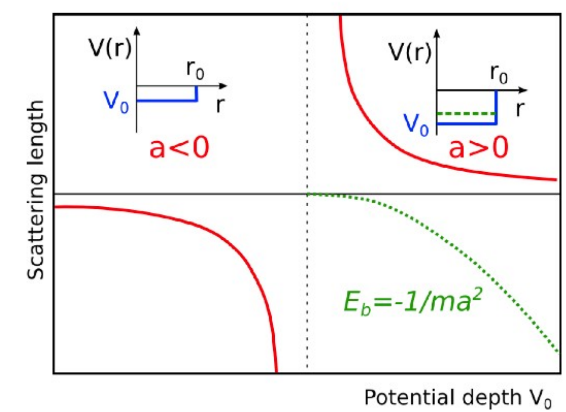
Attractive (AP) and repulsive (RP) polaron branch, molecule-hole continuum (MH).



$$A(\mathbf{q},\omega) = -2\,\mathrm{Im}\,G_X(\mathbf{q},\omega)$$

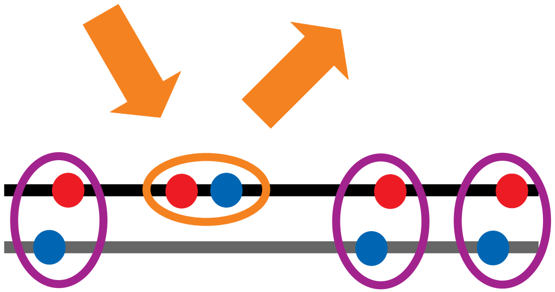
2-body hint for existence of repulsive & attractive branches:

e.g. in 3D



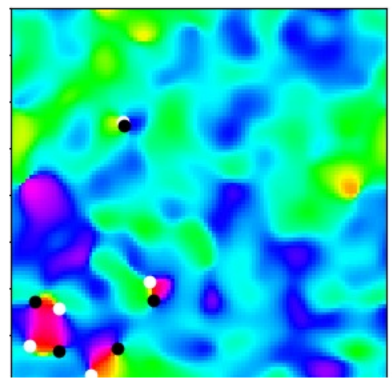
Scope of this talk:

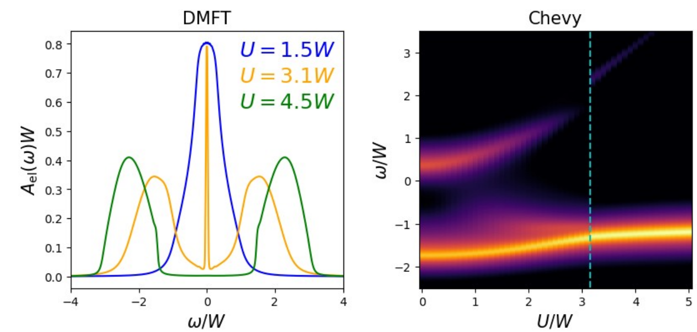
Replace Fermi sea by interesting many-body states!
Insulating states? Detection of nonlocal&topological orders?



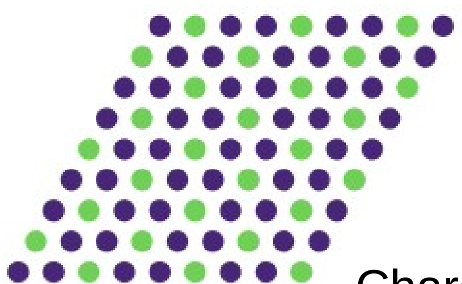
Excitonic insulator
or fermionic superfluids

BKT transition

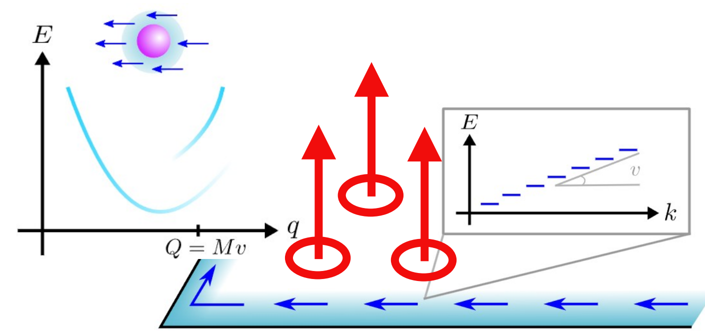




Mott transition



Charge density waves



Topological insulators

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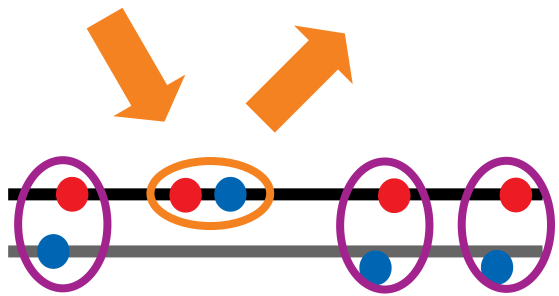
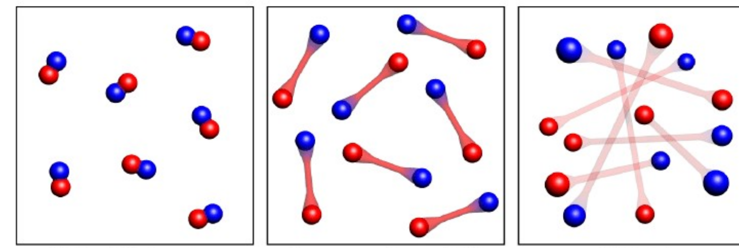
Polarons in bilayer excitonic insulators & Fermi superfluids





Atac Imamoglu (Zurich)
Neil Drummond (Lancaster)
Eugene Demler (Zurich)
Richard Schmidt (Heidelberg)

IA et al., Phys. Rev. B **107**, 155303 (2023)
IA, PRB **107** (10), 104519 (2023)

Polaron across BEC-BCS

fermionic bath *impurity* *impurity-fermion interactions*

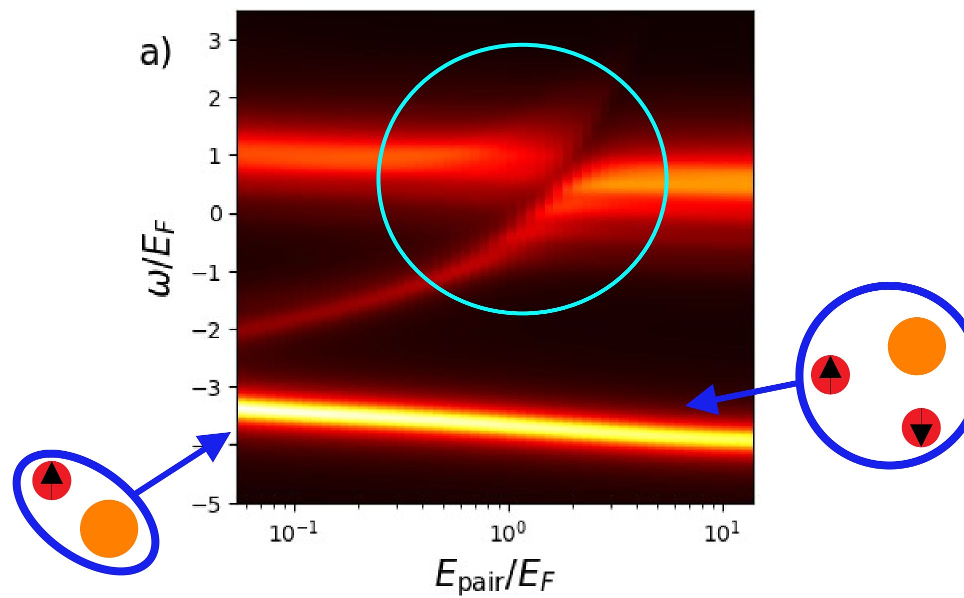
$$H = \sum_{\mathbf{k}\sigma} \xi_k c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + U \sum_{\mathbf{kpq}} c^\dagger_{\mathbf{k+q}\uparrow} c^\dagger_{\mathbf{p-q}\downarrow} c_{\mathbf{p}\downarrow} c_{\mathbf{k}\uparrow} + \sum_{\mathbf{q}} \epsilon^X_{\mathbf{q}} x^\dagger_{\mathbf{q}} x_{\mathbf{q}} + \sum_{\mathbf{kpq}\sigma} g_\sigma x^\dagger_{\mathbf{q-k+p}} x_{\mathbf{q}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{p}\sigma}$$

BCS state is vacuum of Bogoliubov quasiparticles

$$\gamma_{k\sigma} = u_k c_{k\sigma} + v_k c^\dagger_{-k\bar{\sigma}}$$

Interaction with impurity typically depends on spin!

$g_\downarrow = 0$



Generalized Chevy ansatz:

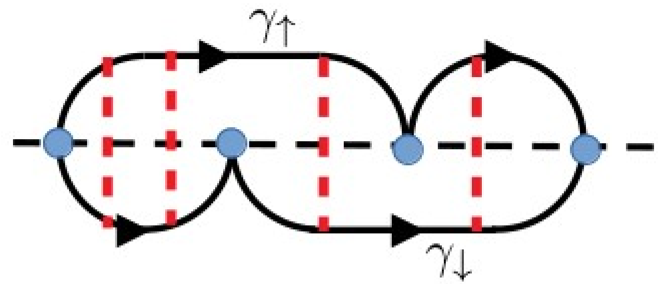
$$|\Psi\rangle = \left(\psi_0 x^\dagger_0 + \sum_{kp} \psi_{kp} x^\dagger_{-k-p} \gamma^\dagger_{k\uparrow} \gamma^\dagger_{p\downarrow}\right) |BCS\rangle$$

Yi et al, PRA **92** (1), 013620 (2015)

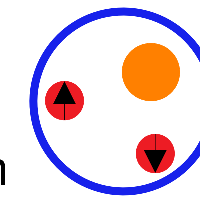
Analysis of Chevy processes

$$|\Psi\rangle = \left(\psi_0 x_0^\dagger + \sum_{kp} \psi_{kp} x^\dagger_{-k-p} \gamma^\dagger_{k\uparrow} \gamma^\dagger_{p\downarrow} \right) |BCS\rangle$$

Typical Chevy diagrams contain **quasi-particle interaction lines...**



… and capture exactly 3-body problem in vacuum

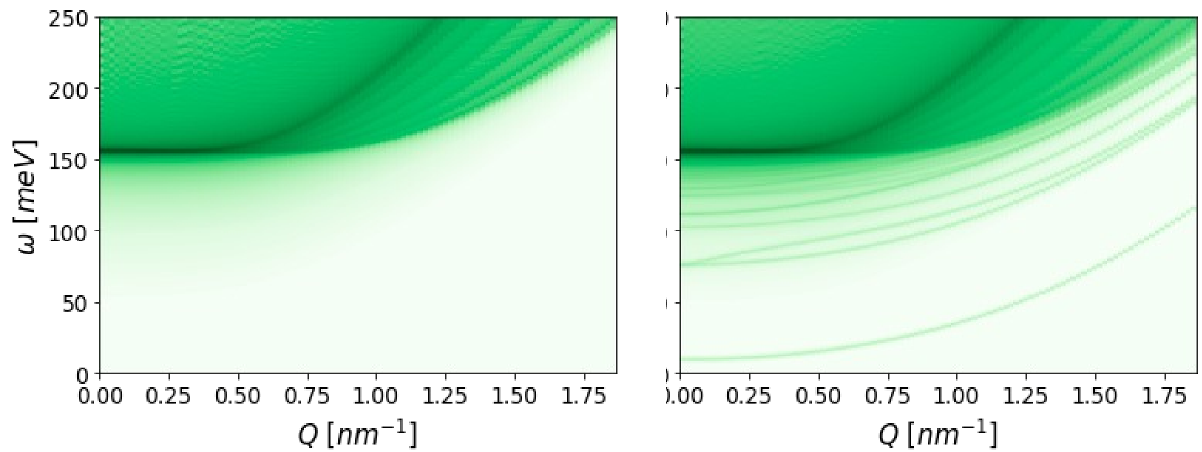


This misses collective modes:

$$H \simeq H_{BCS} = \sum_{\mathbf{k}\sigma} E_k \gamma^\dagger_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma}$$

DOS of excitation modes of EXI (no impurity)

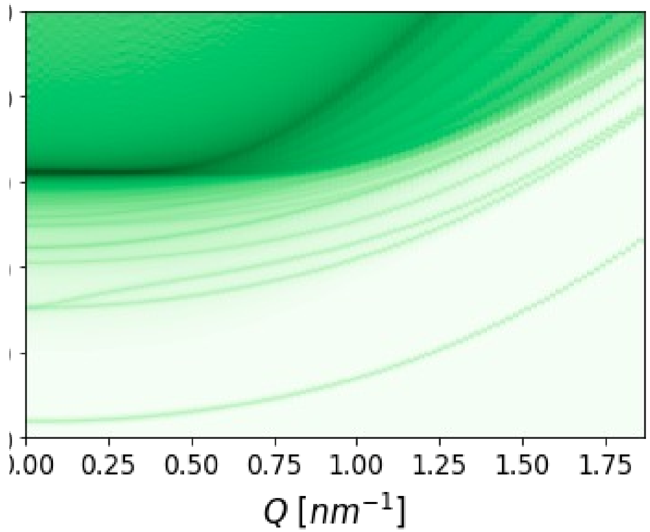
Non-interacting QPs **1PH subspace**



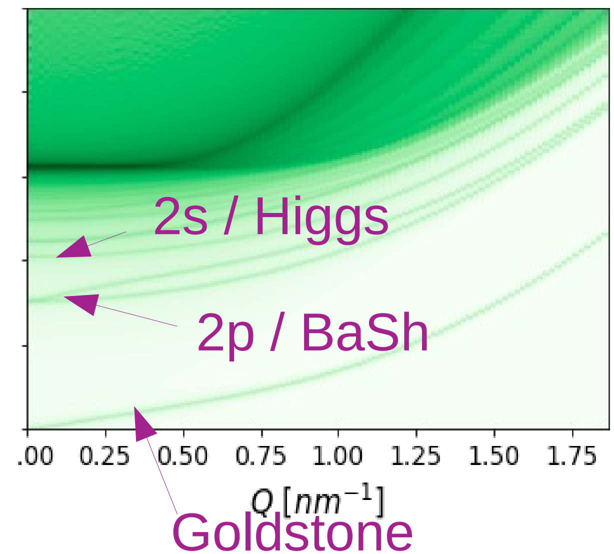
Chevy: what's missing?

DOS of excitation modes of EXI (no impurity)

1PH subspace



TDHF



TDHF = Time dependent Hartree-Fock captures also Goldstone collective mode! (Gaussian fluctuations...)

1ph excitation subspace does not describe correctly Goldstone! It is a problem when gap > binding energy.

Analogy...

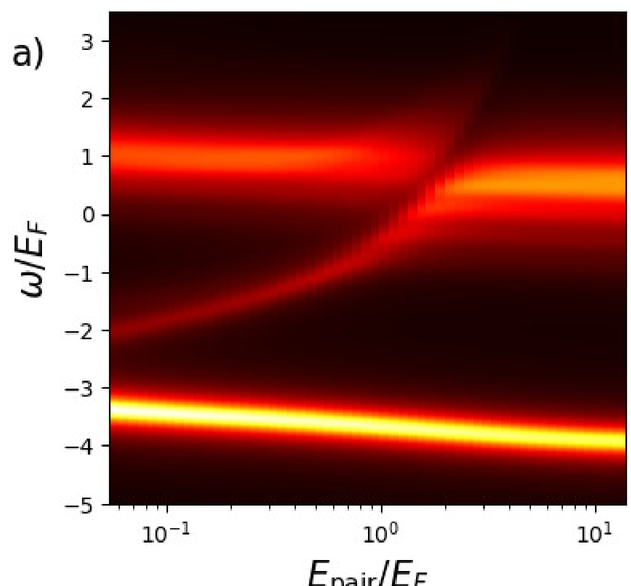
$$\begin{pmatrix} \frac{k^2}{2m} + \mu & \mu \\ -\mu & -\frac{k^2}{2m} - \mu \end{pmatrix}$$

Exact diagonalization

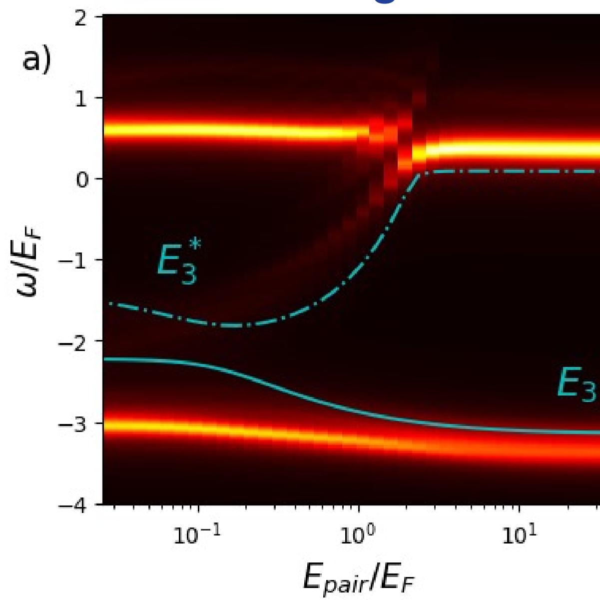
Numerical recipe: sparse real space representation + total momentum conservation + Lanczos Green's function

Nathan Goldman (ULB)

Chevy



ED, with $N_\uparrow = N_\downarrow = 4$ in 4x4 triangular lattice



IA et N Goldman SciPost Physics **16** (2), 056 (2024)

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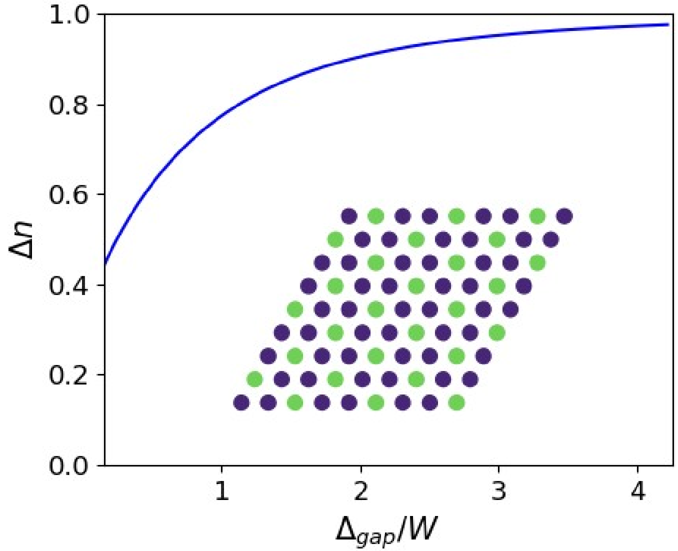
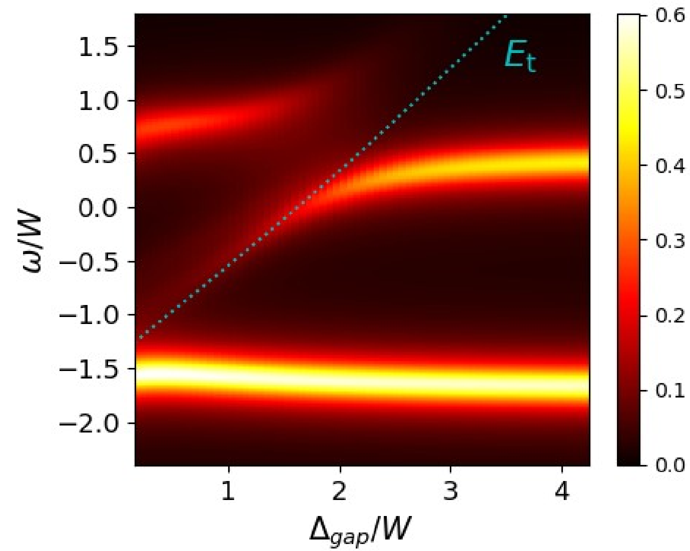
Polarons in CDWs

Motivation: (generalized) Wigner crystals in TMD monolayers and moiré bilayers

Idea: Hartree-Fock for interacting fermions => CDW + Chevy on top of HF.

Main result: clear spectral signature of quasi-particle gap





Nathan Goldman (ULB)
Giacomo Mazza (Pisa)

IA, G Mazza, N Goldman, PRB 110 (23), 235302 (2024)

Polarons in CDWs

Idea: Hartree-Fock for interacting fermions => CDW + Chevy on top of HF.

$$H_{\rm bath} = \sum_{\tilde{k}} \epsilon_{\tilde{k}} c^\dagger_{\tilde{k}} c_{\tilde{k}} + \frac{1}{2\mathcal{V}} \sum_{\tilde{k}\tilde{p}\tilde{q}} V_{\tilde{q}} c^\dagger_{\tilde{k}+\tilde{q}} c^\dagger_{\tilde{p}-\tilde{q}} c_{\tilde{p}} c_{\tilde{k}}$$

Coulomb

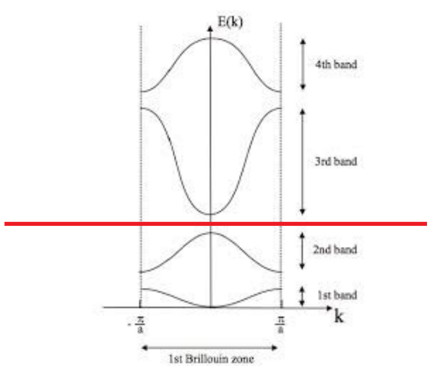
HF + SSB →

$$H^{HF}_{CDW} = \sum_{\vec{q}\alpha} E_{\vec{q}\alpha} d^\dagger_{\vec{q}\alpha} d_{\vec{q}\alpha}$$

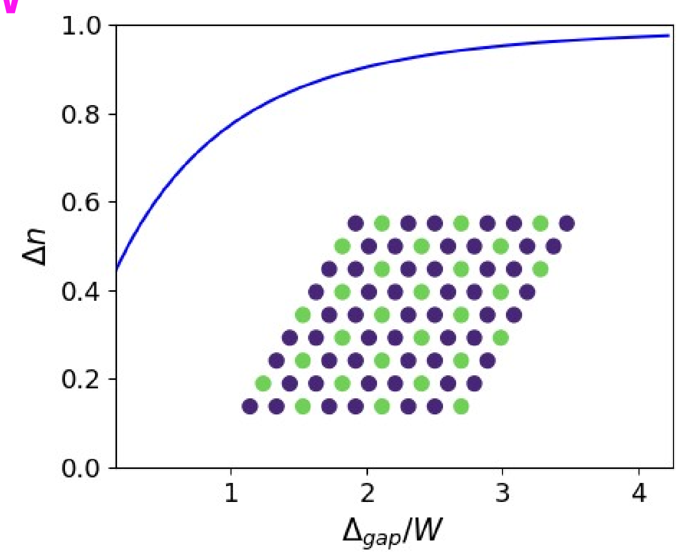
free QPs in HF potential!

$$|\Psi_q\rangle = \left\{ \sum_G \psi_G x^\dagger_{q+G} + \sum_{kpG\alpha\beta} \psi^{\alpha\beta G}_{kp} \; d^\dagger_{k\alpha} d_{p\beta} x^\dagger_{q+p-k+G} \right\} |\mathcal{F}\rangle$$

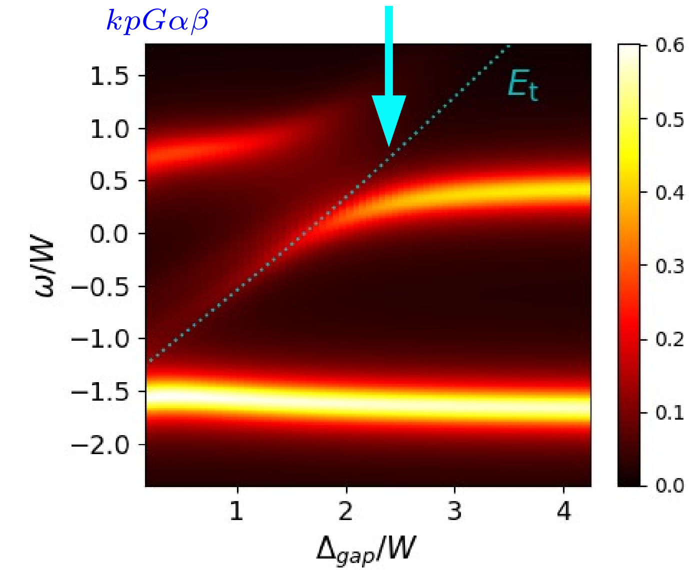
Impurity scatters on CDW



$|\mathcal{F}\rangle$ is fermionic GS = QP bands filled below E_F

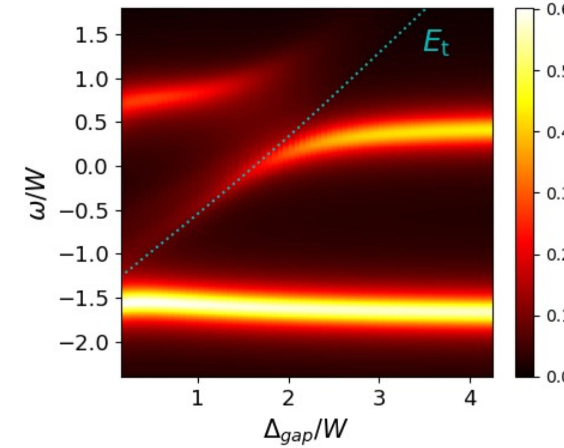


$$|\Psi^t_q\rangle = \sum_{kpG\alpha\beta} \psi^{\alpha\beta G}_{kp} \; d^\dagger_{k\alpha} d_{p\beta} x^\dagger_{q+p-k+G} |\mathcal{F}\rangle$$

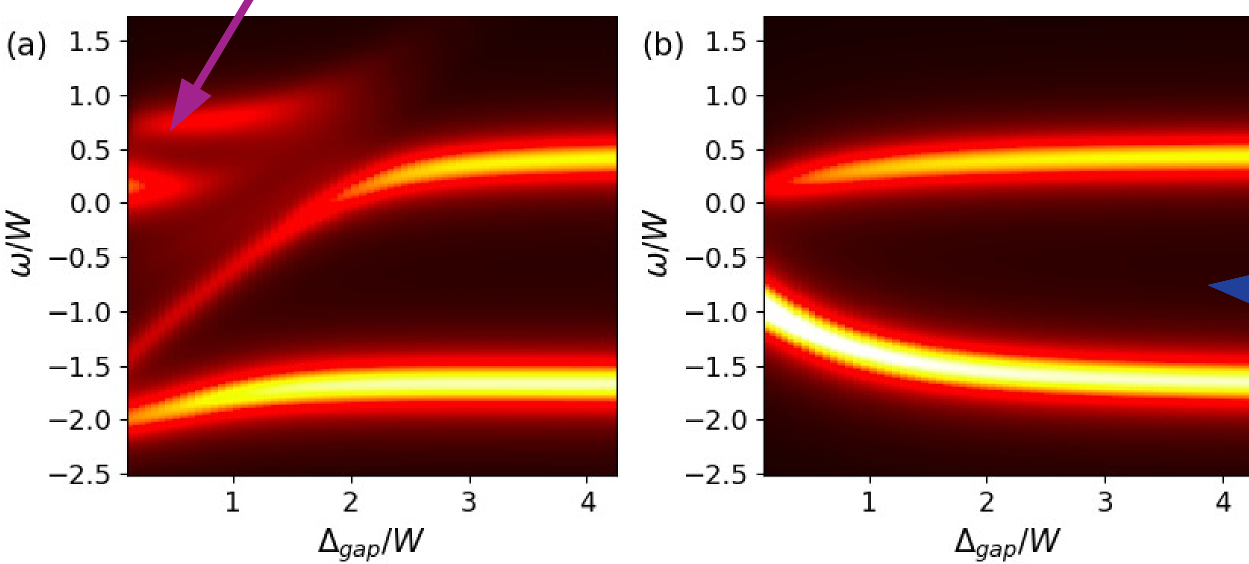


Polarons in CDWs

Idea: Hartree-Fock for interacting fermions => CDW
+ Chevy on top of HF.



Neglecting hole scattering terms is bad!!!
In band systems phase space of hole comparable to phase space of particle!!!



At large QP gap, mean-field scattering against CDW!

$$|\Psi_q^{0PH}\rangle = \sum_G \psi_G x^\dagger_{q+G}|\mathcal{I}\rangle$$

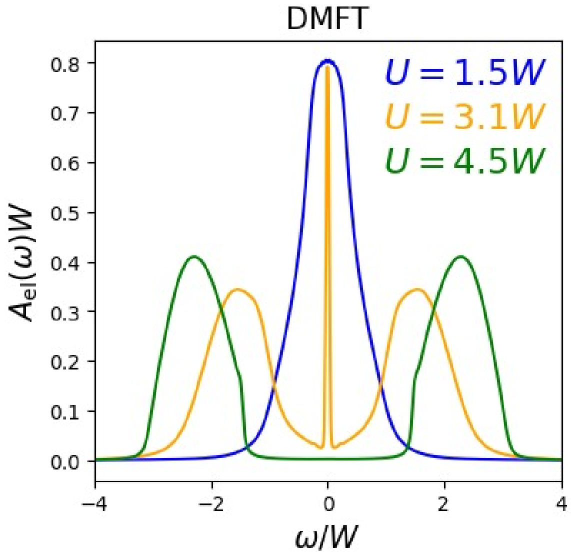
(no PH pairs)

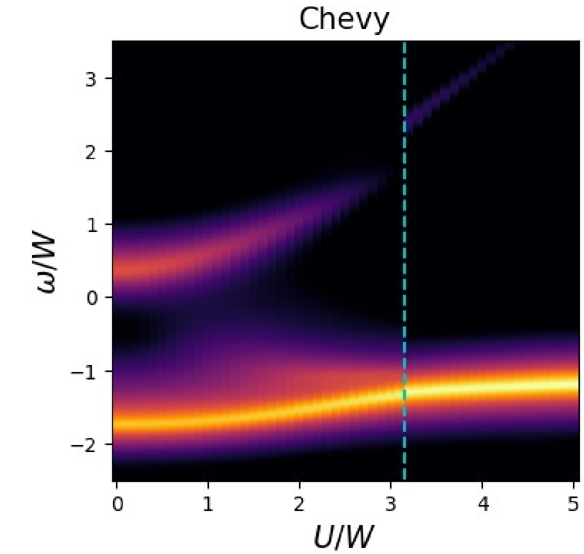
Polaron across Mott transition

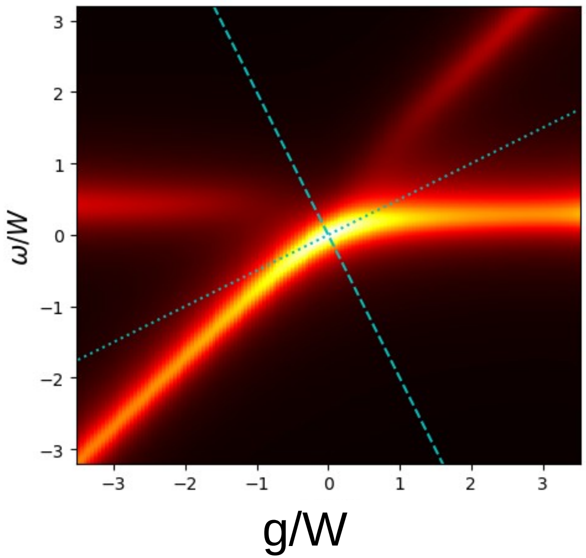
Idea: DMFT DOS + Chevy
(all fermionic vertices neglected...)

Bath = Fermi-Hubbard model,
In DMFT Mott transition at *U ~ 3.1*

Sharp spectral behavior at Mott transition!







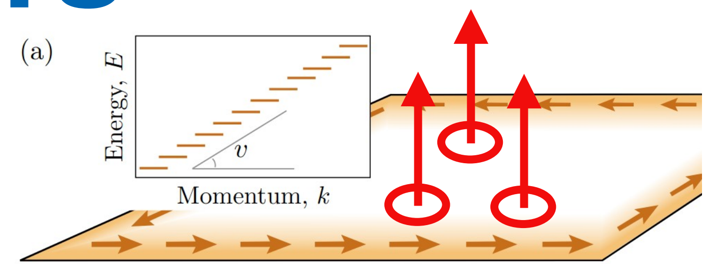
Role of particle-hole symmetry!
Duality between attractive (g<0) and repulsive (g>0) impurity-electron interaction.

IA, G Mazza, N Goldman, (2024) accepted in PRB

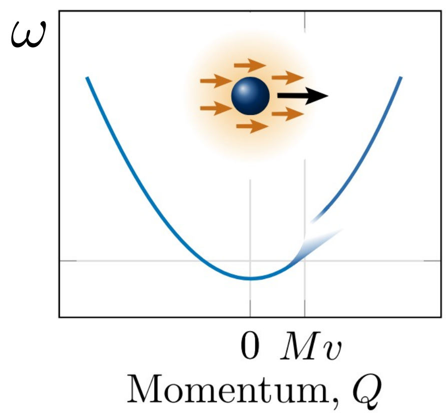
Outline

- Invitation to polarons
- Fermi polaron & Chevy ansatz approach
- Polarons in fermionic superfluids
- Polarons in insulators (CDW & Mott transition)
- **Chiral polarons**
- Repulsive impurity across BKT & BEC transitions
- Conclusions

Chiral polarons in topological insulators



Idea: impurity dressed by excitations of chiral edge mode







Also: development of Tensor Network methods for polarons in fractional Chern insulators!

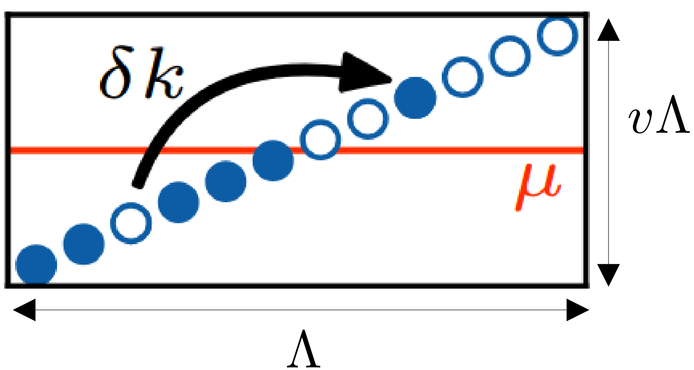
Amit Vashist, IA, Laurens Vanderstraeten, Georg Bruun (Aahrus), Oriana Diessel (Harvard), Nathan Goldman, arXiv:2407.19093

Chiral polarons: 1D model

Idea: impurity dressed by excitations of chiral edge mode

1D effective model of chiral bath:

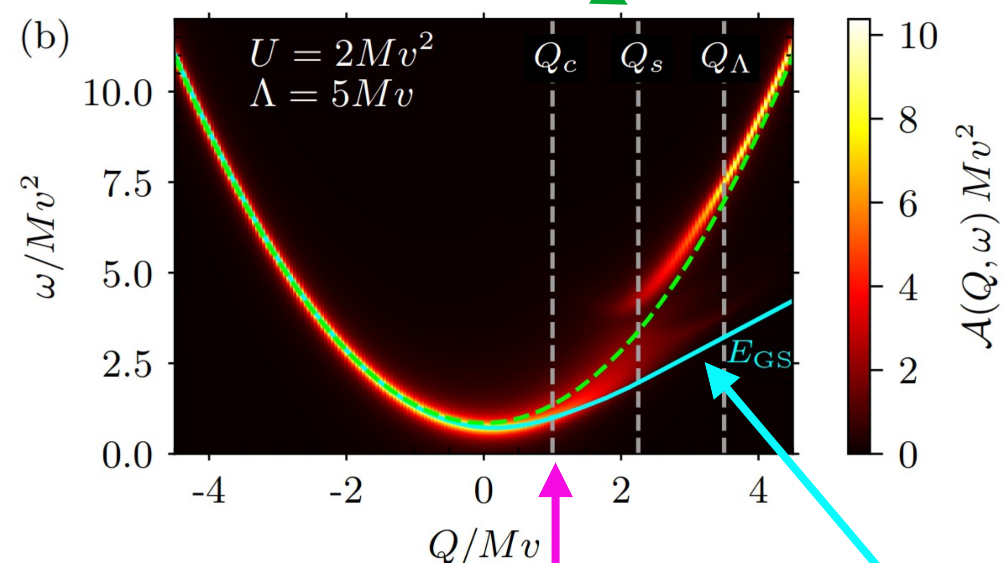
$$\hat{H}_{\text{bath}} = \hat{H}_{\text{chiral}} = \sum_k [vk - \mu]\hat{c}_k^\dagger \hat{c}_k$$



Chirality

➡ **asymmetry in momentum resolved polaron spectrum**

Splitting point $Q_c = Mv + \Lambda/4$ depends on size of topological gap



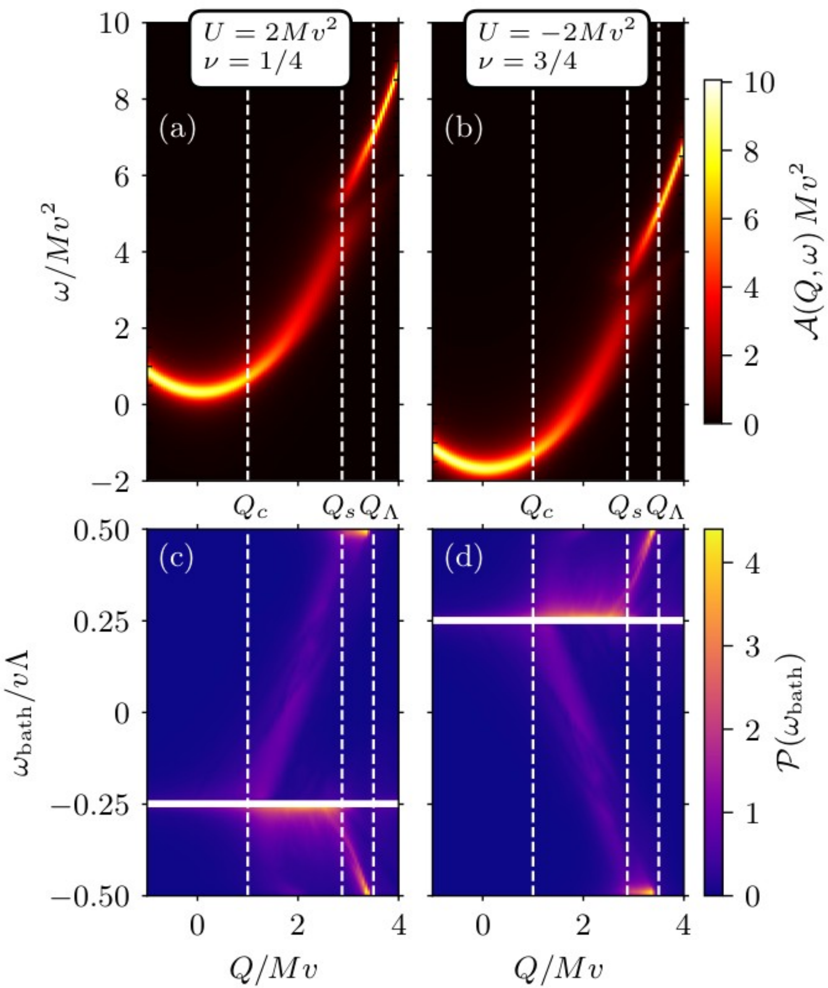
GS dark for Q>Mv

Q_c = Mv special: resonant Impurity-bath scattering

Chevy ansatz (benchmarked with ED)

arXiv:2407.19093

Chiral polarons: particle & hole



Chevy wavefunction: let's look at particle & hole occupations in bright eigenstates.

Origin of Q_s (=splitting) point:
e.g. at *U>0* corresponds to saturation of hole momentum.

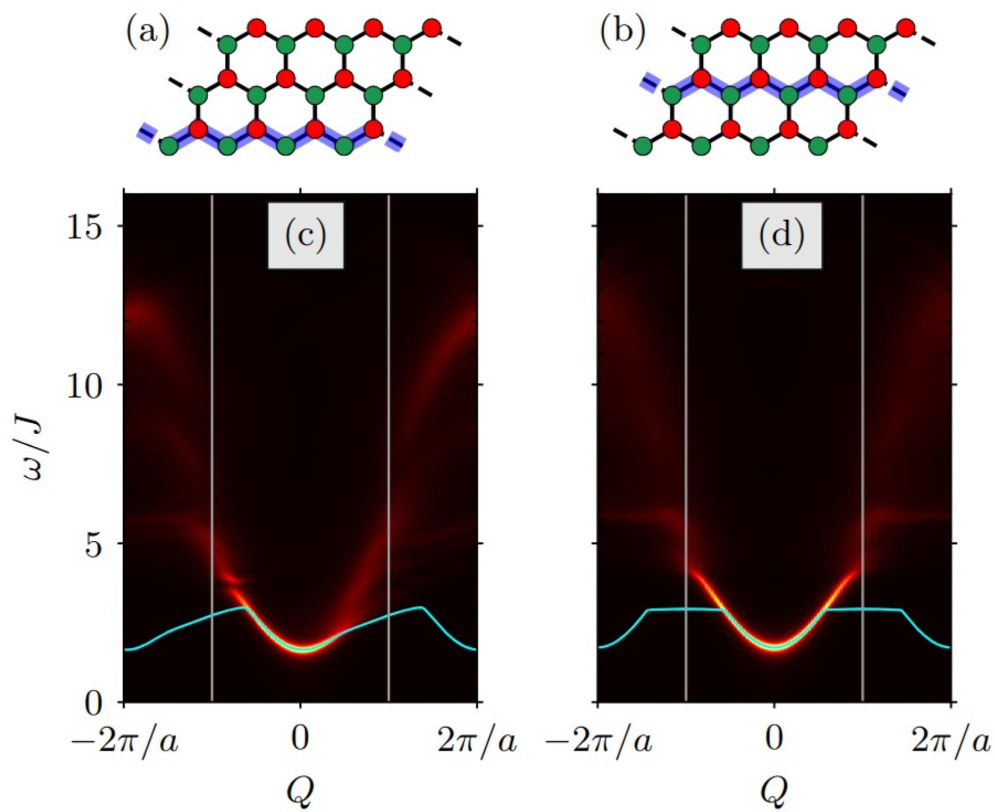
Duality: $U \to -U, \quad \nu \to 1-\nu$

Spectra are the same but for mean-field shift!

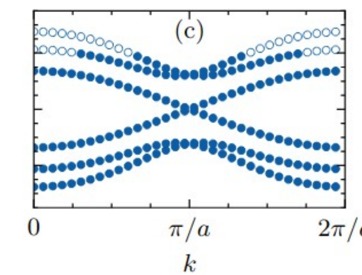
arXiv:2407.19093

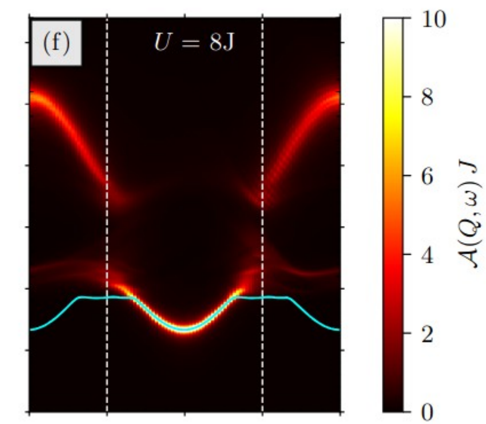
Chiral polarons: Haldane

2D Haldane bath at half-filling
Chiral polaron if impurity confined on edge



No chiral polaron in metallic phase



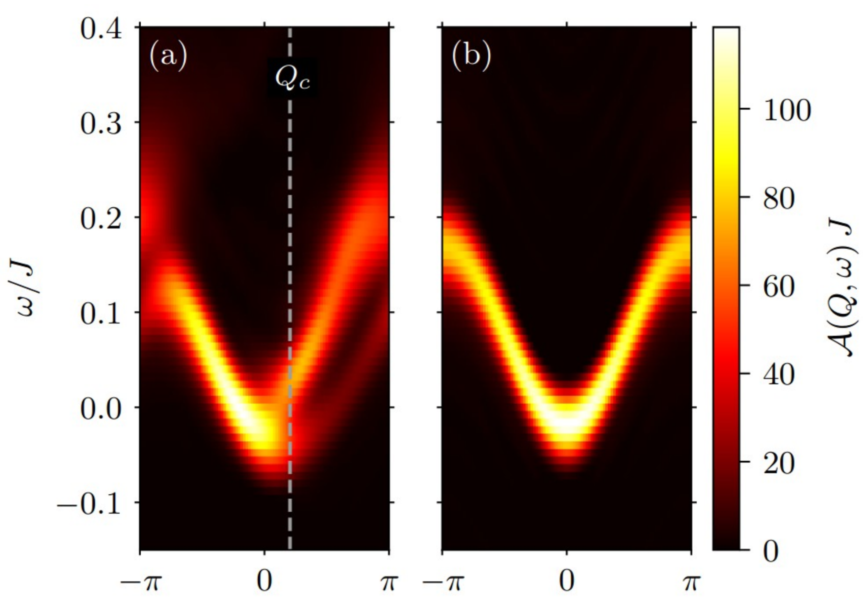


arXiv:2407.19093

Chiral polarons: fractional Chern insulators

First results for Harper-Hofstadter hard-core bosons at filling 1/2 (density 1/8)

On edge   vs   in bulk



Use of a few advanced Tensor Networks tricks:

- VUMPS (exploit translational invariance)
- Charge conservation
- Time-dependent variational principle
- MPO approximation of evolution operator
- Adaptive window of perturbed tensors
- Post-processing in time-domain
- ...

arXiv:2407.19093

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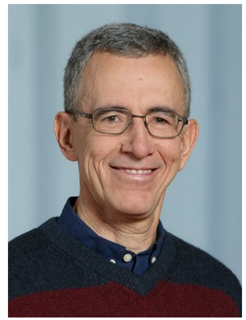
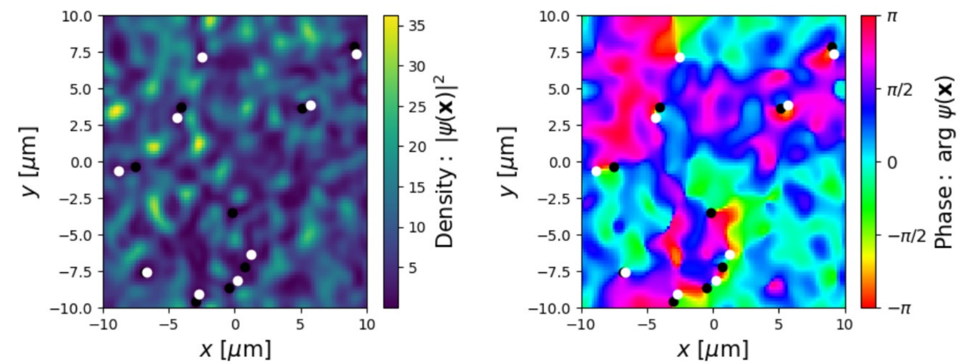
Quantum impurity across BKT

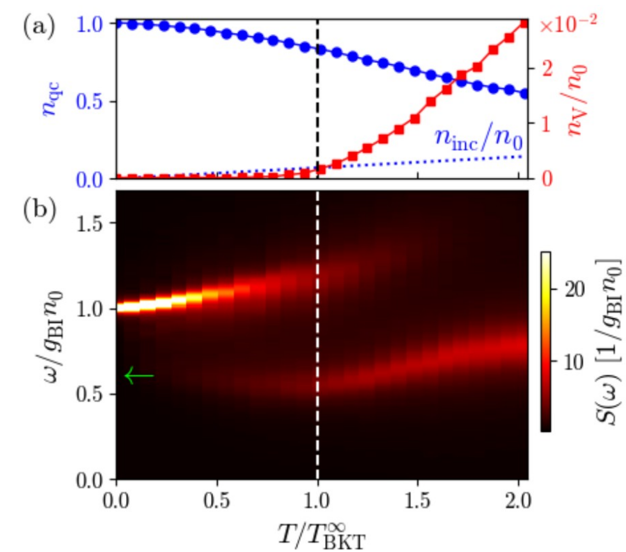






P Comaron (Lecce), N Goldman (ULB),
A Imamoglu (ETH), IA, arXiv:2412.08546

Motivations:
1) can we optically detect BKT transition in EXI?
2) Bose polaron at finite T?



Classical analogue: imaging of vortices in Helium via microparticles (PNAS, **105**, 13707 (2008)).

Quantum impurity across BKT

$$i\partial_t \psi = \mathcal{P}(1 - i\gamma) \left\{ -\frac{1}{2m}\nabla^2 + g_{BB}|\psi|^2 - \mu \right\} \psi + \eta$$

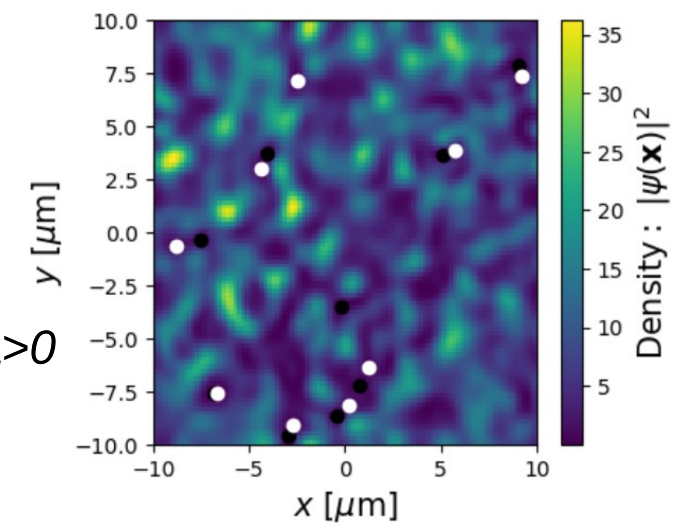
Stochastic Projected Gross-Pitaevskii equation for Bose gas

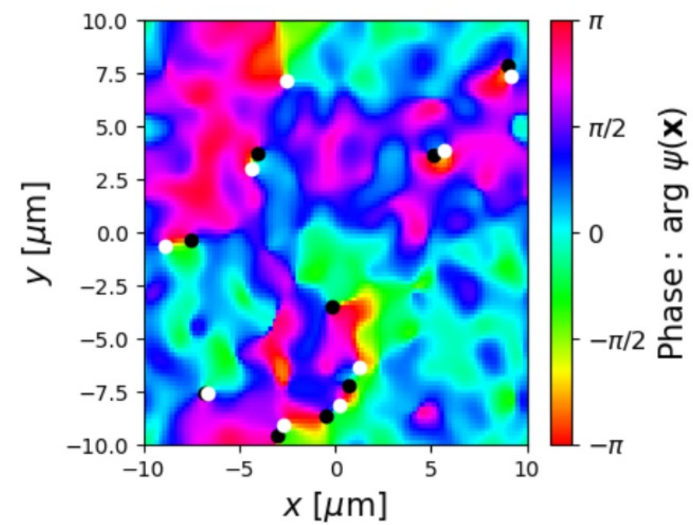
Noise & relaxation satisfying FDT: $\langle \eta^*(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle = 2\gamma T \delta(t-t')\delta(\mathbf{x}-\mathbf{x}')$

$$i\partial_t \Psi = \left\{ -\frac{1}{2M}\nabla^2 + g_{BI}|\psi|^2 \right\} \Psi$$

Schrödinger eq. in time-dependent potential for the impurity

Setting: impurity in a finite T Bose bath will bind to vortices and density deeps, for <u>repulsive</u> Bose-impurity interactions $g_{BI}>0$





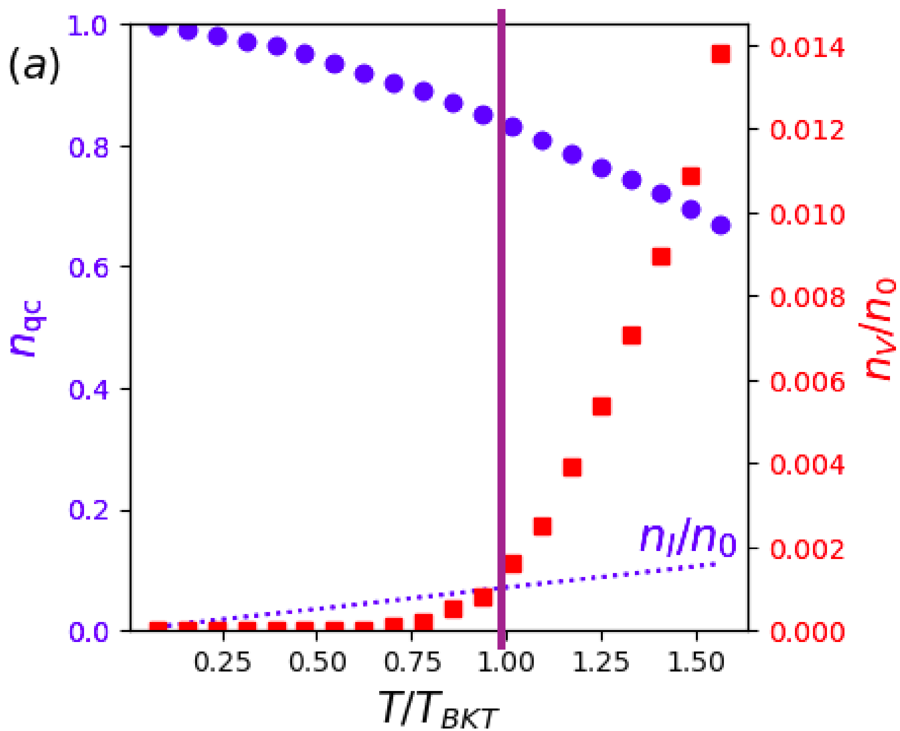
Quantum impurity across BKT

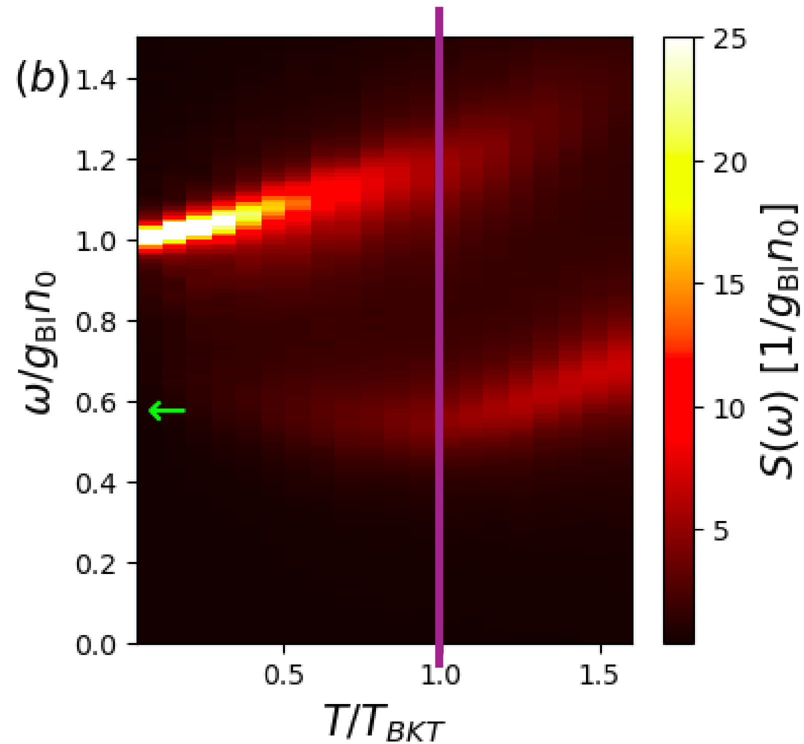
Quasi-condensate fraction is smooth across BKT transition!

$$n_{\rm qc} = \sqrt{2\langle\langle|\psi|^2\rangle\rangle^2 - \langle\langle|\psi|^4\rangle\rangle} / \langle\langle|\psi|^2\rangle\rangle$$

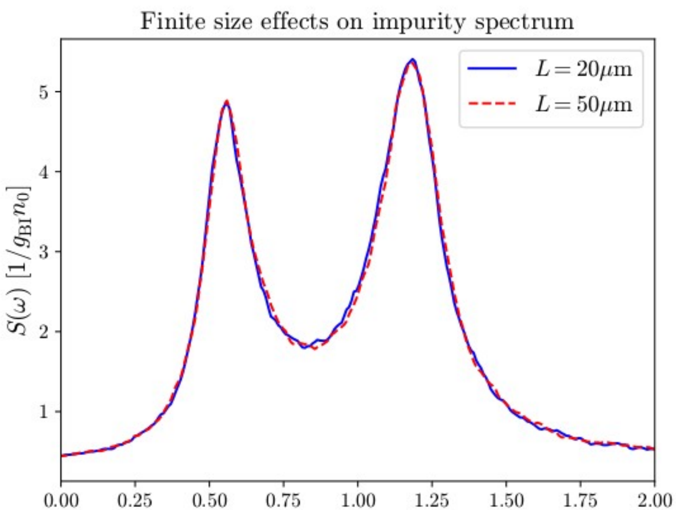
Vortex unbinding at

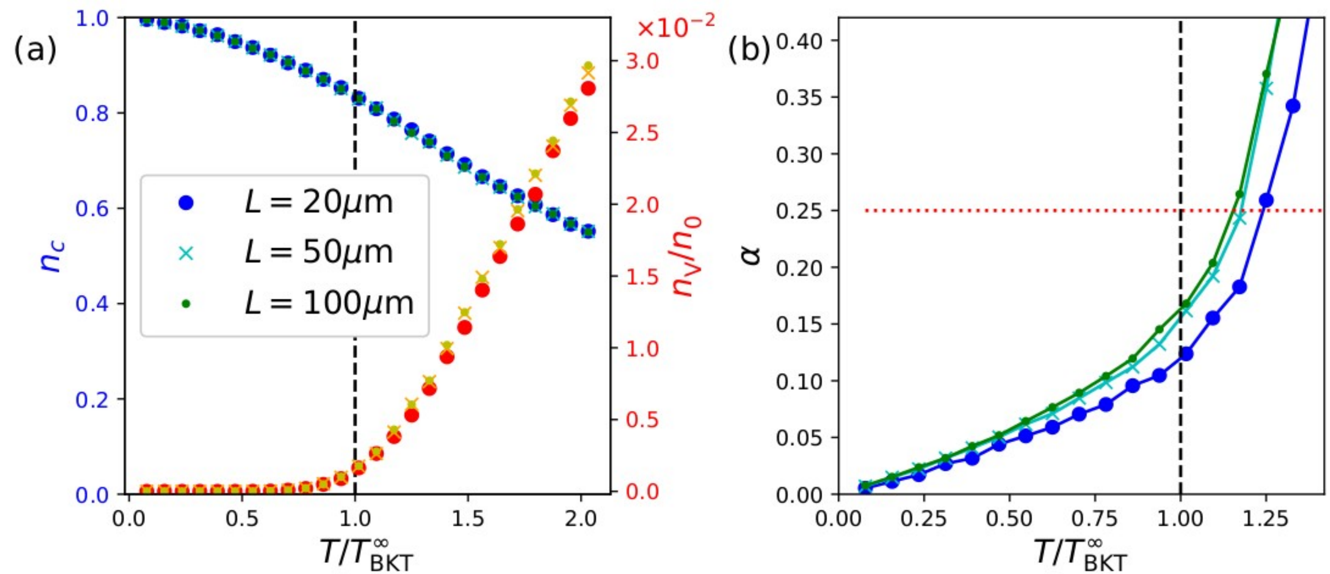
$$T_{\rm BKT} \simeq \frac{2\pi n_0}{m \log(380/m g_{\rm BB})}$$





Locality vs non-locality





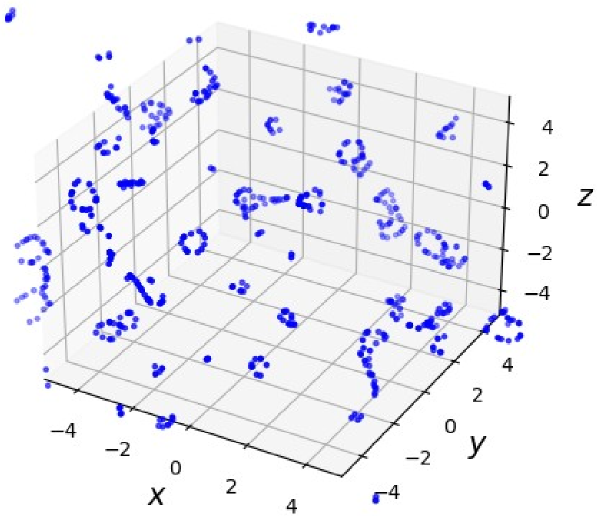
The polaron spectrum is sensitive only to local properties.

Superfluidity, or quasi-long-range order has logarithmic finite size corrections

Polaron spectroscopy can detect BKT transition only INDIRECTLY.

$$\langle\langle \psi^*(\mathbf{x})\psi(\mathbf{x}')\rangle\rangle \propto |\mathbf{x}-\mathbf{x}'|^{-\alpha}$$

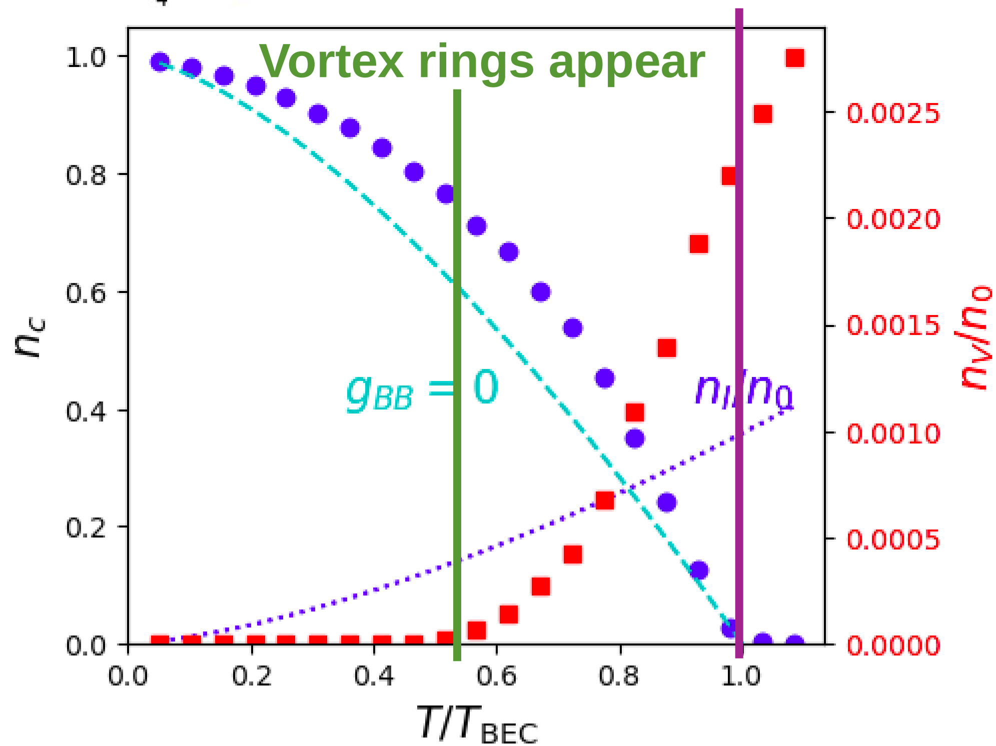
And in 3D?

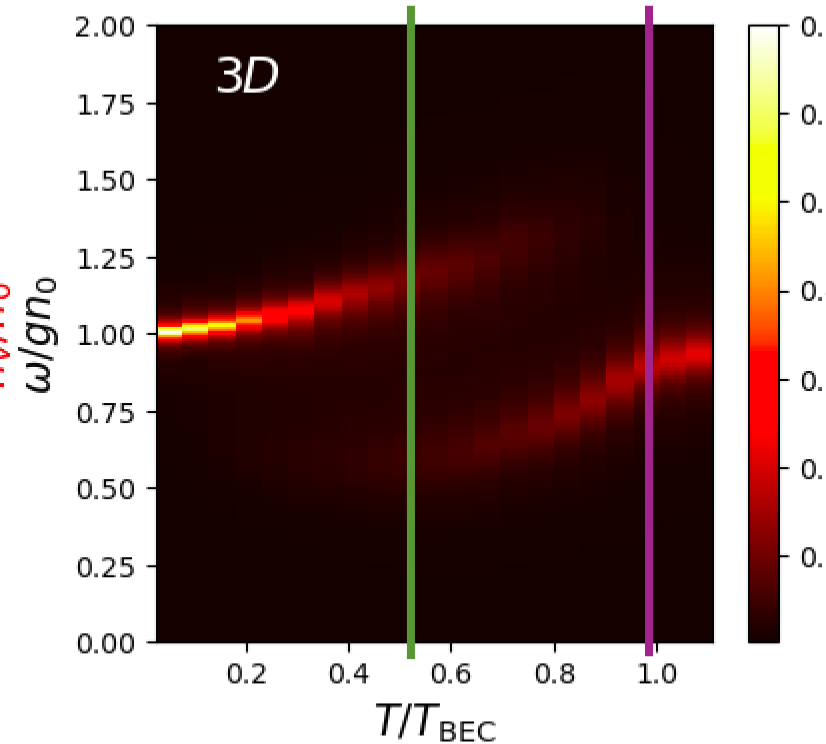


$$n_c(g_{\rm BB}=0) = 1-(T/T_{\rm BEC})^{3/2}$$

$$T_{\rm BEC} \simeq 3.3125\ n^{2/3}/m$$

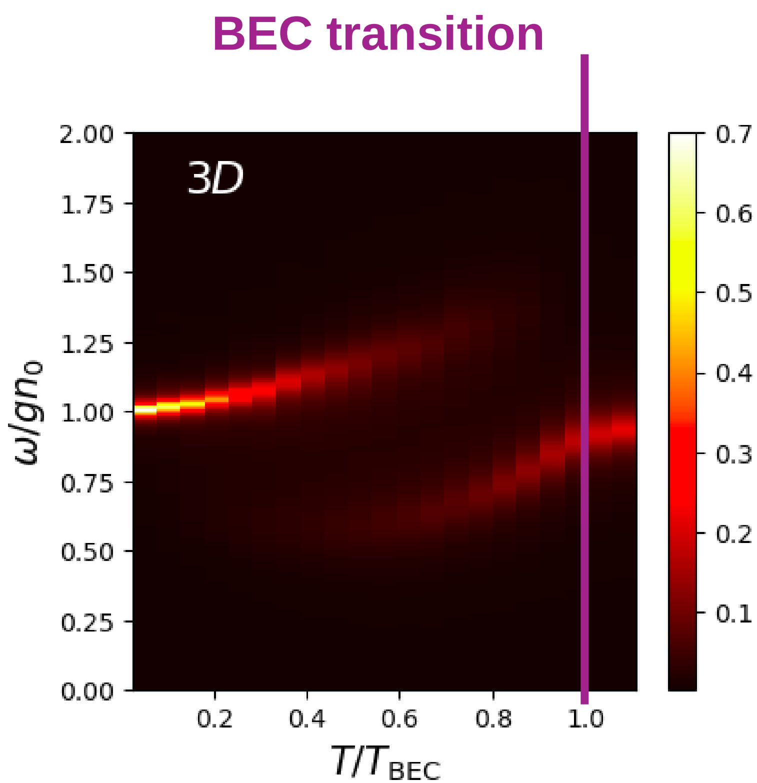
BEC transition

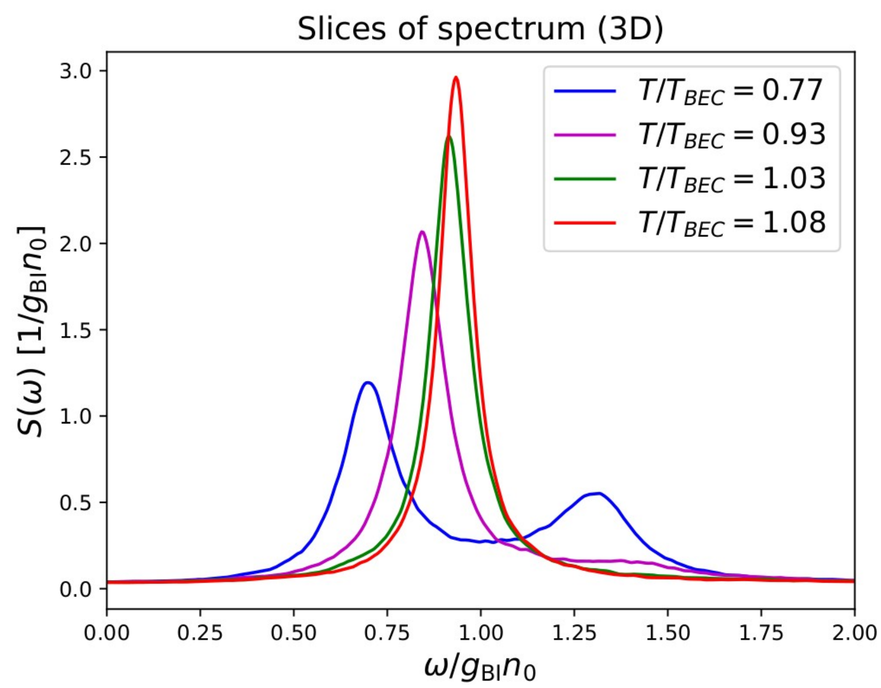




And in 3D?

Bimodal structure rapidly lost above T_{BEC}





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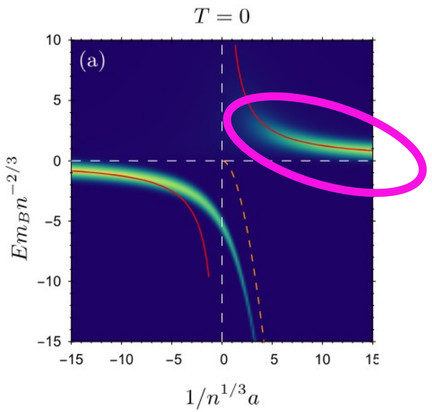
Directions

Properly including Goldstone & phonons in treatment of superfluid & CDWs

Propose new ways of experimentally tuning trion E_B in TMDs

Finite-T Bose polaron: splitting in repulsive branch?

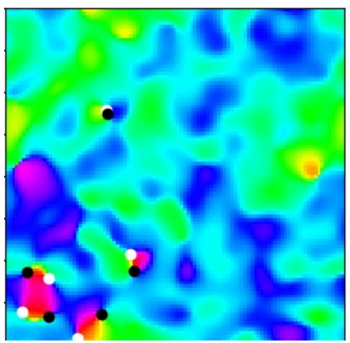


Polaron-polaron interactions
[Baroni et al., Nature Physics 20 (1), 68-73 (2024)]

…….

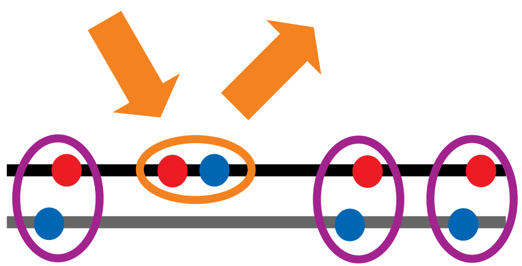
Summary

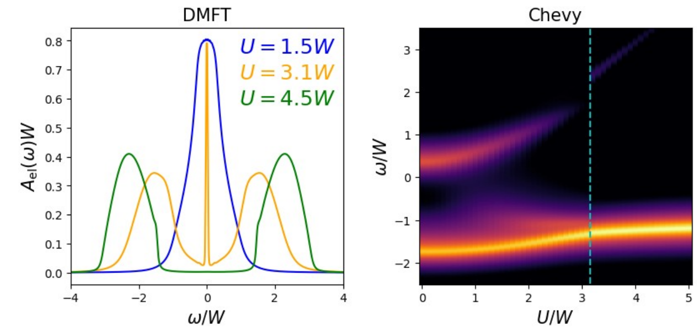
Replace Fermi sea with interesting many-body states!
“Properly done” Chevy generally gives good results,
in agreement with Exact Diagonalization & Matrix Product States



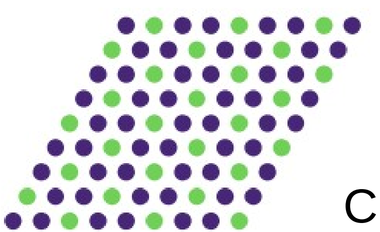
BKT transition

EXI & fermionic superfluids



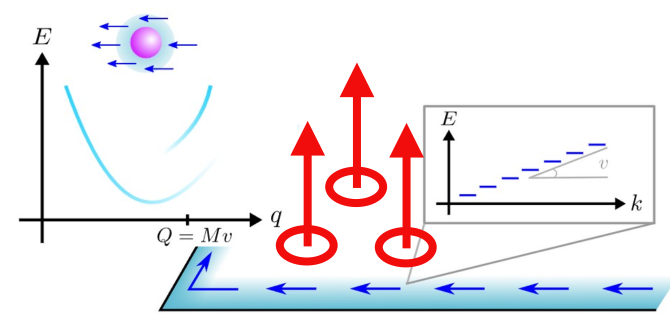


Mott transition



Charge density waves

Thank you for the attention!



Topological insulators

Supplementary Slides

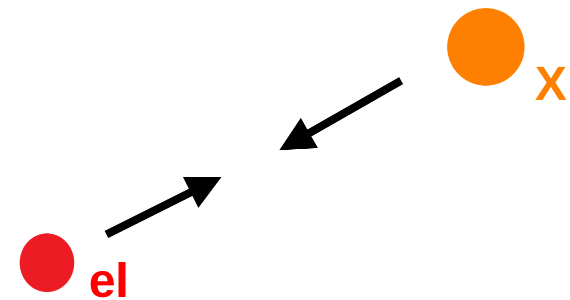
2-body T-matrix

Let's study 2-body scattering in vacuum

(sloppy notation, Matsubara frequencies needed to do it properly...)

$$T(q) \equiv g + g \sum_k G_X^0(q-k) G_{el}^0(k) T(q)$$

$$T(\mathbf{q}, \omega) = \left[\frac{1}{g} - \sum_{\mathbf{k}} \frac{1}{\omega - \frac{\mathbf{k}^2}{2m} - \epsilon_{\mathbf{q}-\mathbf{k}}} \right]^{-1}$$



Diagrammatic sketch (ladder resummation):

Legend: G_X^0 G_{el}^0 g

$$G_X^0(\mathbf{q}, \omega) = \frac{1}{\omega - \epsilon_{\mathbf{q}}^X}$$

$$G_{el}^0(\mathbf{k}, \omega) = \frac{1}{\omega - \xi_{\mathbf{k}}^X}$$

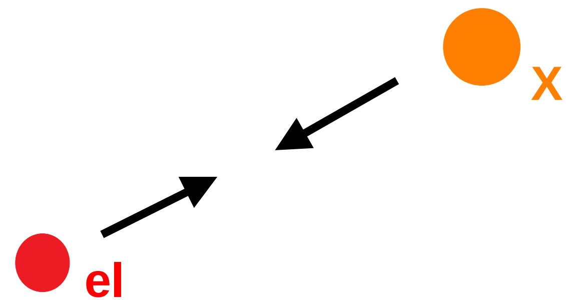
2-body T-matrix

Let's study 2-body scattering in vacuum

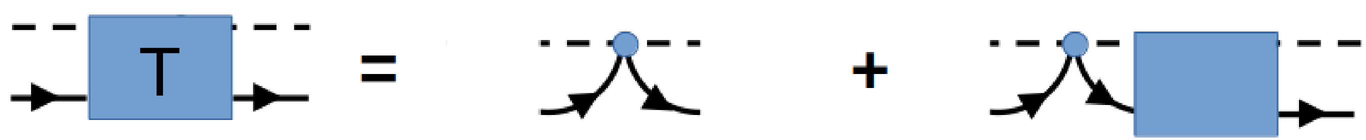
(sloppy notation, Matsubara frequencies needed to do it properly...)

$$T(q) \equiv g + g \sum_k G^0_X(q-k) G^0_{el}(k) T(q)$$

$$T(\mathbf{q},\omega) = \left[\frac{1}{g} - \sum_{\mathbf{k}} \frac{1}{\omega - \frac{\mathbf{k}^2}{2m} - \epsilon_{\mathbf{q}-\mathbf{k}}} \right]^{-1}$$



Diagrammatic sketch (ladder resummation):



Legend:

$$G^0_X(\mathbf{q},\omega) = \frac{1}{\omega - \epsilon^X_{\mathbf{q}}}$$

$$G^0_{el}(\mathbf{k},\omega) = \frac{1}{\omega - \xi^X_{\mathbf{k}}}$$

2-body T-matrix

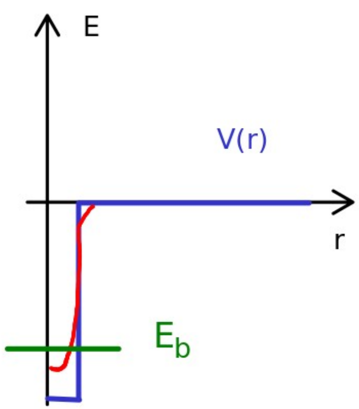
$$T(\mathbf{q},\omega) = \left[\frac{1}{g} - \sum_{\mathbf{k}} \frac{1}{\omega - \frac{\mathbf{k}^2}{2m} - \epsilon_{\mathbf{q}-\mathbf{k}}}\right]^{-1}$$

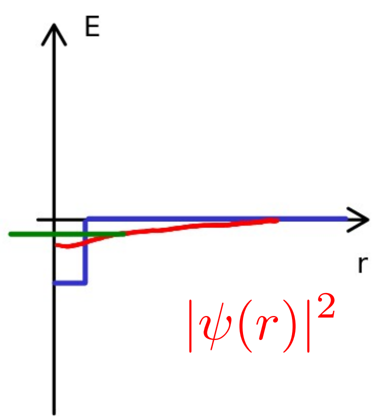
At this level this is just solution of Schrödinger equation in momentum space with contact interactions!

A bound state may exist with energy $E_B(g, E_{\mathrm{cutoff}})$ (see later for precise relation!)

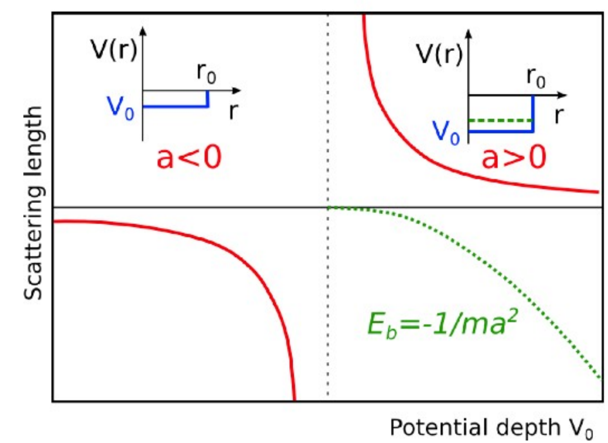
This shows up as pole of T-matrix

2-body hint for existence of repulsive & attractive branches:





P.e. in 3D:



T-matrix approach to polarons

Impurity in a Fermi bath:
$$H = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + \sum_{\mathbf{q}} \epsilon^X_{\mathbf{q}} x^\dagger_{\mathbf{q}} x_{\mathbf{q}} + g \sum_{\mathbf{kpq}} x^\dagger_{\mathbf{q-k+p}} x_{\mathbf{q}} c^\dagger_{\mathbf{k}} c_{\mathbf{p}}$$

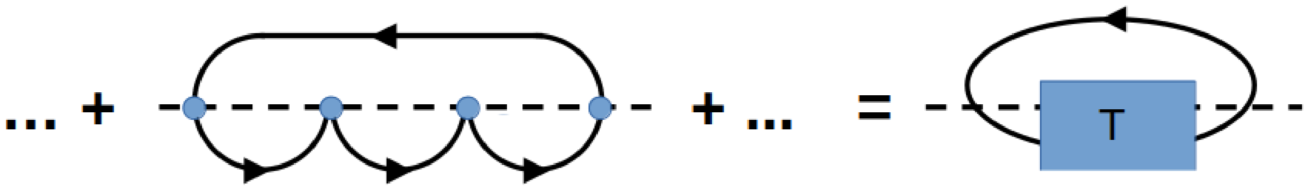
Impurity Green's function:
$$G_X(\mathbf{q},\omega) = \langle FS| x_{\mathbf{q}} \frac{1}{\omega - H + i0^+} x^\dagger_{\mathbf{q}} |FS\rangle = \frac{1}{\omega - \epsilon^X_{\mathbf{q}} - \Sigma_X(\mathbf{q},\omega)}$$

self-energy

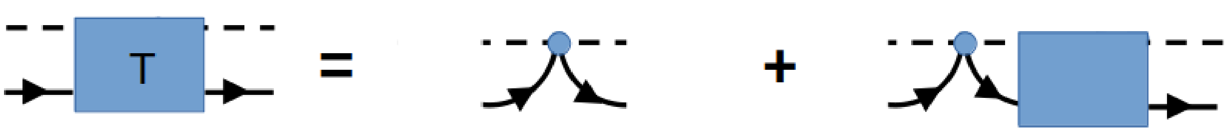
$$\Sigma_X(\mathbf{q},\omega) \simeq \sum_{|\mathbf{p}|<k_F} T(\mathbf{q}+\mathbf{p}, \omega + \xi_{\mathbf{p}})$$

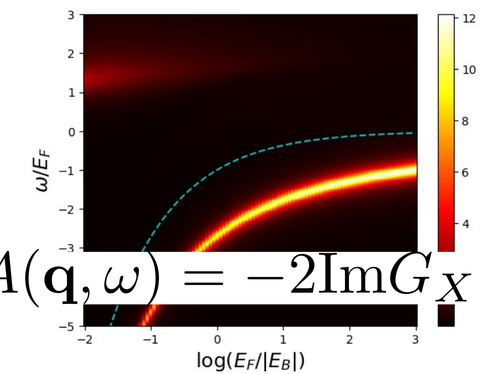
$$\Lambda(\mathbf{q},\omega) = \left[\frac{1}{g} - \sum_{|\mathbf{k}|>k_F} \frac{1}{\omega - \frac{\mathbf{k}^2}{2m} - \epsilon_{\mathbf{q-k}}} \right]^{-1}$$

2-body T-matrix on top of Fermi sea (Pauli blocking)



Ladder resummation:





$$A(\mathbf{q},\omega) = -2\mathrm{Im}G_X(\mathbf{q},\omega)$$

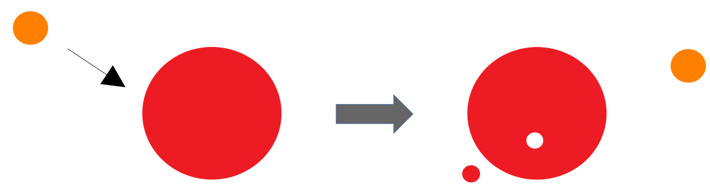
Chevy ansatz approach

$$H = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + \sum_{\mathbf{q}} \epsilon^X_{\mathbf{q}} x^\dagger_{\mathbf{q}} x_{\mathbf{q}} + g \sum_{\mathbf{kpq}} x^\dagger_{\mathbf{q}-\mathbf{k}+\mathbf{p}} x_{\mathbf{q}} c^\dagger_{\mathbf{k}} c_{\mathbf{p}}$$

Schrödinger equation in Chevy subspace

$$|\Psi_{\mathbf{Q}}\rangle = \left\{ \psi_0 x^\dagger_{\mathbf{Q}} + \sum_{\mathbf{kp}} \psi_{\mathbf{kp}} x^\dagger_{\mathbf{Q}-\mathbf{k}+\mathbf{p}} c^\dagger_{\mathbf{k}} c_{\mathbf{p}} \right\} |FS\rangle$$

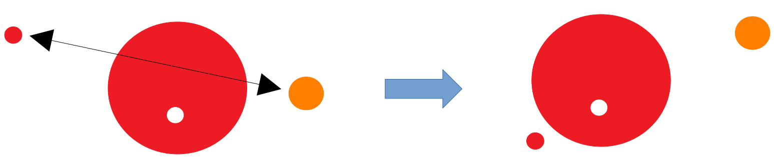
$$i\partial_t \psi_0 = (\epsilon_{\mathbf{Q}} + gn)\psi_0 + \boxed{g \sum_{\mathbf{kp}} \psi_{\mathbf{kp}}}$$



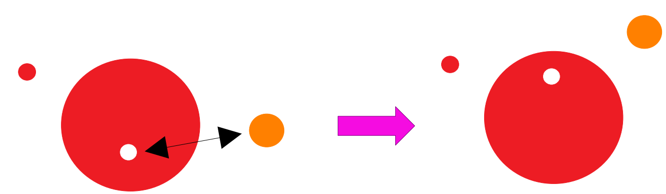
p-h creation

$$i\partial_t \psi_{\mathbf{kp}} = (\epsilon_{\mathbf{Q}-\mathbf{k}+\mathbf{p}} + \xi_{\mathbf{k}} - \xi_{\mathbf{p}} + gn)\psi_{\mathbf{kp}} + g\psi_0 + \boxed{g \sum_{|\mathbf{k}'|>k_F} \psi_{\mathbf{k}'\mathbf{p}}} \boxed{- g \sum_{|\mathbf{p}'|<k_F} \psi_{\mathbf{kp}'}}$$

hole-X scattering

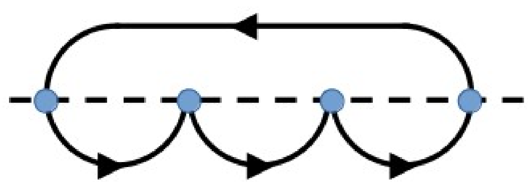


p-X scattering

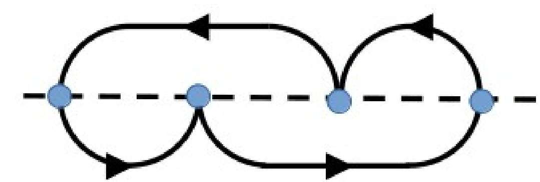


Chevy ansatz vs T-matrix

Typical ladder diagram



Typical Chevy diagram



So, in principle Chevy accounts for more diagrams than T-matrix…
… however, hole-X scattering term is typically negligible due to finite phase space for hole scattering ~ k_F
Moreover, for contact interactions and in the continuous 2D or 3D space *g* has to be renormalized and goes to 0 for infinite cutoff & fixed E_b

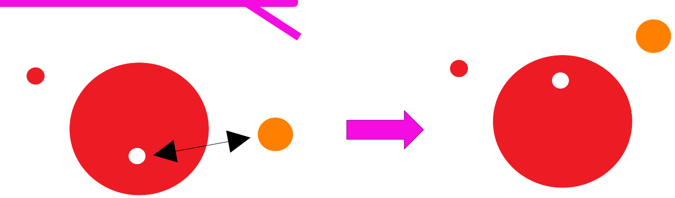
p.e in 2D

$$\frac{1}{g} = -\frac{m_{\text{red}}}{2\pi}\log\left(\frac{E_{\text{cutoff}}}{|E_b|}\right)$$

$$i\partial_t\psi_{\mathbf{kp}} = (\epsilon_{\mathbf{Q}-\mathbf{k}+\mathbf{p}} + \xi_{\mathbf{k}} - \xi_{\mathbf{p}} + gn)\psi_{\mathbf{kp}} + g\psi_0 + g\sum_{|\mathbf{k}'|>k_F}\psi_{\mathbf{k}'\mathbf{p}} - g\sum_{|\mathbf{p}'|<k_F}\psi_{\mathbf{kp}'}$$

hole-X scattering

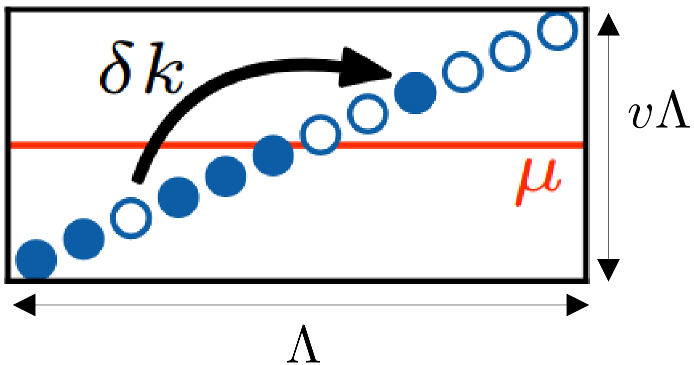
In this case: T-matrix = Chevy !!!



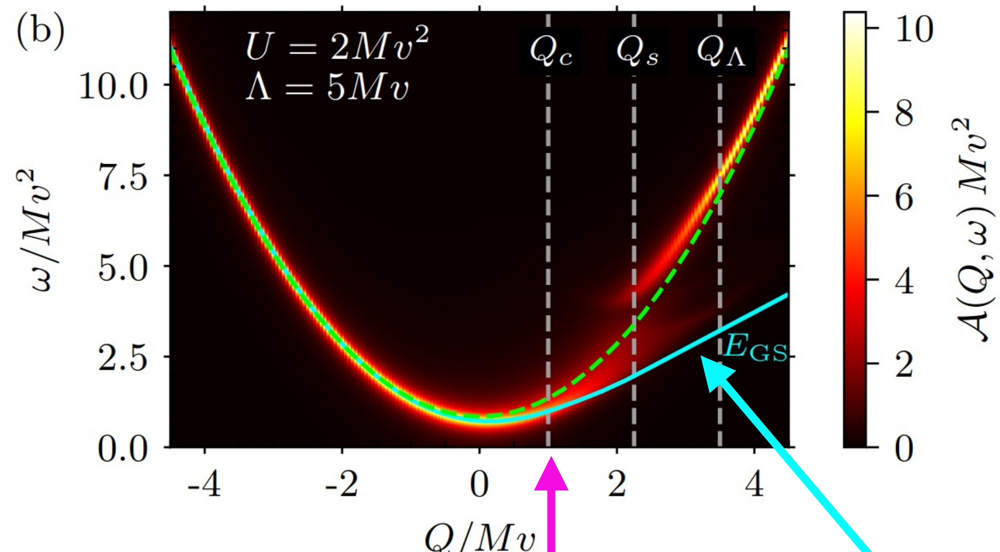
Combescot et al., PRL **98**, 180402 (2007)

Chiral polarons: 1D model

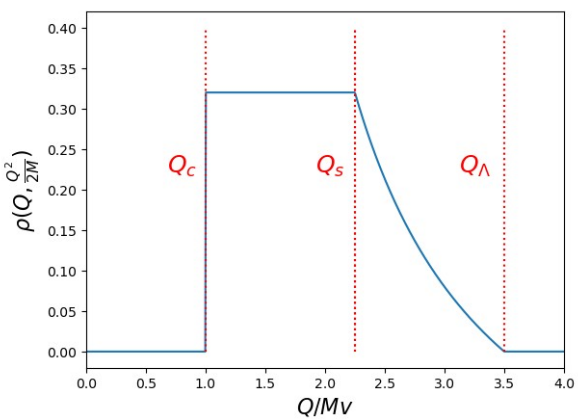
Idea: impurity dressed by excitations of chiral edge mode



Chirality ➡ **asymmetry in momentum resolved polaron spectrum**



DOS of bath
1ph excitations



GS dark for Q>Mv

Q_c = Mv special: resonant Impurity-bath scattering

arXiv:2407.19093

Impurity across BKT

Setting: impurity in a finite T Bose bath will bind to vortices, for repulsive Bose-impurity interactions $g_{BI}>0$

Method: Stochastic Projected Gross-Pitaevskii for the bath

$$i\partial_t \psi = \mathcal{P}(1-i\gamma)\left\{-\frac{1}{2m}\nabla^2 + g_{BB}|\psi|^2 - \mu\right\}\psi + \eta$$

Incoherent density

Projection: $\mathcal{P} = \theta(\varepsilon_{\rm cut} - \frac{\mathbf{k}^2}{2m})$ $\quad \varepsilon_{\rm cut} = \mu + T\log 2$

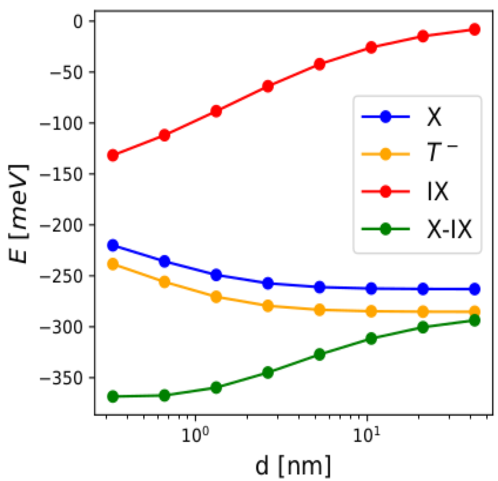
$$n_{\rm I} = \int \frac{d\mathbf{k}}{(2\pi)^d}\frac{\theta(|\mathbf{k}| - k_{\rm cut})}{e^{\mathbf{k}^2/2mT} - 1}$$

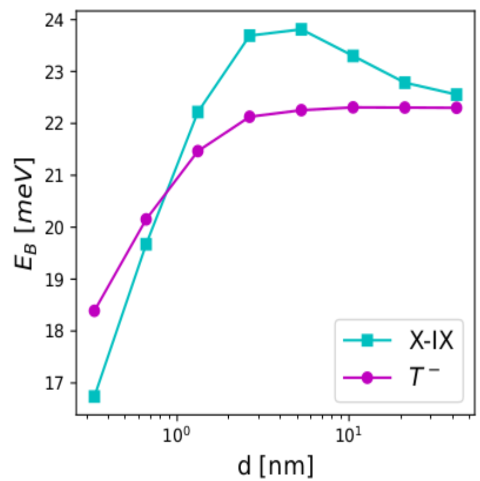
Noise & relaxation satisfying FDT: $\langle \eta^*(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle = 2\gamma T\delta(t-t')\delta(\mathbf{x}-\mathbf{x}')$

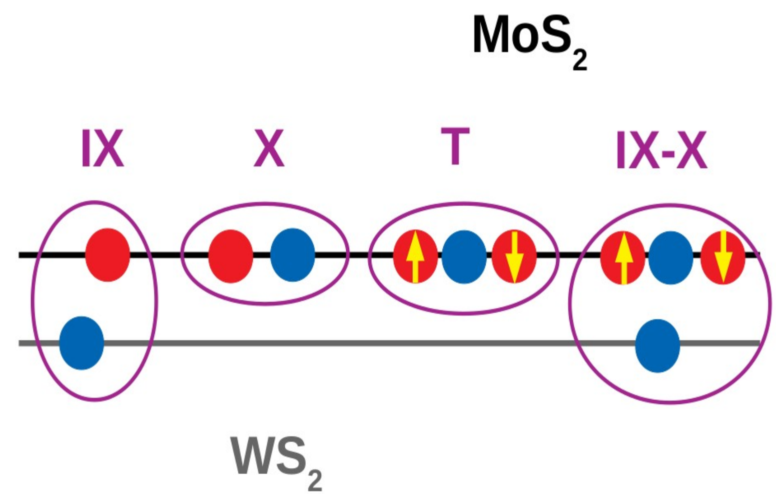
Schrödinger eq. in time dependent potential for the impurity

$$i\partial_t \Psi = \left\{-\frac{1}{2M}\nabla^2 + g_{BI}|\psi|^2\right\}\Psi$$

[Few-body problem in EXI (Diffusion QMC)]







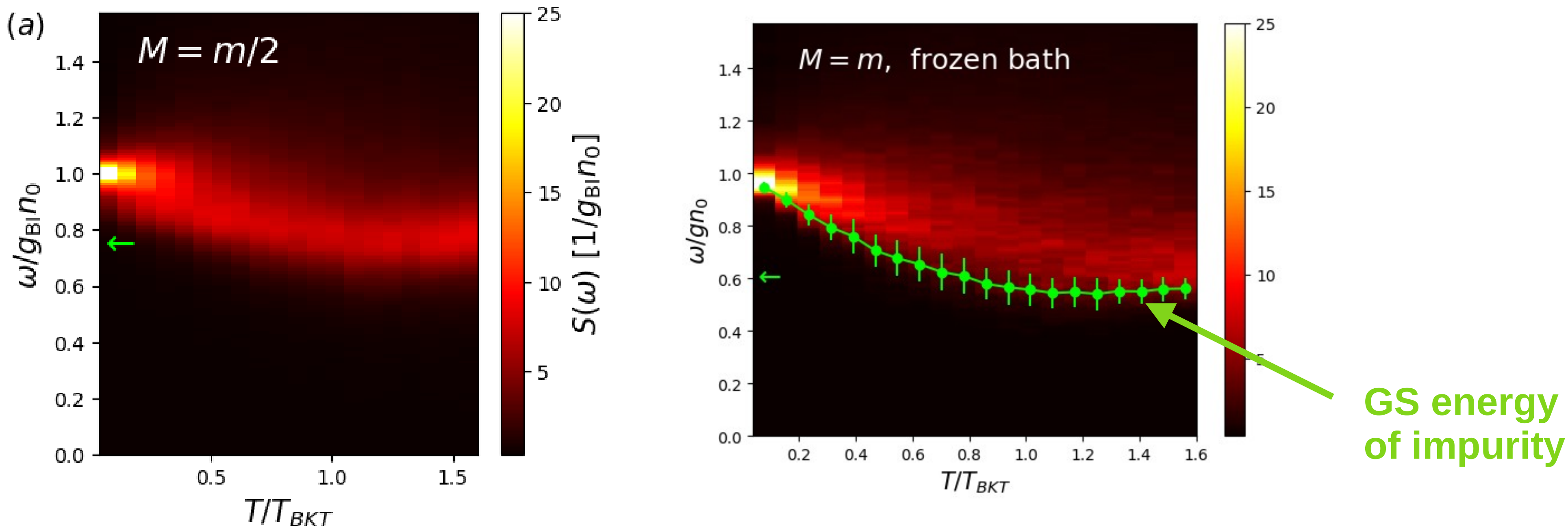
interlayer distance

Goal: to find binding energies.
Cutoff given by excitonic Bohr radius.

IA et al., Phys. Rev. B **107**, 155303 (2023)

Light impurity

Varying the impurity mass!



Heavy impurity

Varying the impurity mass!