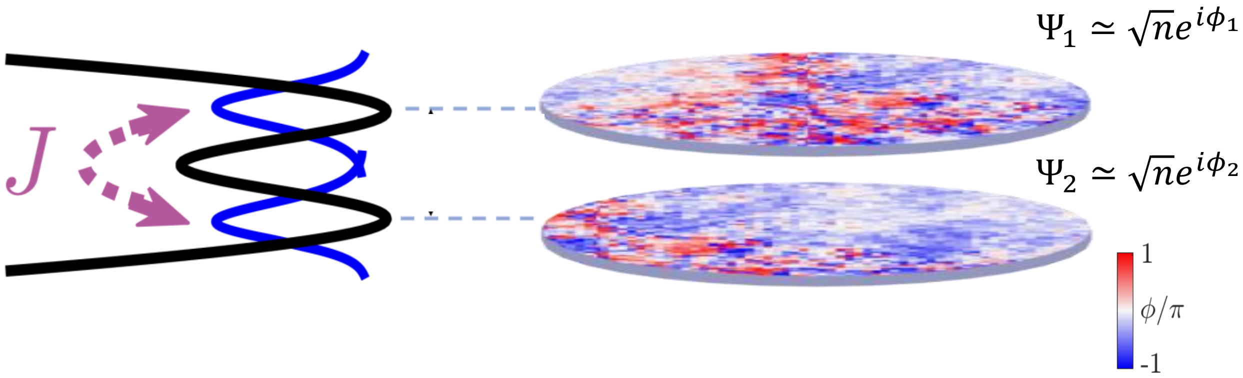
Observation of a Bilayer Superfluid with Interlayer Coherence

14 Nov 2024, Universitat Politècnica de Catalunya

Erik Rydow



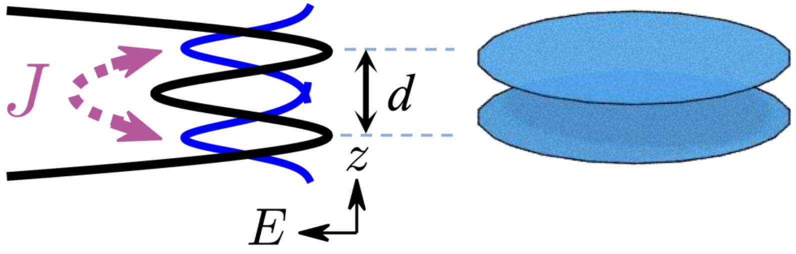
Erik Rydow (PhD student), Shinichi Sunami (Postdoc/Junior Research Fellow),
Abel Beregi (Postdoc), En Chang (PhD student), Chris Foot (PI)

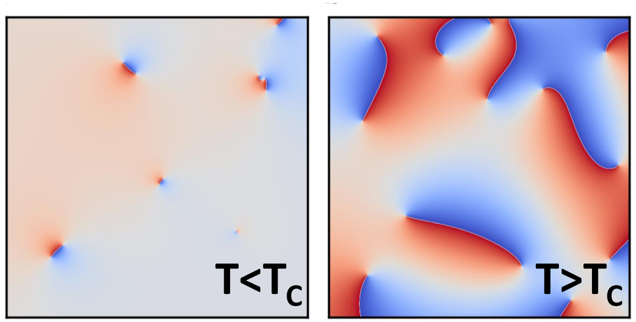
E. Rydow et al., arxiv.org/abs/2410.22326 (2024)
Brief review of our bilayer experiment: A. Beregi, C. Foot and S. Sunami arXiv:2406.04080 (2024)

Oxford Physics
UNIVERSITY OF OXFORD
UKRI
Engineering and Physical Sciences Research Council

Overview

- Part 1: Coupled bilayer 2D quantum gases





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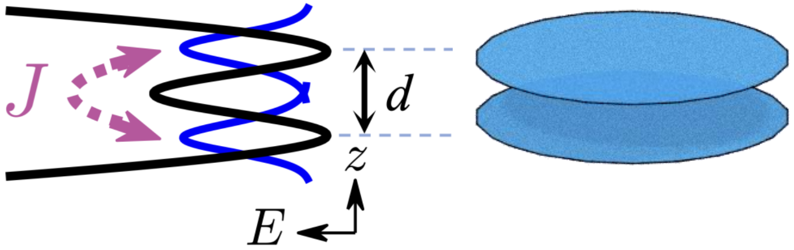
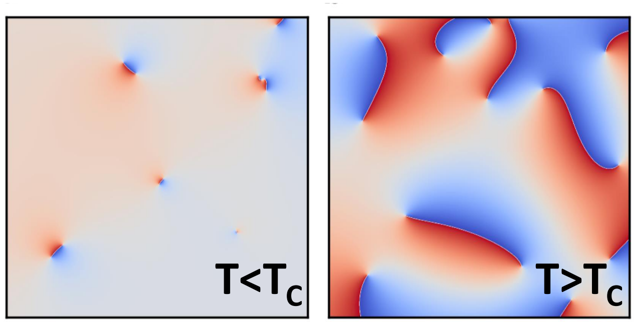
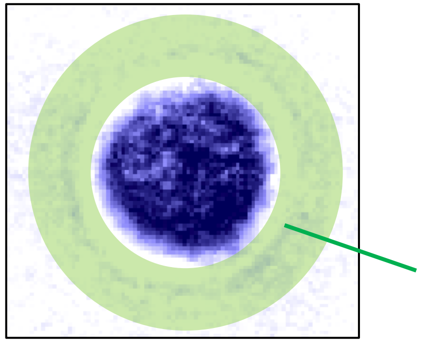
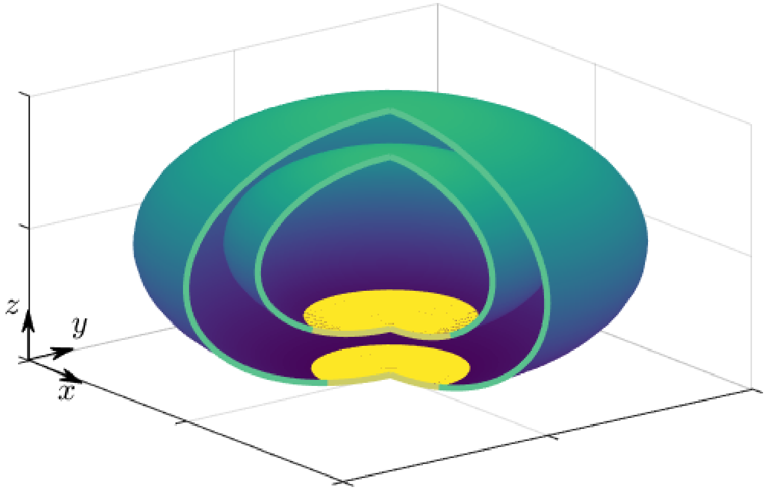




- Part 2: Realising 2D quantum gases with controllable coupling


box potential



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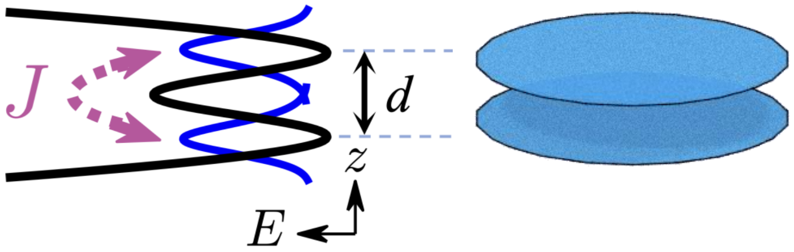

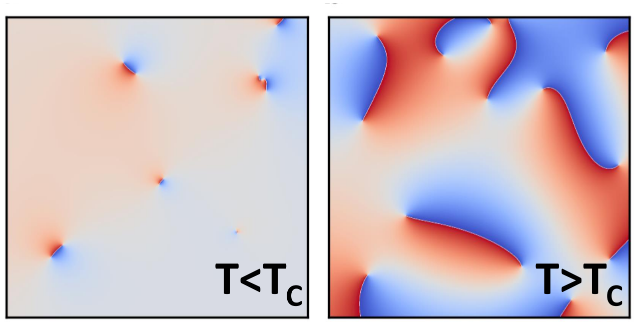

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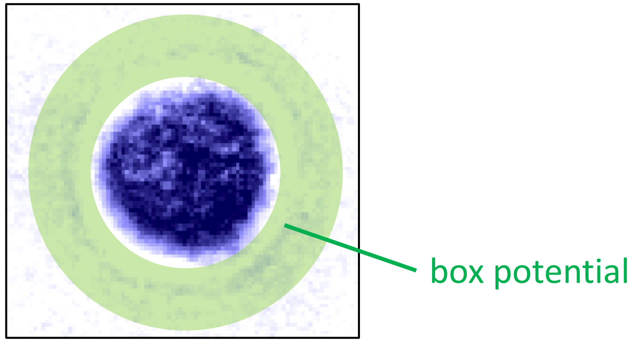
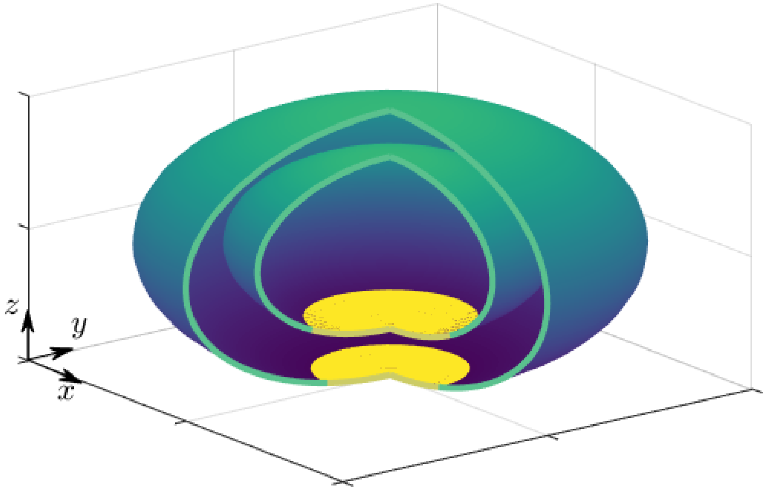
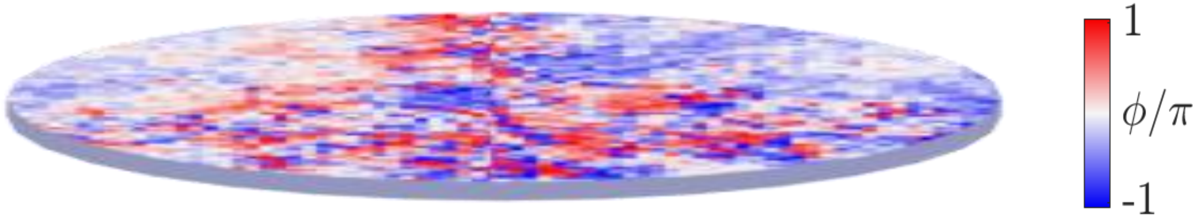




- Part 3: Summary of results

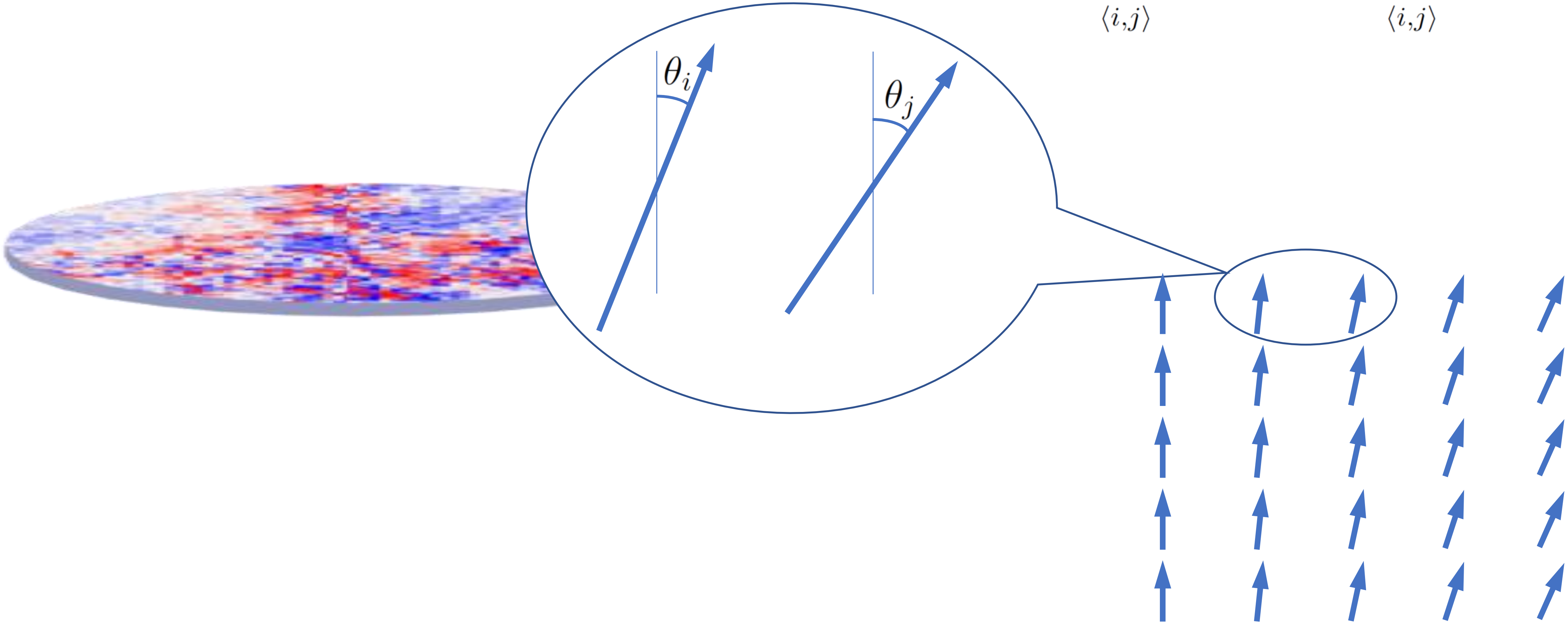
2D Bose gas

$$\psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}e^{i\theta(\mathbf{r},t)}$$

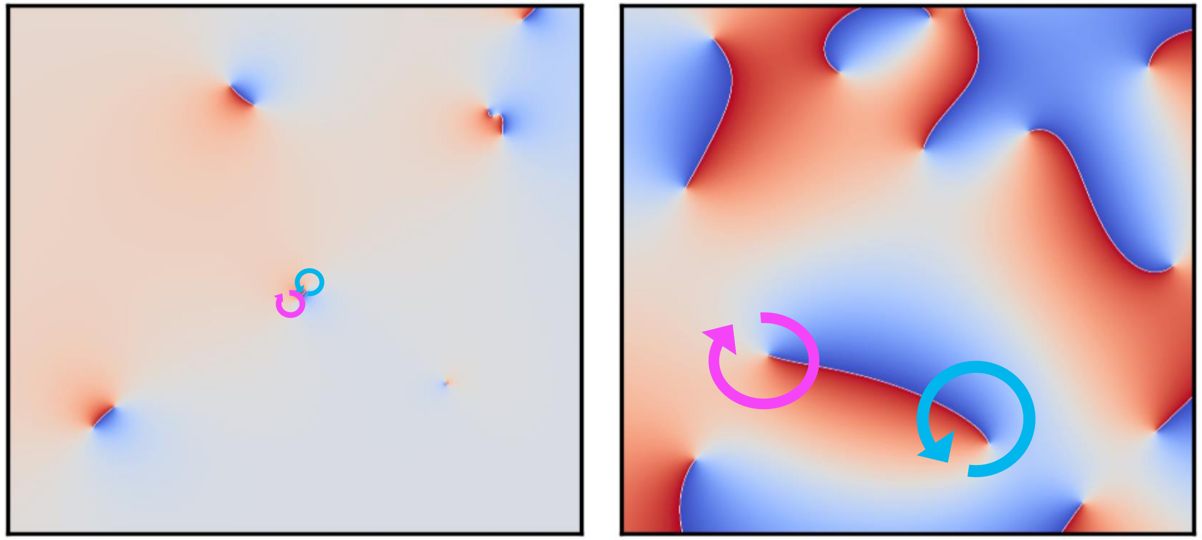


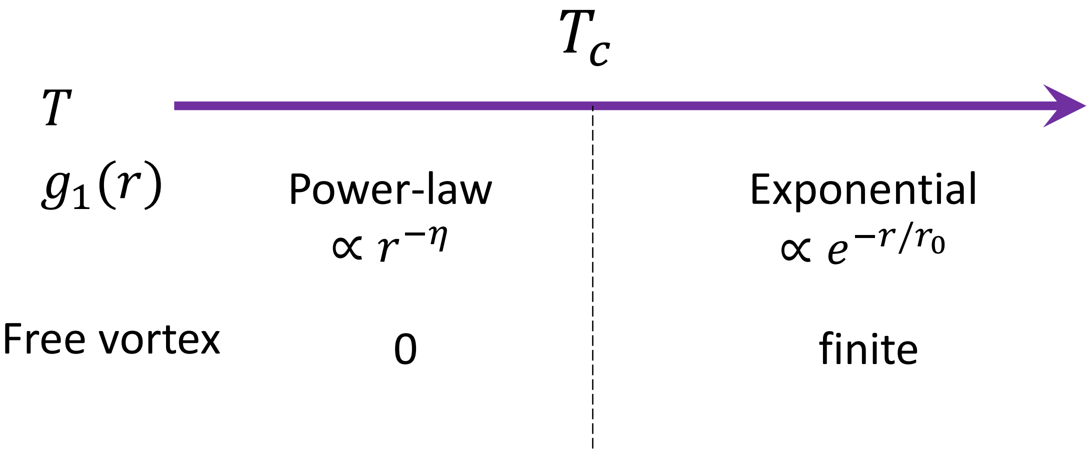
2D Bose gas ⇔ 2D XY model

$$\psi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}e^{i\theta(\mathbf{r},t)} \quad \Longleftrightarrow \quad \mathcal{H} = -J\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j = -J\sum_{\langle i,j\rangle}\cos(\theta_i-\theta_j),$$



Intro 1-1: BKT transition





T_c

T		
$g_1(r)$	Power-law $\propto r^{-\eta}$	Exponential $\propto e^{-r/r_0}$
Free vortex	0	finite

- No spontaneous symmetry breaking in 1D and 2D
  Mermin and Wagner (1966), Hohenberg (1967)

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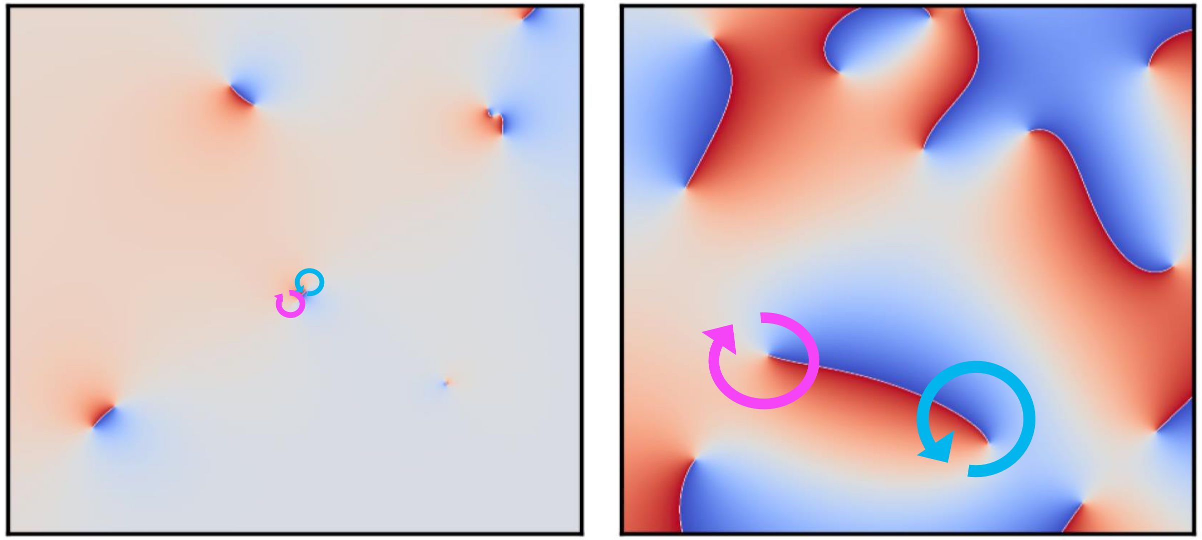

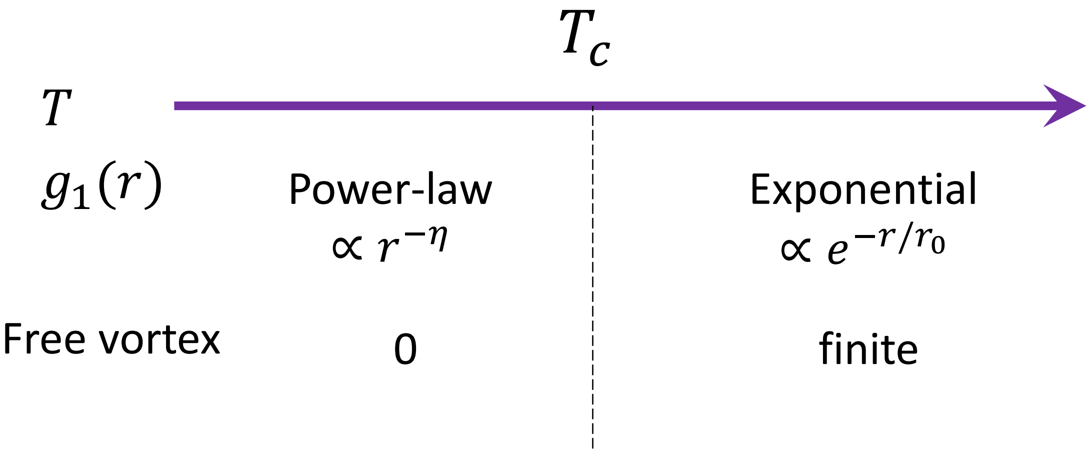

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  Berezinskii (1972), Kosterlitz and Thouless (1972)

  Bishop and Reppy (1978): liquid He film experiment

  - Topological phase transition
  - Quasi-long-range order, power-law decay

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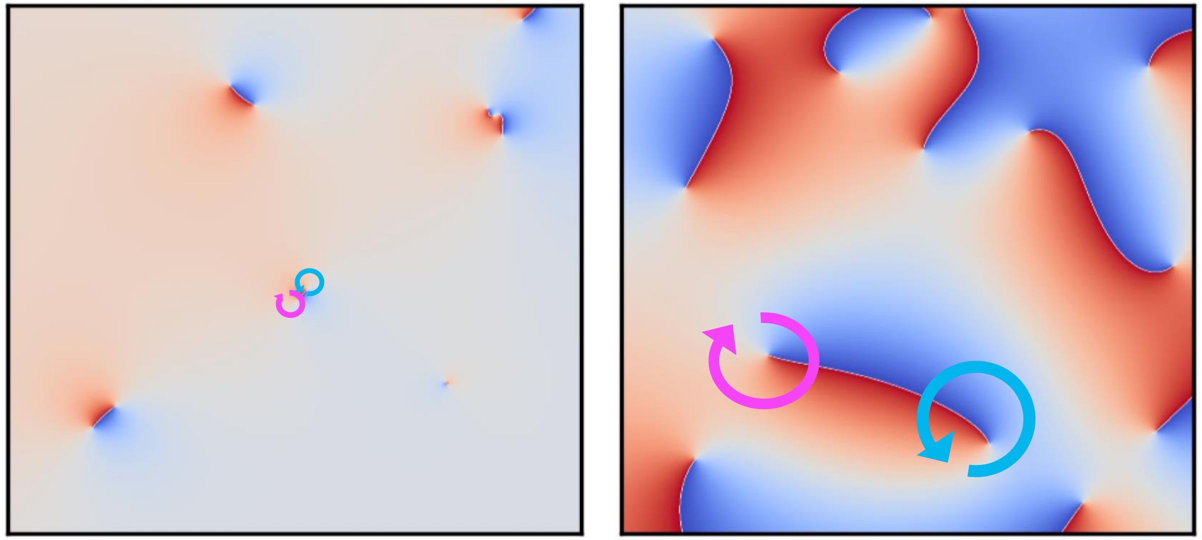

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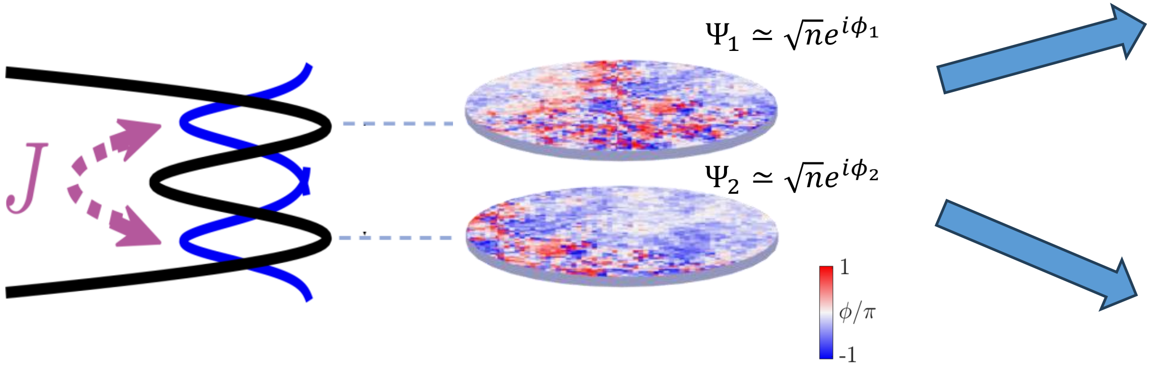
Cold-atom experiments:

- Hadzibabic (2006), Desbuquois (2012), many more

Intro 1-2: bilayer phases

bilayer



Relative phase

$$\phi_1 - \phi_2$$

$$\phi_1 + \phi_2$$

Common phase

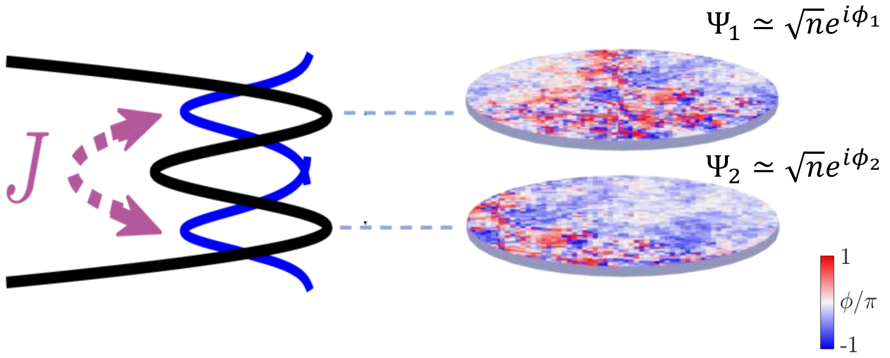
$$H = K\int d^2\boldsymbol{r}(\nabla\phi_1(\boldsymbol{r}))^2 + K\int d^2\boldsymbol{r}(\nabla\phi_2(\boldsymbol{r}))^2$$

Intro 1-2: bilayer phases

bilayer

$\Psi_1 \simeq \sqrt{n}e^{i\phi_1}$

$\Psi_2 \simeq \sqrt{n}e^{i\phi_2}$



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$$\boxed{+J\int d^2\boldsymbol{r}\cos(\phi_1(\boldsymbol{r}) - \phi_2(\boldsymbol{r}))}$$

Intro 1-2: bilayer phases

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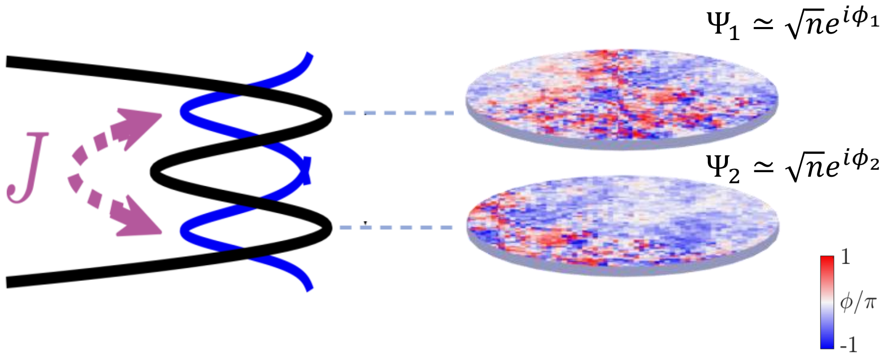

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$$\theta(\boldsymbol{r}) = \phi_1(\boldsymbol{r}) - \phi_2(\boldsymbol{r})$$

$$\varphi(\boldsymbol{r}) = \phi_1(\boldsymbol{r}) + \phi_2(\boldsymbol{r})$$

Intro 1-2: bilayer phases

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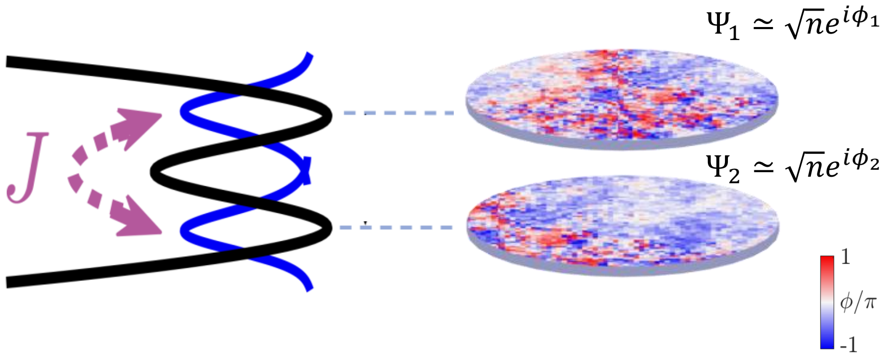

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$$\boxed{+J\int d^2\boldsymbol{r}\cos(\phi_1(\boldsymbol{r}) - \phi_2(\boldsymbol{r}))}$$

$$\xrightarrow[\varphi(\boldsymbol{r}) = \phi_1(\boldsymbol{r}) + \phi_2(\boldsymbol{r})]{\theta(\boldsymbol{r}) = \phi_1(\boldsymbol{r}) - \phi_2(\boldsymbol{r})}$$

$$H = \frac{K}{2}\int d^2\boldsymbol{r}(\nabla\varphi(\boldsymbol{r}))^2$$
$$+\frac{K}{2}\int d^2\boldsymbol{r}(\nabla\theta(\boldsymbol{r}))^2 + \boxed{J\int d^2\boldsymbol{r}\cos(\theta(\boldsymbol{r}))}$$

Experiment: 2D quantum gases

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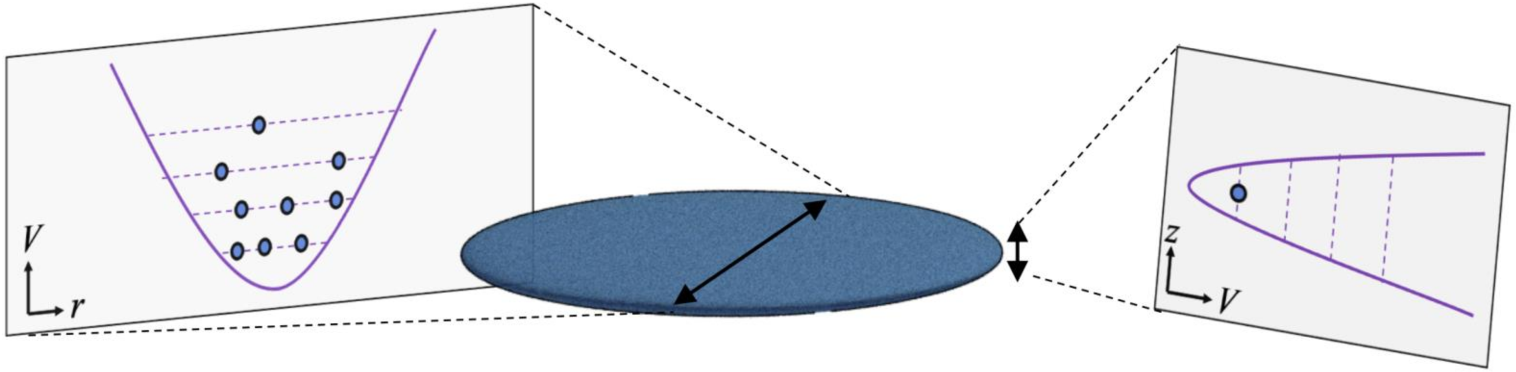
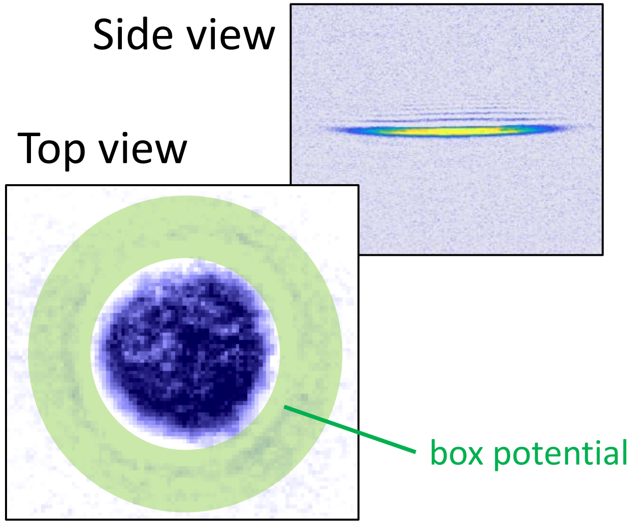
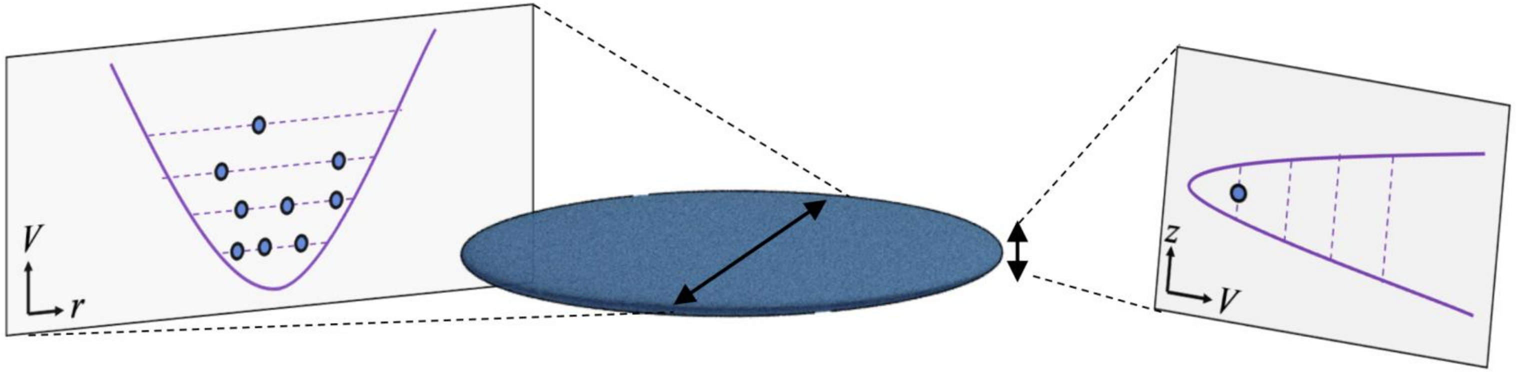


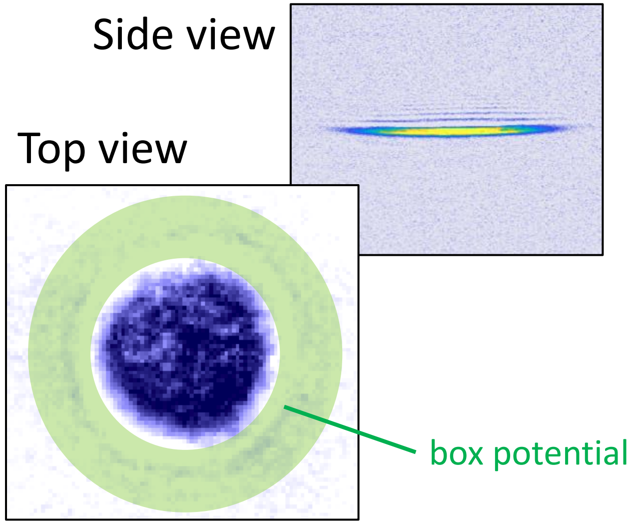


2D condition: $k_B T, \mu < \hbar\omega_z$

$\omega_z/2\pi > 1\text{kHz},\ T < 50\text{ nK},\ \mu < 500\text{Hz}$

Experiment: 2D quantum gases





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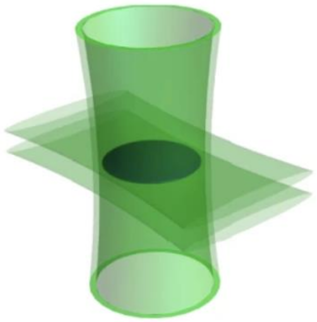
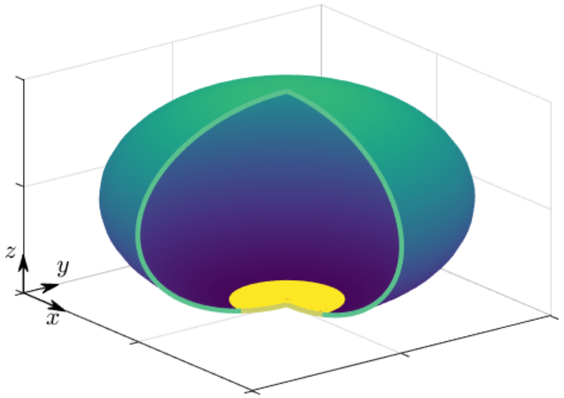
Optical traps



Cambridge, MIT, Chicago,
Paris, Heidelberg, Seoul, …

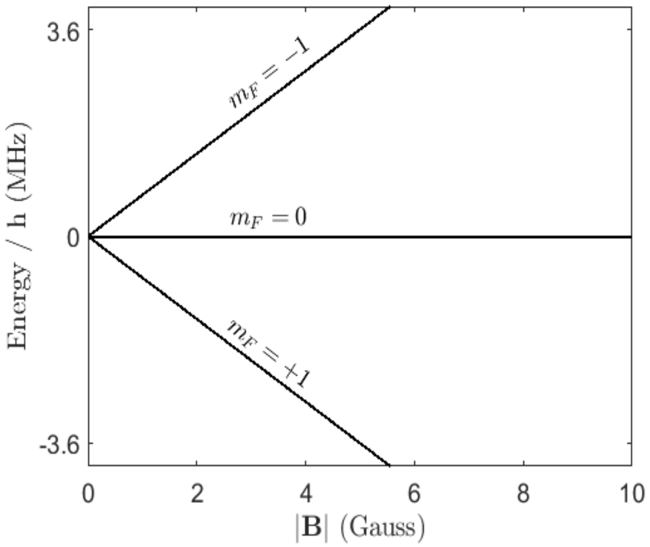
Fig from Nat. Phys. 17, 1334 (2021)

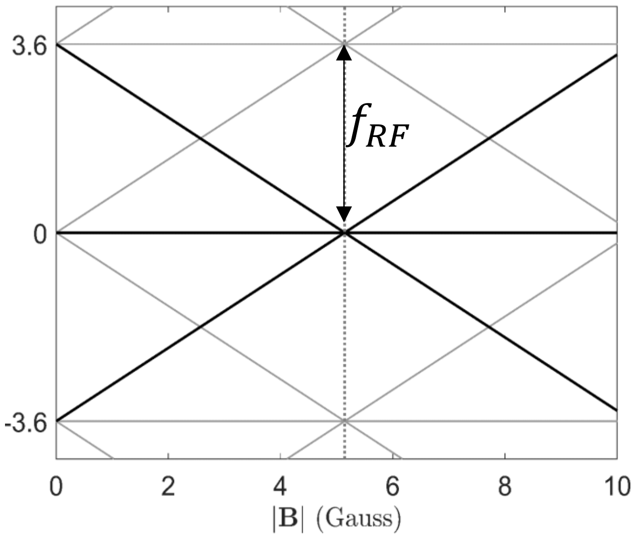
Magnetic traps (RF dressing)

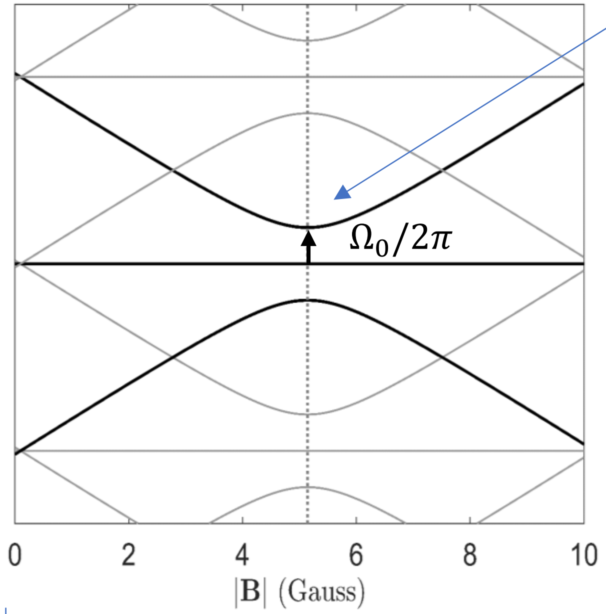


Paris, Oxford, …

RF-dressed trapping







Potential minima

Bare states:

$$|m_F\rangle$$

Zeeman states of F = 1 atom:

$$E = m_F g_F \mu_B |B|$$

Uncoupled states:

$$|\boldsymbol{m_F}\rangle \otimes |\boldsymbol{N}\rangle$$

Zeeman state + RF Fock state

$$E = m_F g_F \mu_B |B| + h f_{RF} N$$

Coupled(dressed) states:

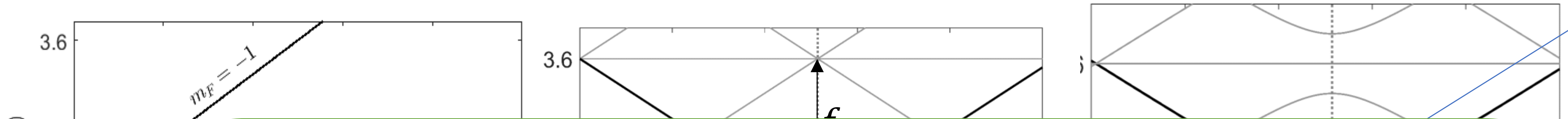
$$|\tilde{\boldsymbol{N}}, \tilde{\boldsymbol{m}}\rangle$$

Dressed eigenstates

$$E = \tilde{m}\hbar\sqrt{\Omega_0^2 + \delta^2} + h f_{RF} \tilde{N}$$

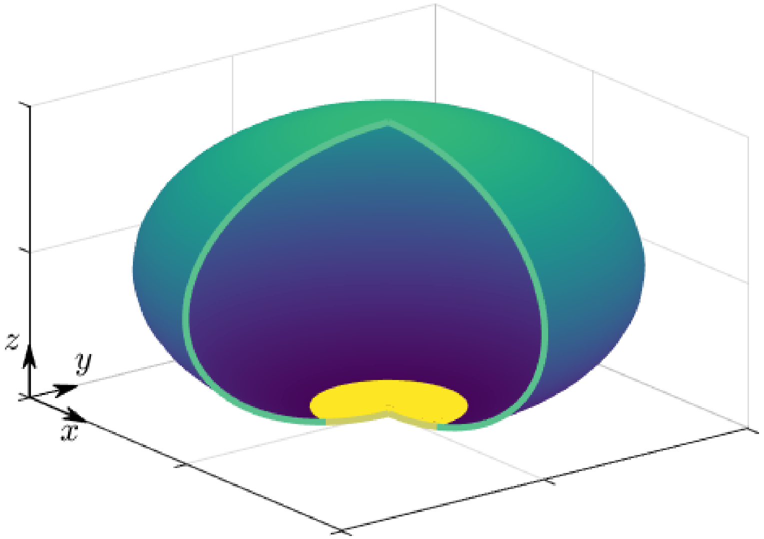
Sunami, Beregi, Foot, AVS Quantum (2024)

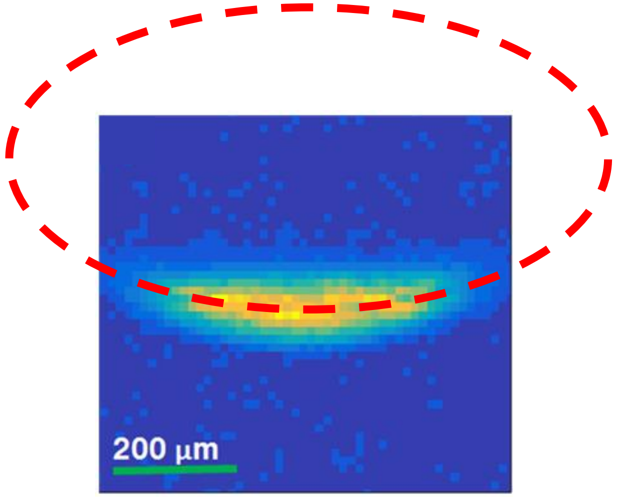
RF-dressed trapping



Potential minima

With quadrupole static field: *shell* potential





Bare s

Zeema

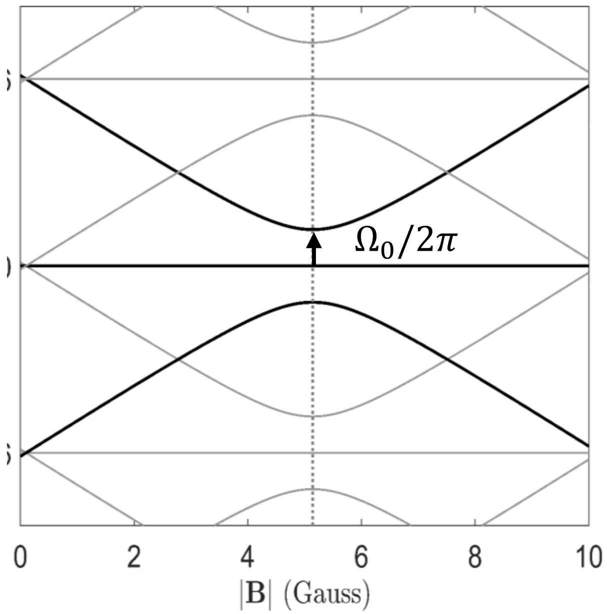
$$E = m_F g_F \mu_B |B|$$

$$E = m_F g_F \mu_B |B| + h f_{RF} N$$

$$E = \tilde{m}\hbar\sqrt{\Omega_0^2 + \delta^2} + h f_{RF} \tilde{N}$$

Sunami, Beregi, Foot, AVS Quantum (2024)

Multiple-RF-dressed trapping



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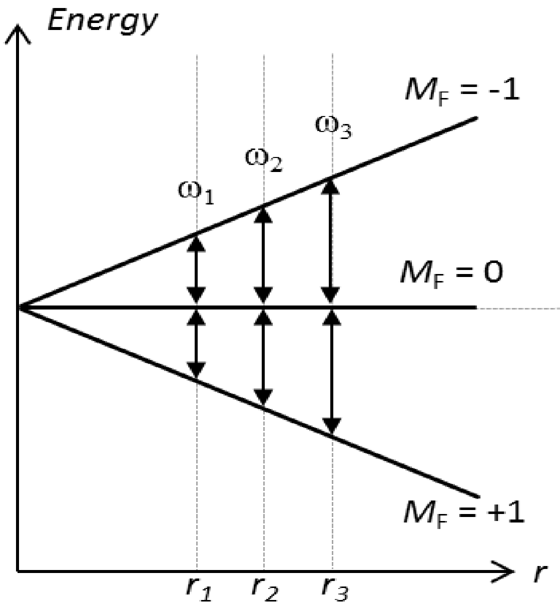
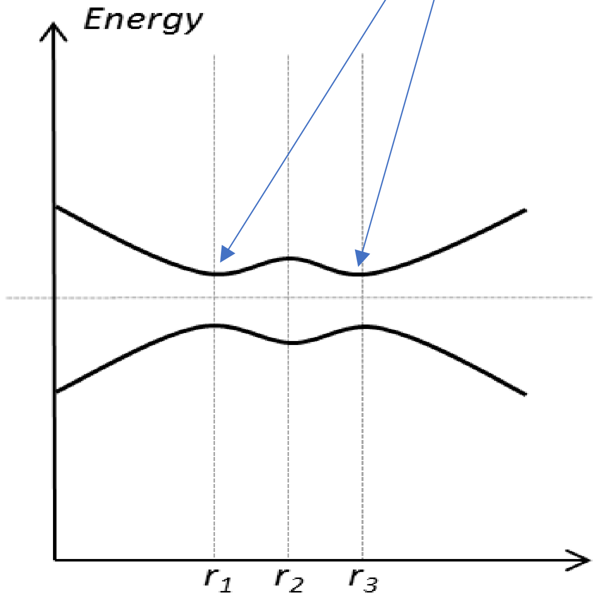
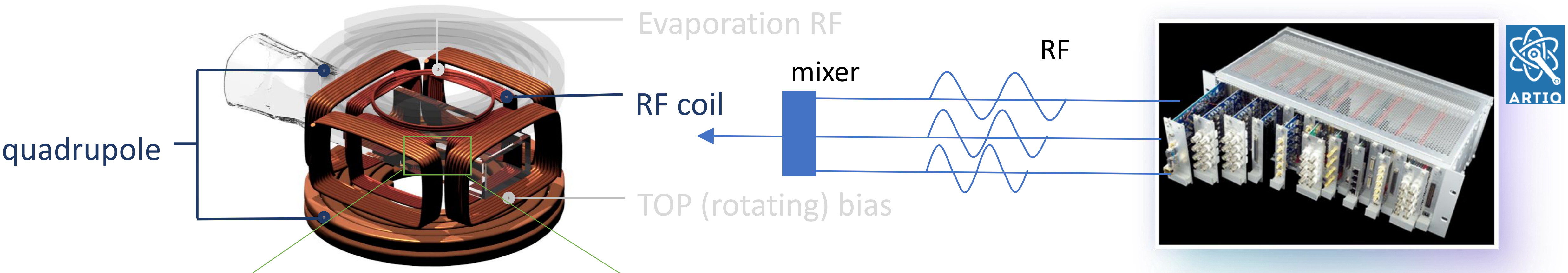


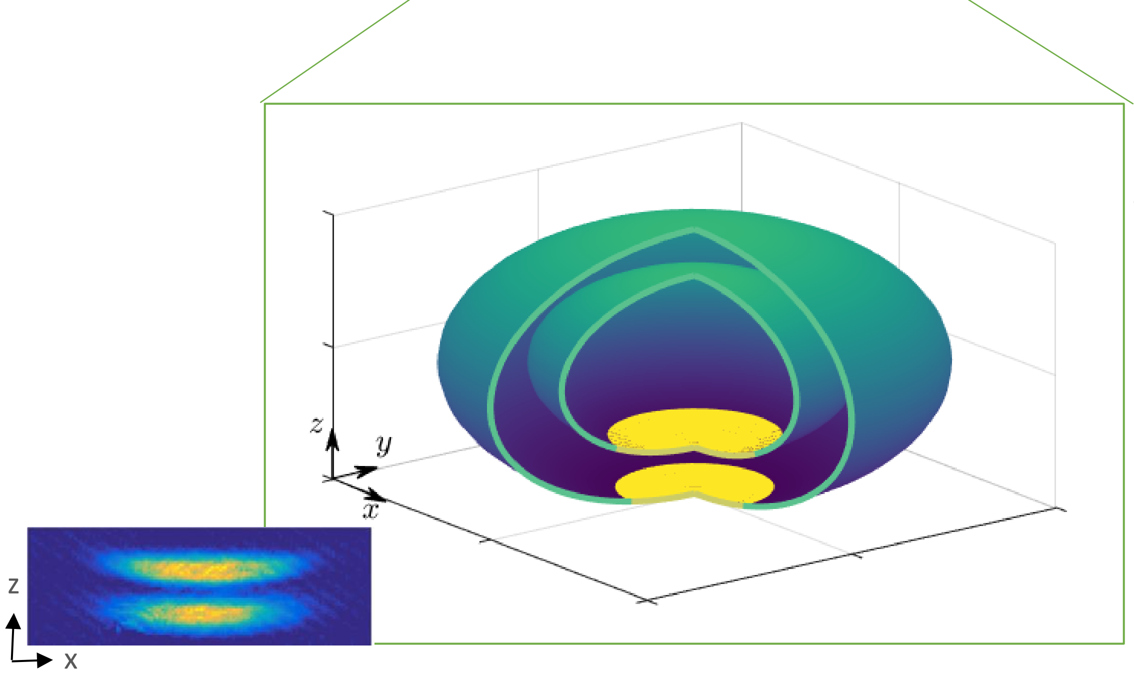


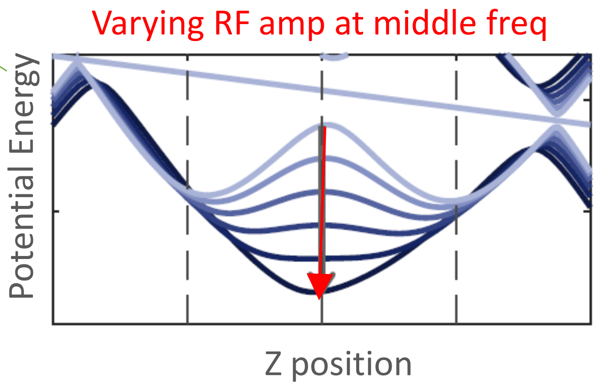
Potential minima

Sunami, Beregi, Foot, AVS Quantum (2024)

Experiment: **bilayer** 2D quantum gases





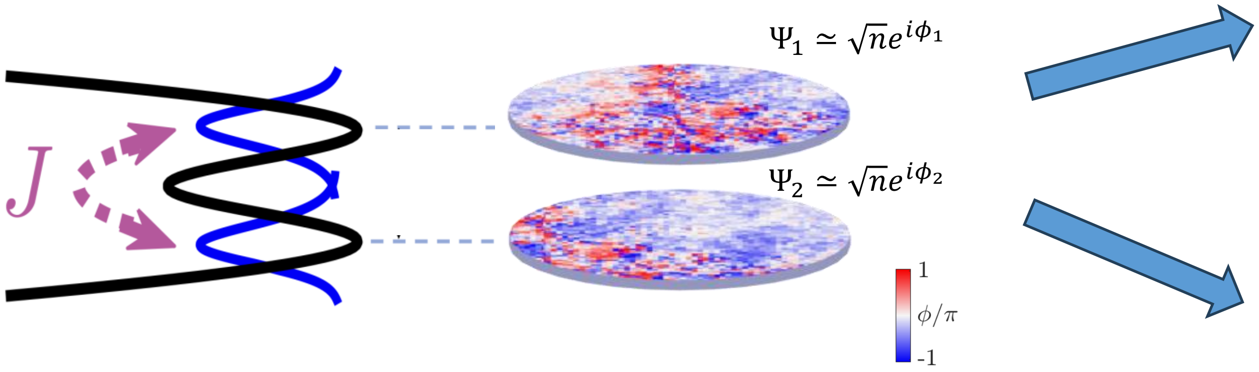

Varying RF amp at middle freq

Controllability:

- RF frequencies
  → **<u>well separation</u>**
- RF amplitudes
  → trap freq, trap shape
- RF polarizations
  → species-selectivity

Harte, Bentine, … , Foot, PRA(2018)

Experiment: **bilayer** 2D quantum gases



bilayer

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$\Psi_2 \simeq \sqrt{n}e^{i\phi_2}$

Relative phase

$\phi_1 - \phi_2$

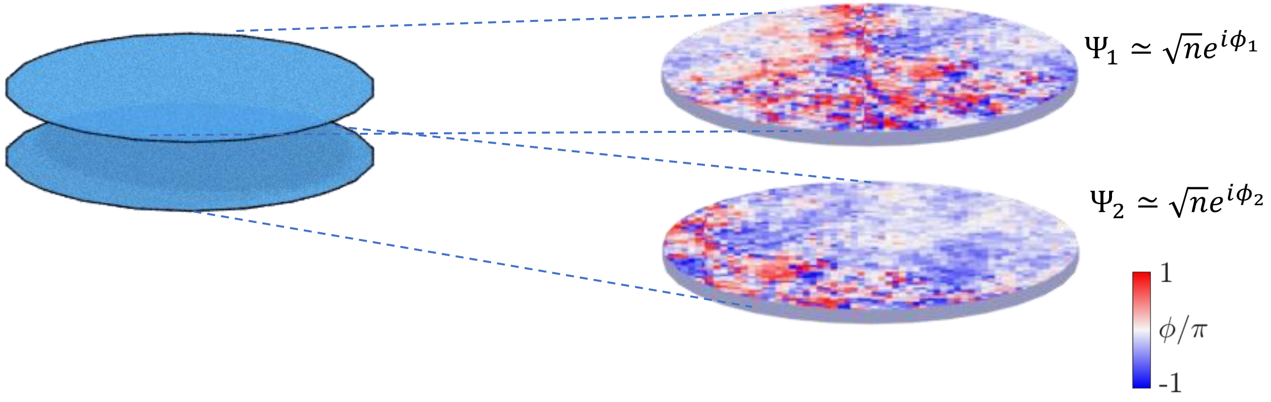
$\phi_1 + \phi_2$

Common phase

Need to probe two **complementary modes**

Relative phase: interferometry

Phase fluctuation



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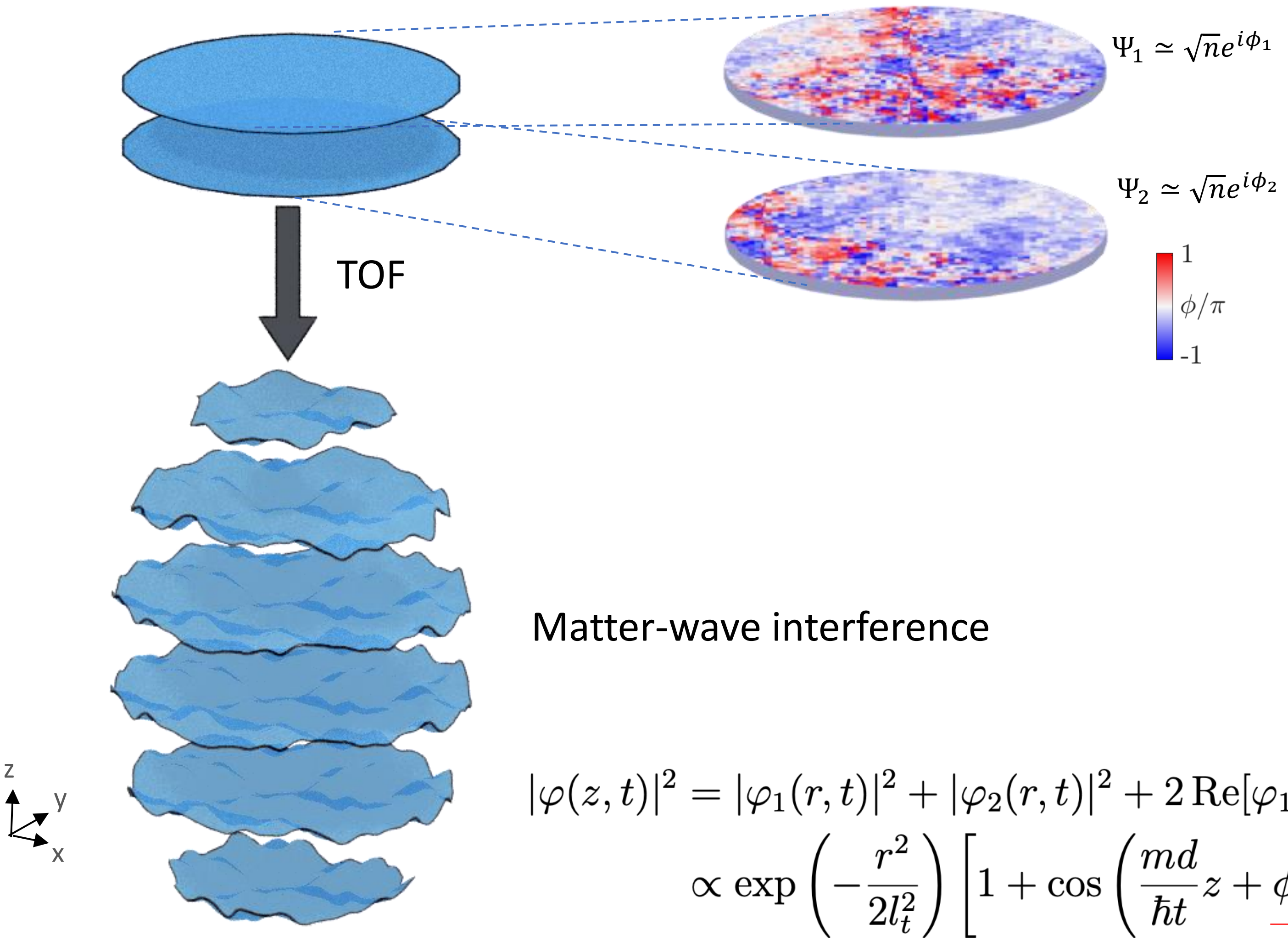
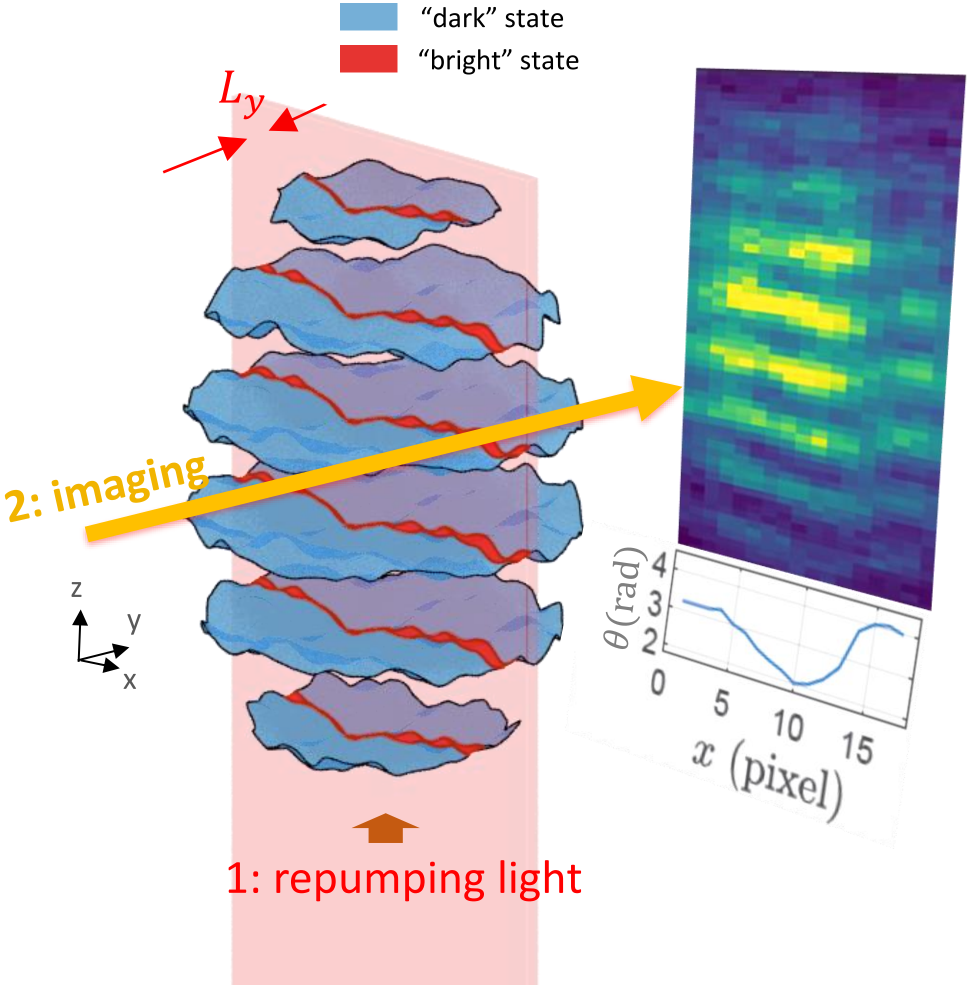
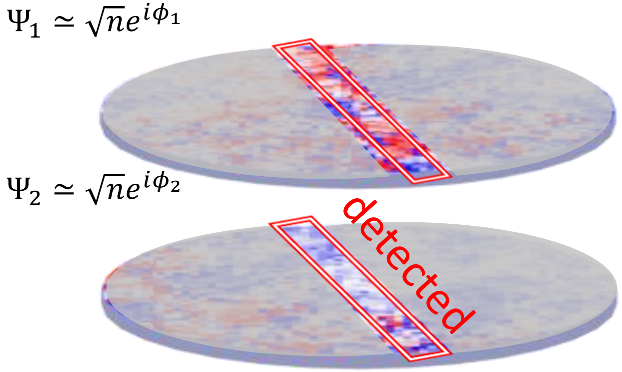


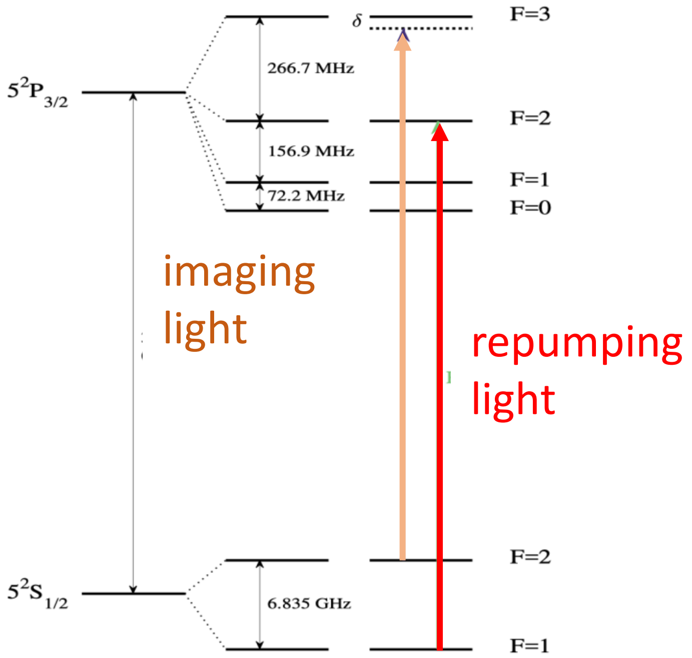
Matter-wave interference

$$
\begin{aligned}
|\varphi(z,t)|^2 &= |\varphi_1(r,t)|^2 + |\varphi_2(r,t)|^2 + 2\,\mathrm{Re}[\varphi_1(r,t)^*\varphi_2(r,t)], \\
&\propto \exp\left(-\frac{r^2}{2l_t^2}\right)\left[1+\cos\left(\frac{md}{\hbar t}z + \underset{\text{Relative phase}}{\underline{\phi_1-\phi_2}}\right)\right],
\end{aligned}
$$

Relative phase: interferometry

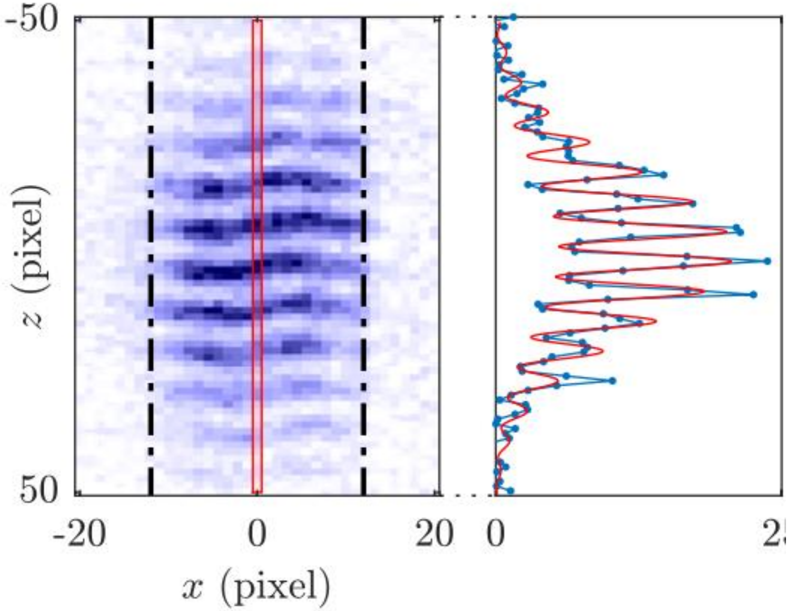




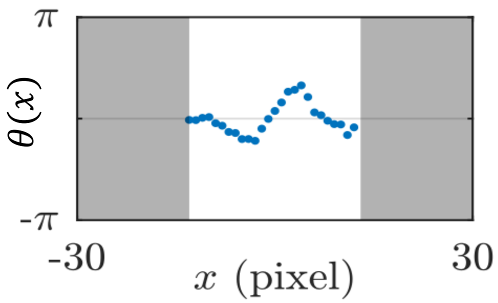


Relative phase: interferometry

Fit image columns

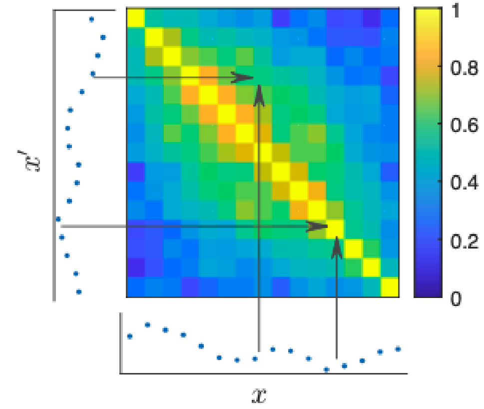


Fit results



Correlate and average (N>50)

$$C(\boldsymbol{r},\boldsymbol{r}') = \langle e^{i\theta(\boldsymbol{r})-i\theta(\boldsymbol{r}')}\rangle$$



Spatial average

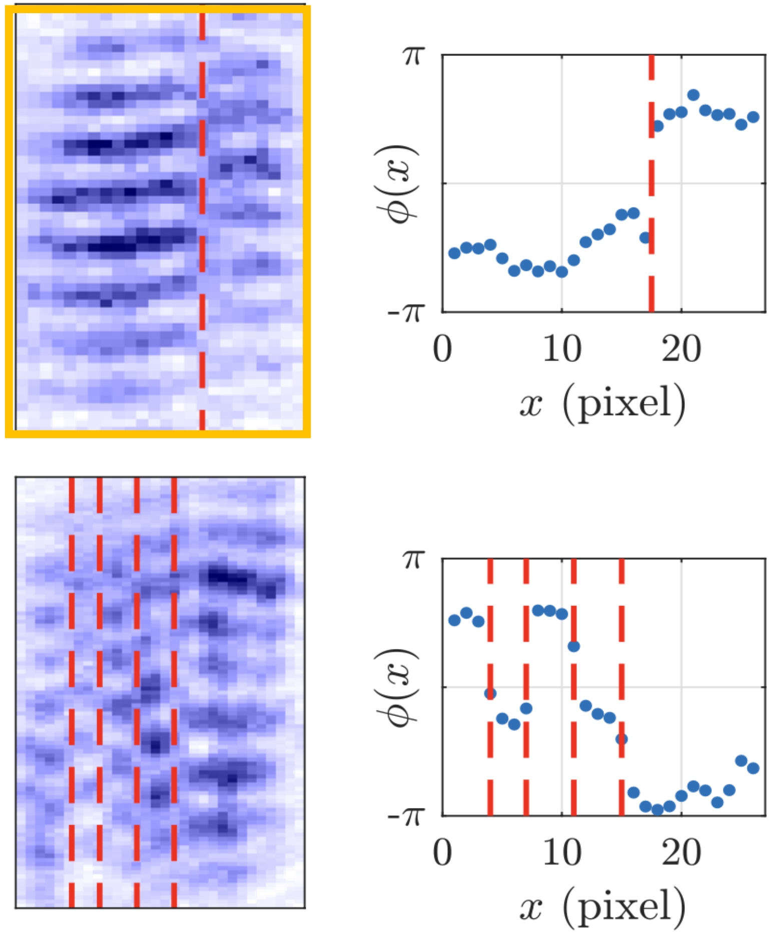
$$C(\overline{x}) = \mathrm{Re}[\langle e^{i[\theta(x)-\theta(x-\overline{x})]}\rangle]$$

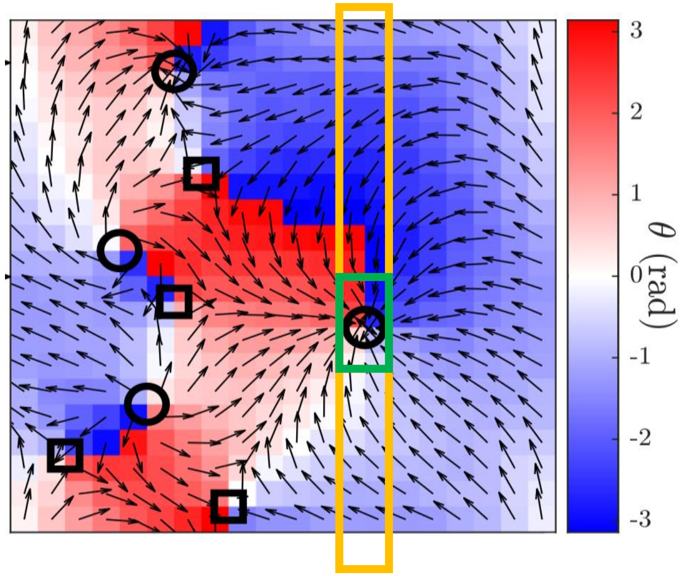
relative phase correlation function

c.f. series of work by
Vienna group (1D)

S. Sunami et al., PRL (2022)

Bonus: vortex detection



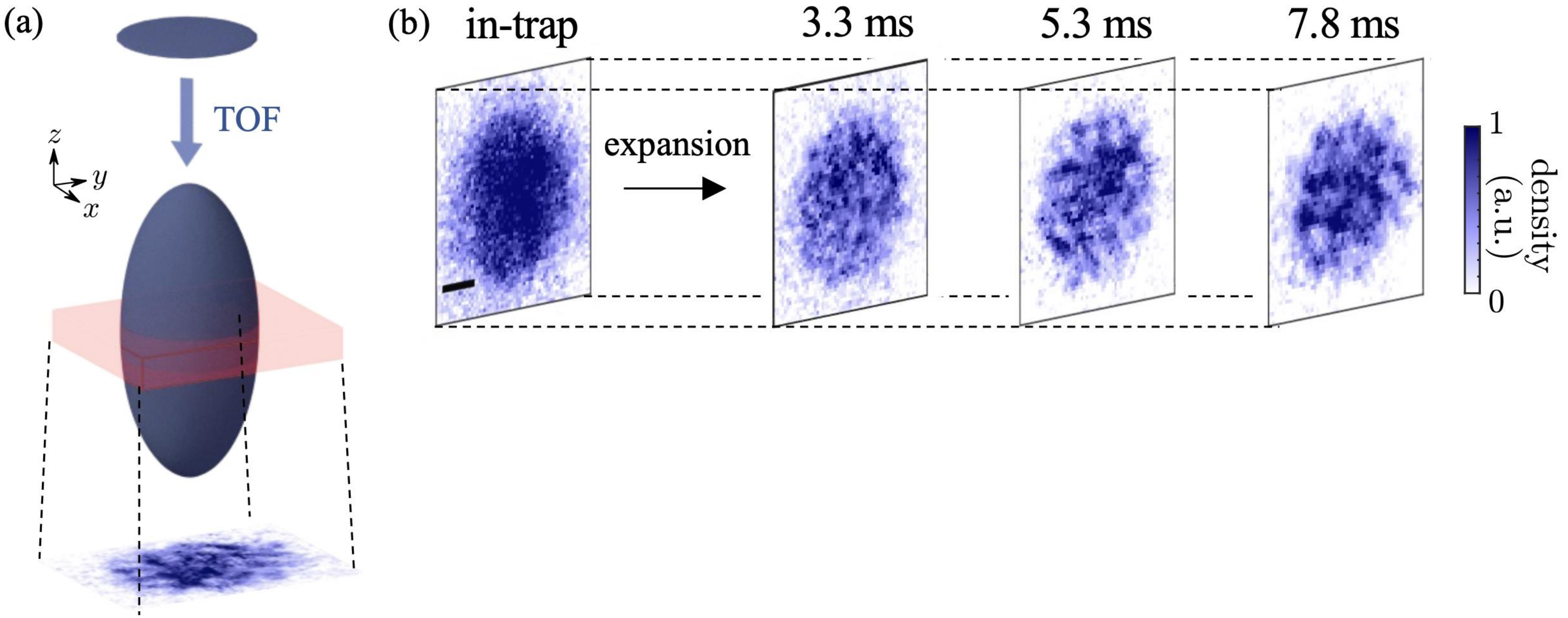


S. Sunami et al., PRL (2022)

Common phase: noise correlations

Sunami, Singh, Rydow, Beregi, Chang, Mathey, Foot (2024) arxiv.org/abs/2406.03491

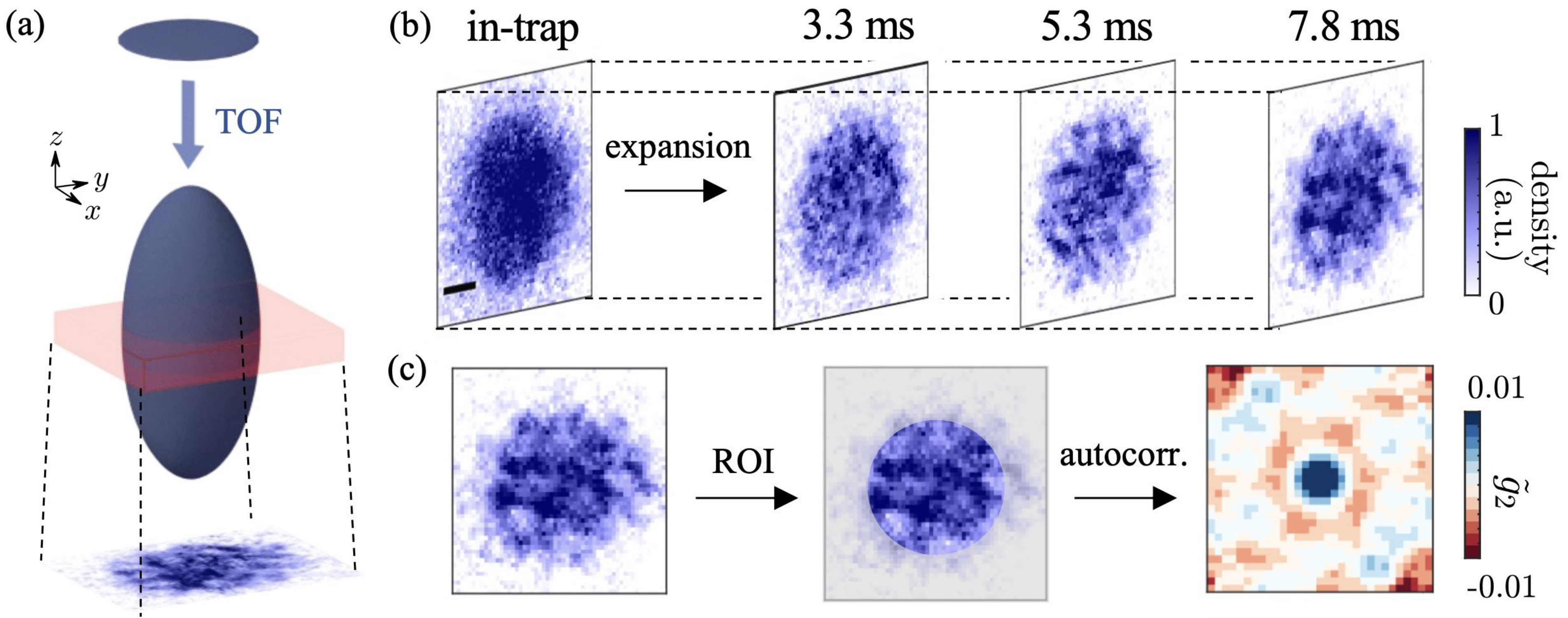
Common phase: noise correlations



c.f. related work by
Schmiedmayer (1D)
Shin (2D, spectrum only)
Bloch (optical lattice)

Sunami, Singh, Rydow, Beregi, Chang, Mathey, Foot (2024) arxiv.org/abs/2406.03491

Common phase: noise correlations



$$\hat{n}(\mathbf{r},t) = \underline{\hat{\psi}^{\dagger}(\mathbf{r},t)}\hat{\psi}(\mathbf{r},t)$$

Bosonic field operator after TOF

$$\frac{\langle \hat{n}(\mathbf{r},t)\hat{n}(0,t)\rangle}{n_0^2} - 1$$

c.f. related work by
Schmiedmayer (1D)
Shin (2D, spectrum only)
Bloch (optical lattice)

Sunami, Singh, Rydow, Beregi, Chang, Mathey, Foot (2024) arxiv.org/abs/2406.03491

Noise correlations (single-layer)

Singh et al., PRA (2014)

$$g_2(\boldsymbol{r},t) = \frac{1}{2\pi^2}\int d^2\boldsymbol{q}\int d^2\boldsymbol{R}\cos(\boldsymbol{q}\cdot\boldsymbol{r})\cos(\boldsymbol{q}\cdot\boldsymbol{R})\left\langle\hat{\Psi}^\dagger(\hbar\boldsymbol{q}t/m)\hat{\Psi}^\dagger(\boldsymbol{R})\hat{\Psi}(\boldsymbol{R}+\hbar\boldsymbol{q}t/m)\hat{\Psi}(\boldsymbol{0})\right\rangle$$

V. Singh, L. Mathey, Phys. Rev. A 89, 053612 (2014)

Noise correlations (single-layer)

Singh et al., PRA (2014)

Oscillatory

$$g_2(\boldsymbol{r},t) = \frac{1}{2\pi^2}\int d^2\boldsymbol{q}\int d^2\boldsymbol{R}\,\boxed{\cos(\boldsymbol{q}\cdot\boldsymbol{r})\cos(\boldsymbol{q}\cdot\boldsymbol{R})}\left\langle \hat{\Psi}^\dagger(\hbar\boldsymbol{q}t/m)\hat{\Psi}^\dagger(\boldsymbol{R})\hat{\Psi}(\boldsymbol{R}+\hbar\boldsymbol{q}t/m)\hat{\Psi}(\mathbf{0})\right\rangle$$

V. Singh, L. Mathey, Phys. Rev. A 89, 053612 (2014)

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$$g_2(\boldsymbol{r},t) = \frac{1}{2\pi^2}\int d^2\boldsymbol{q}\int d^2\boldsymbol{R}\,\overset{\text{Oscillatory}}{\boxed{\cos(\boldsymbol{q}\cdot\boldsymbol{r})\cos(\boldsymbol{q}\cdot\boldsymbol{R})}}\,\overset{\text{Decay}}{\boxed{\left\langle \hat{\Psi}^\dagger(\hbar\boldsymbol{q}t/m)\hat{\Psi}^\dagger(\boldsymbol{R})\hat{\Psi}(\boldsymbol{R}+\hbar\boldsymbol{q}t/m)\hat{\Psi}(\boldsymbol{0})\right\rangle}}$$

V. Singh, L. Mathey, Phys. Rev. A 89, 053612 (2014)

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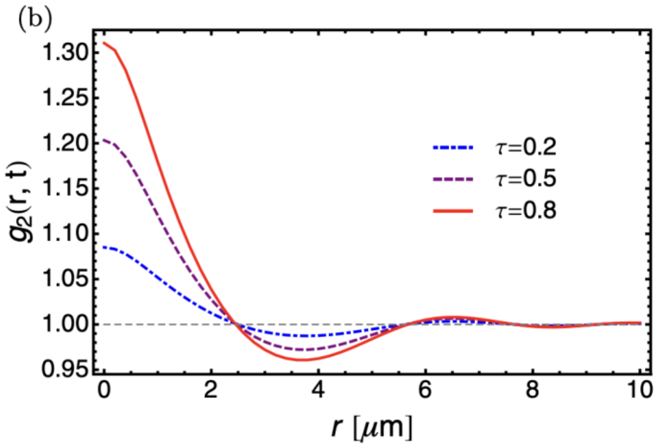
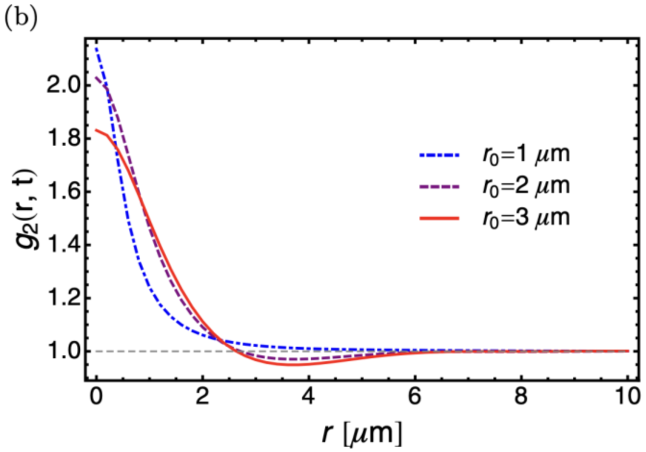
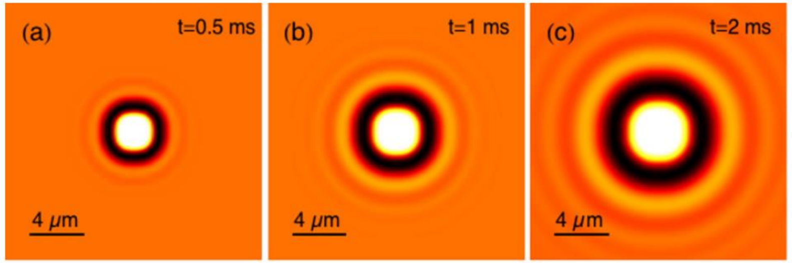
SF



Normal



SF



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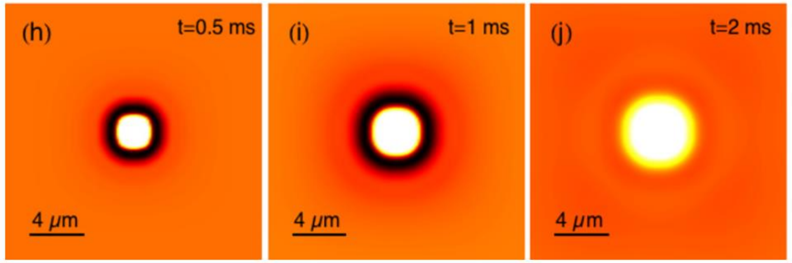
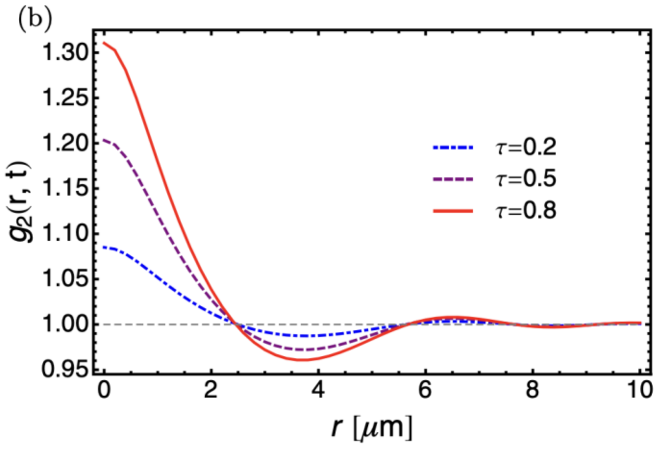


Noise correlations (single-layer)

Singh et al., PRA (2014)

SF



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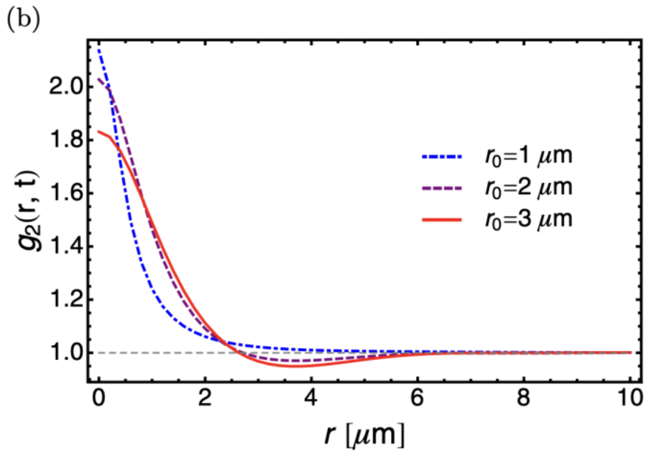


Density-density correlation function after TOF expansion

$$\boxed{\frac{\langle \hat{n}(\mathbf{r},t)\hat{n}(0,t)\rangle}{n_0^2}} = g_2(\mathbf{r},t) = \frac{1}{(2\pi)^2}\int d^2\mathbf{q}\int d^2\mathbf{R}\cos\mathbf{q}\cdot\mathbf{r}\cos\mathbf{q}\cdot\mathbf{R} \times\left(\frac{\mathcal{F}_i(\mathbf{q}_t)^2\boxed{\mathcal{F}_i(\mathbf{R})}^2}{\mathcal{F}_i(\mathbf{R}-\mathbf{q}_t)\mathcal{F}_i(\mathbf{R}+\mathbf{q}_t)}\right),$$

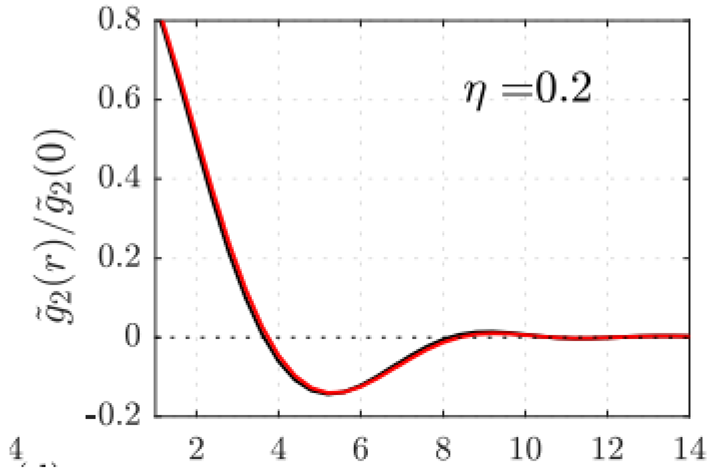
In-trap 1st-order correlation function

One-to-one mapping between g_1 and g_2 of expanded cloud

Noise correlations (bilayer-layer)

Singh et al., PRA (2014)

SF



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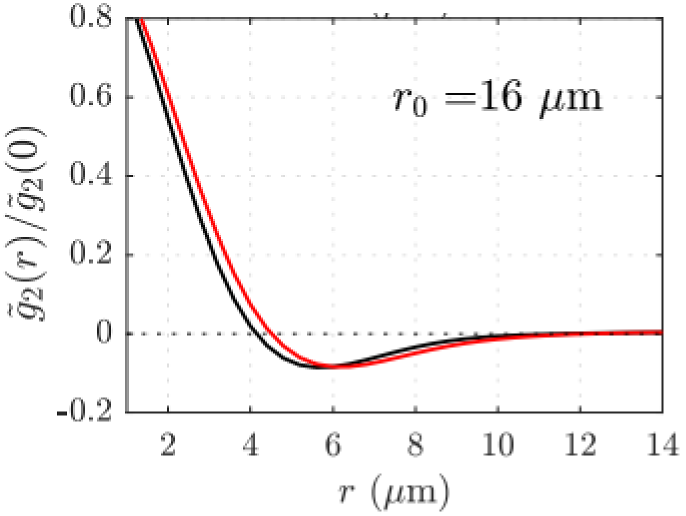


Density-density correlation function after TOF expansion

$$\boxed{\frac{\langle \hat{n}(\mathbf{r},t)\hat{n}(0,t)\rangle}{n_0^2}} = g_2(\mathbf{r},t) = \frac{1}{(2\pi)^2}\int d^2\mathbf{q}\int d^2\mathbf{R}\cos\mathbf{q}\cdot\mathbf{r}\cos\mathbf{q}\cdot\mathbf{R}$$

$$\times \boxed{\frac{\mathcal{F}_{\mathrm{com}}(\boldsymbol{q}_t)^2\mathcal{F}_{\mathrm{com}}(\boldsymbol{R})^2}{\mathcal{F}_{\mathrm{com}}(\boldsymbol{R}-\boldsymbol{q}_t)\mathcal{F}_{\mathrm{com}}(\boldsymbol{R}+\boldsymbol{q}_t)}}\boxed{\mathcal{F}_{\mathrm{rel}}(\boldsymbol{q}_t)^2}$$

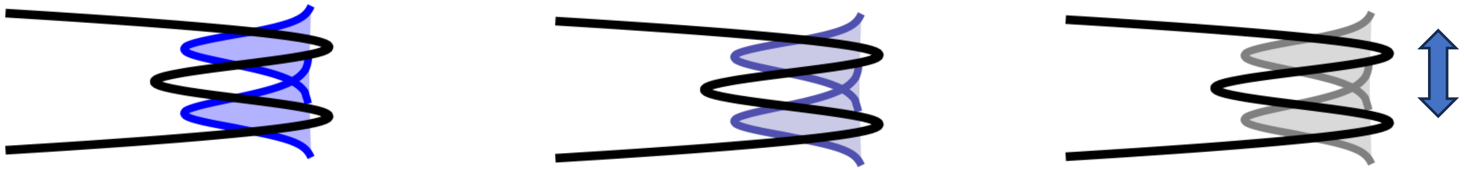
Common-phase correlations

Relative-phase effect
(known from interference measurements)

Sunami, Singh, Rydow, Beregi, Chang, Mathey, Foot (2024) arxiv.org/abs/2406.03491

Coupled-Bilayer: 2D Sweep

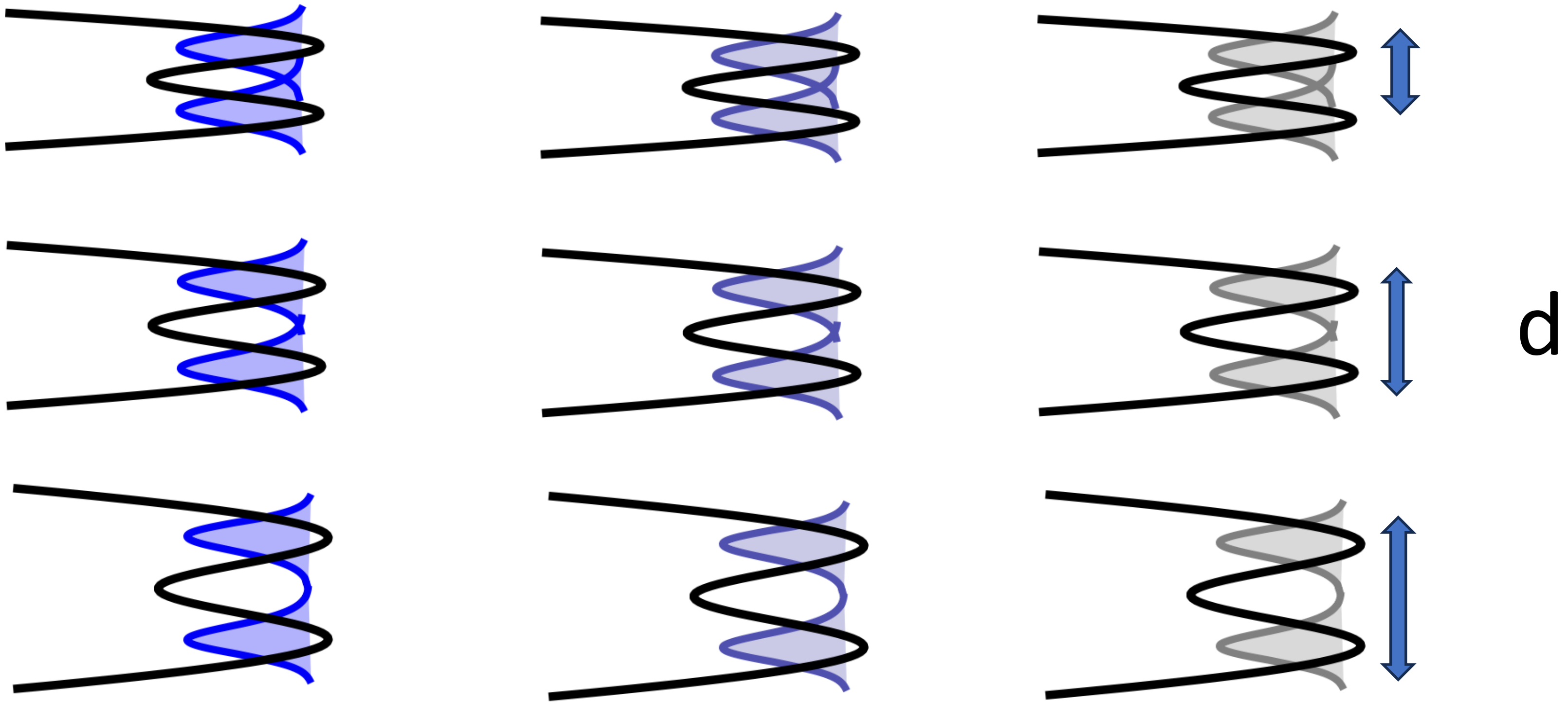
$$N \propto D$$



S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

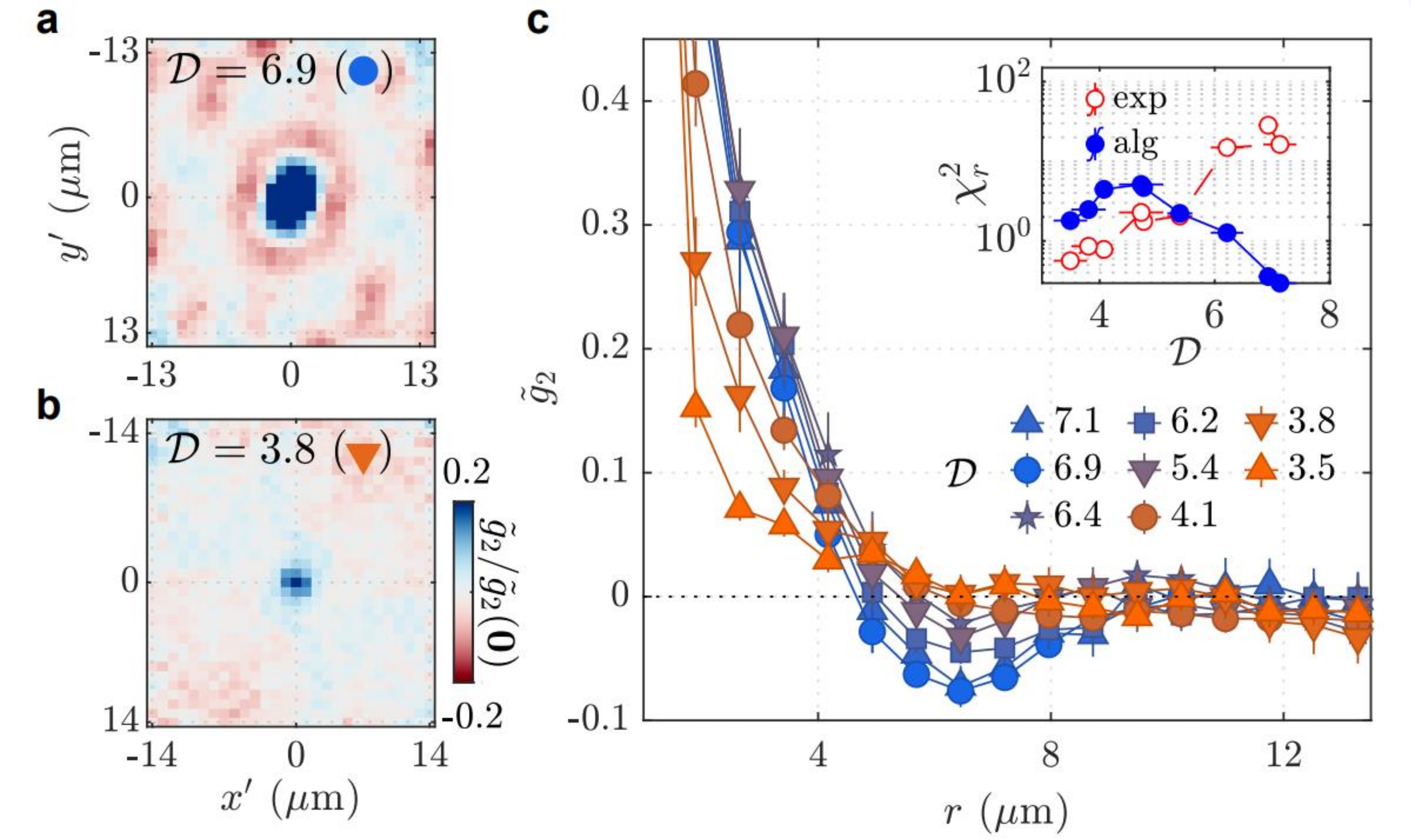
Coupled-Bilayer: 2D Sweep

$$N \propto D$$



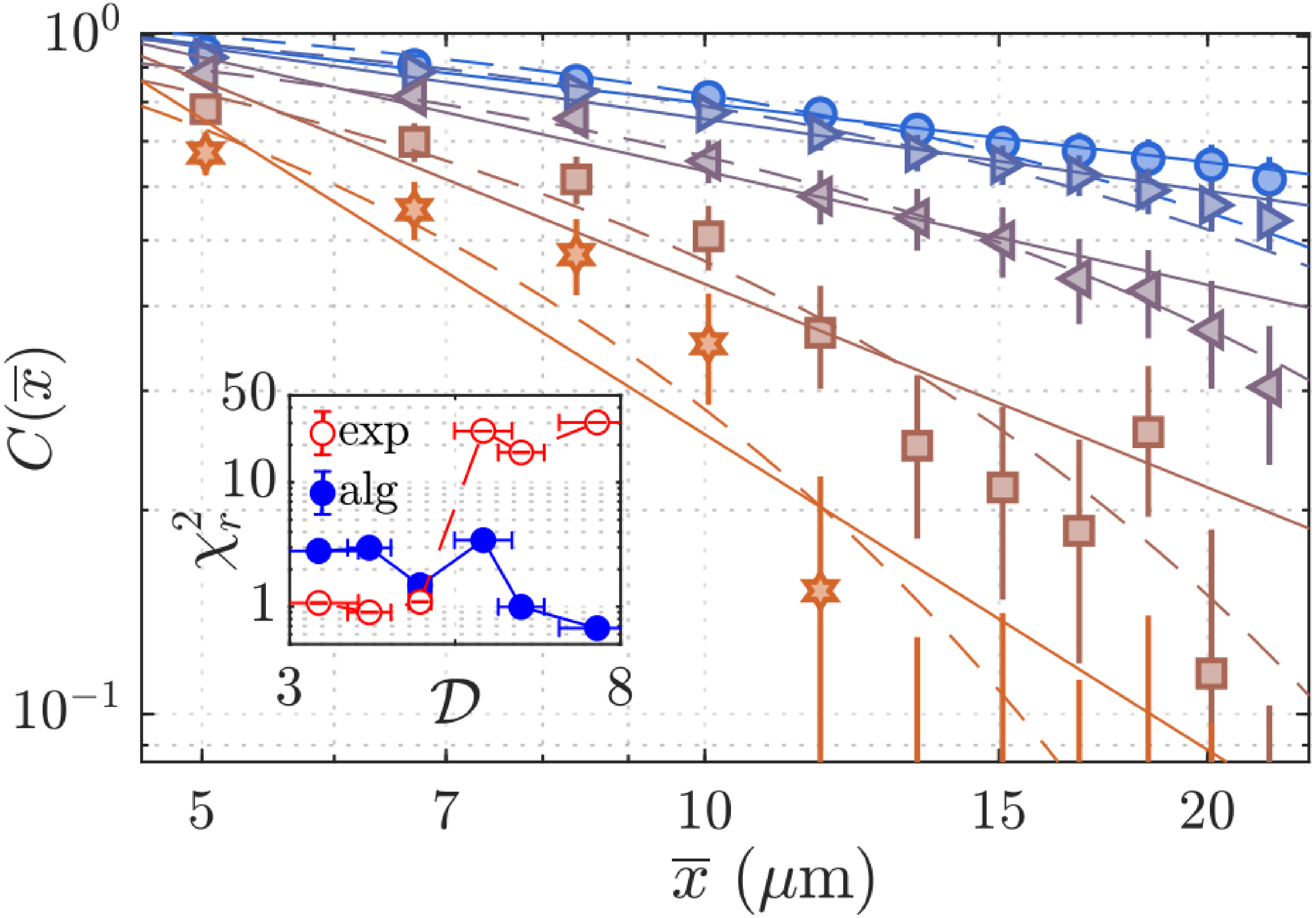
S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

Coupled-Bilayer: Common phase



S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

Coupled-Bilayer: Relative phase



$D = 7.6$

$D = 5.9$

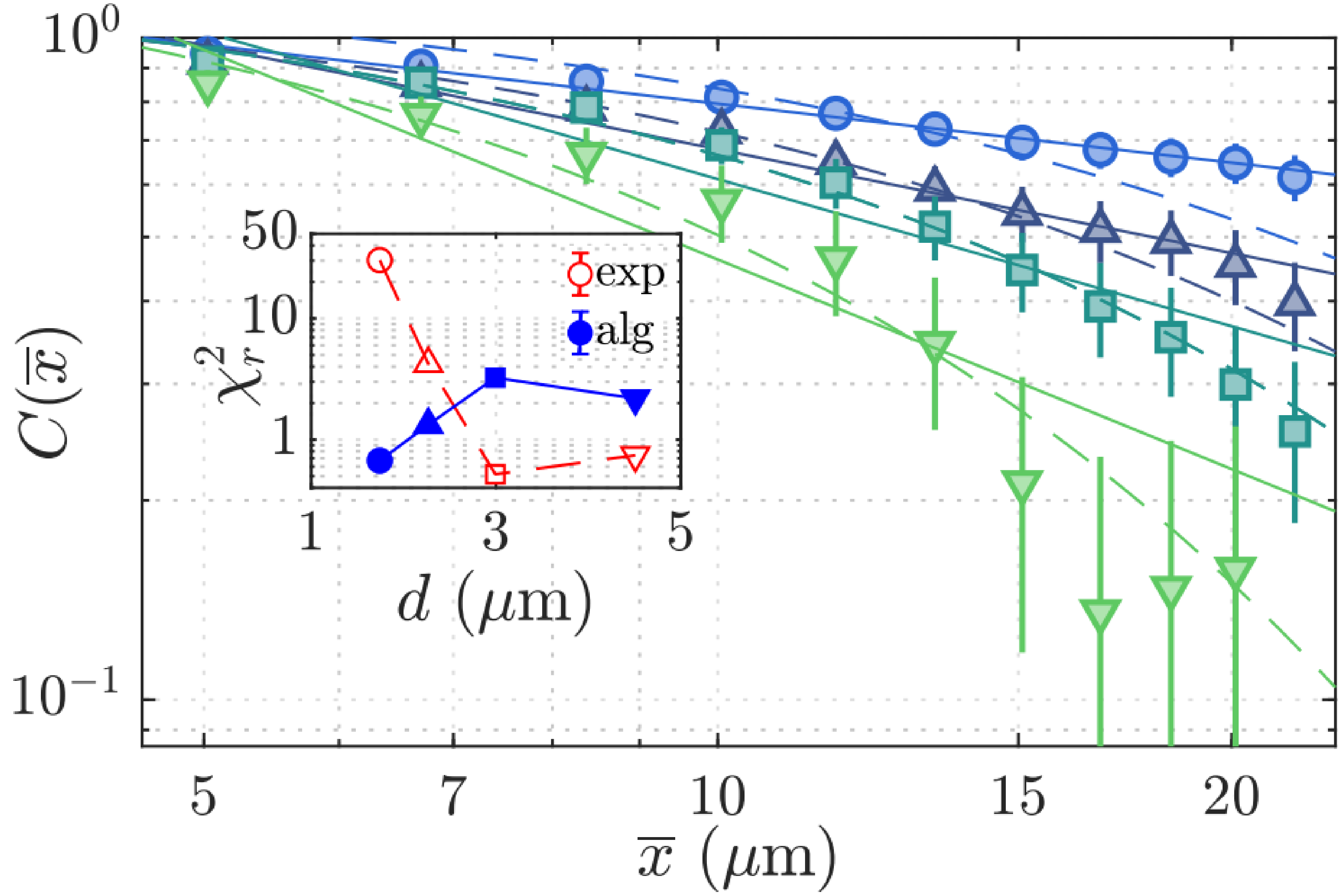
$D = 5.0$

$D = 4.2$

$D = 3.4$

S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

Coupled-Bilayer: Relative phase



$J/\hbar$ = 30 Hz

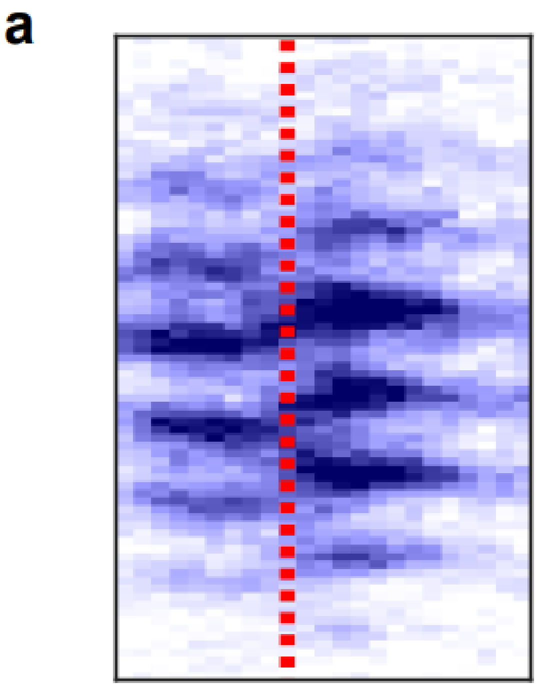
$J/\hbar$ = 7 Hz

$J/\hbar$ = 1 Hz

$J/\hbar$ = 0.05 Hz

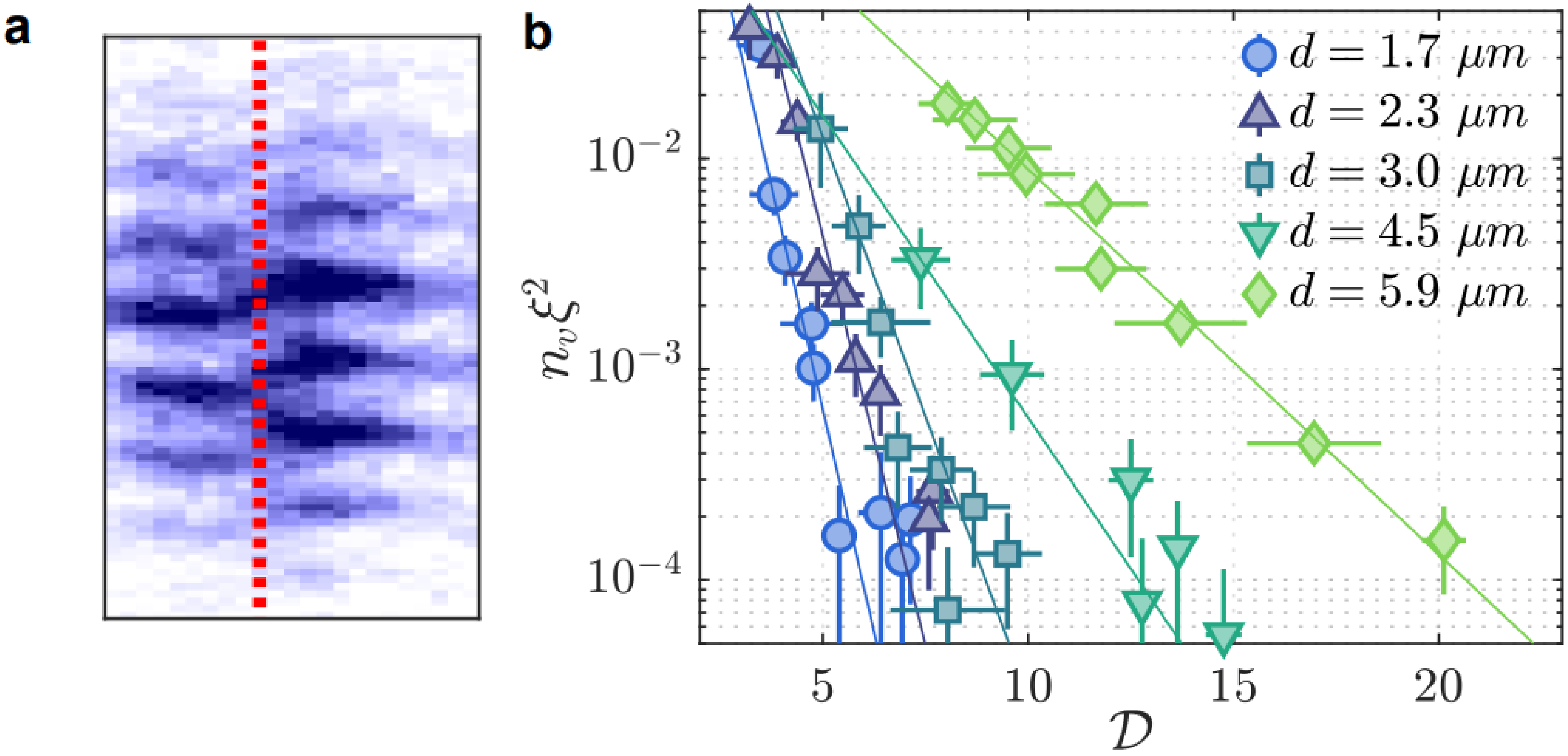
S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

Coupled-Bilayer: Vortex Suppression



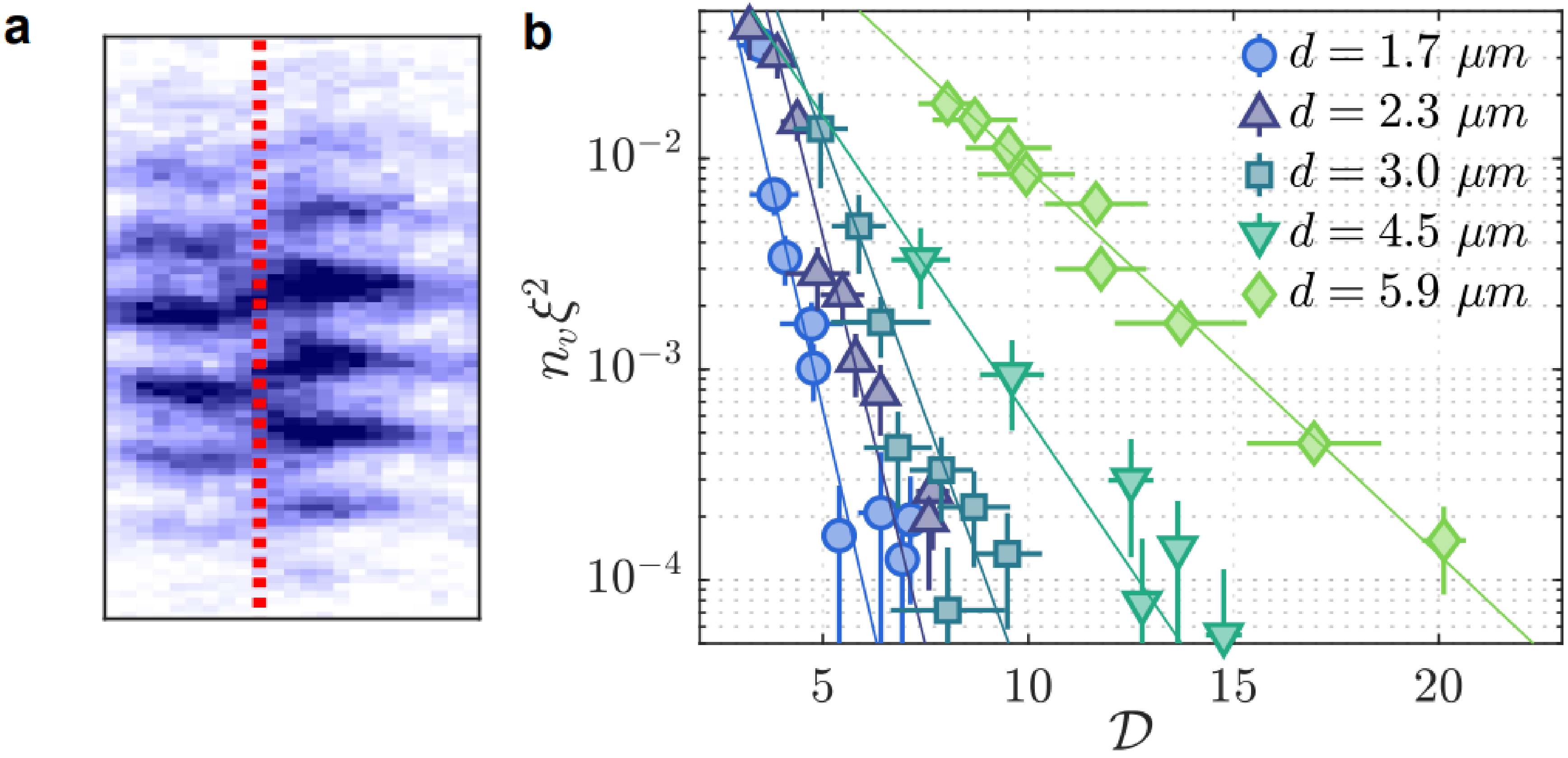
S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

Coupled-Bilayer: Vortex Suppression



S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

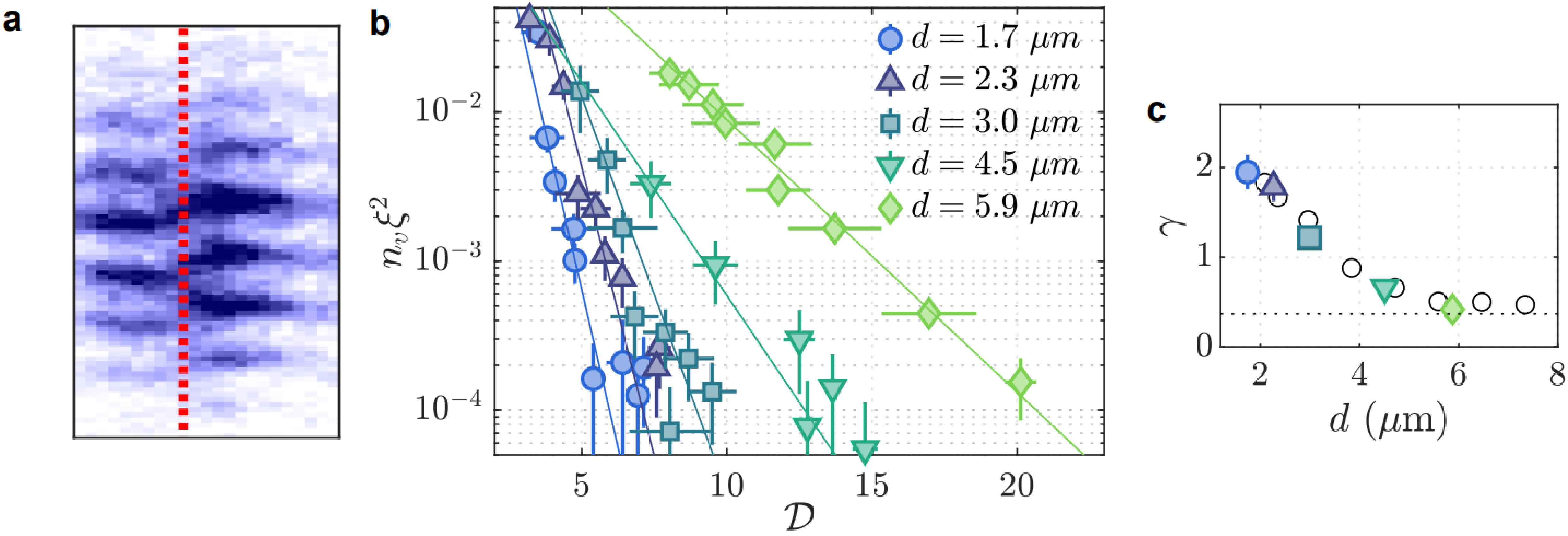
Coupled-Bilayer: Vortex Suppression



$$f(\mathcal{D}) = A\exp(-\gamma\mathcal{D})$$

S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

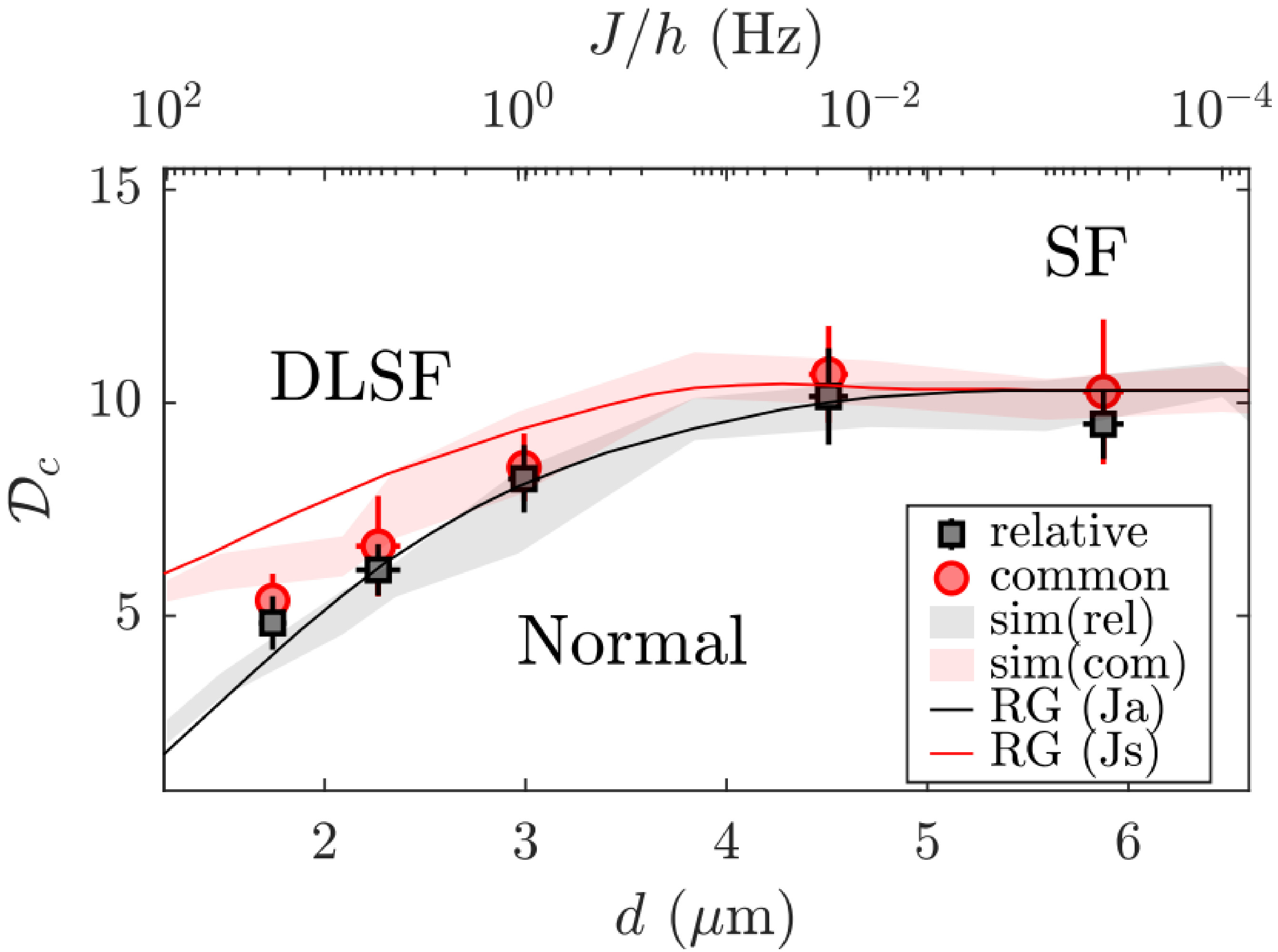
Coupled-Bilayer: Vortex Suppression



$$f(\mathcal{D}) = A \exp(-\gamma \mathcal{D})$$

S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

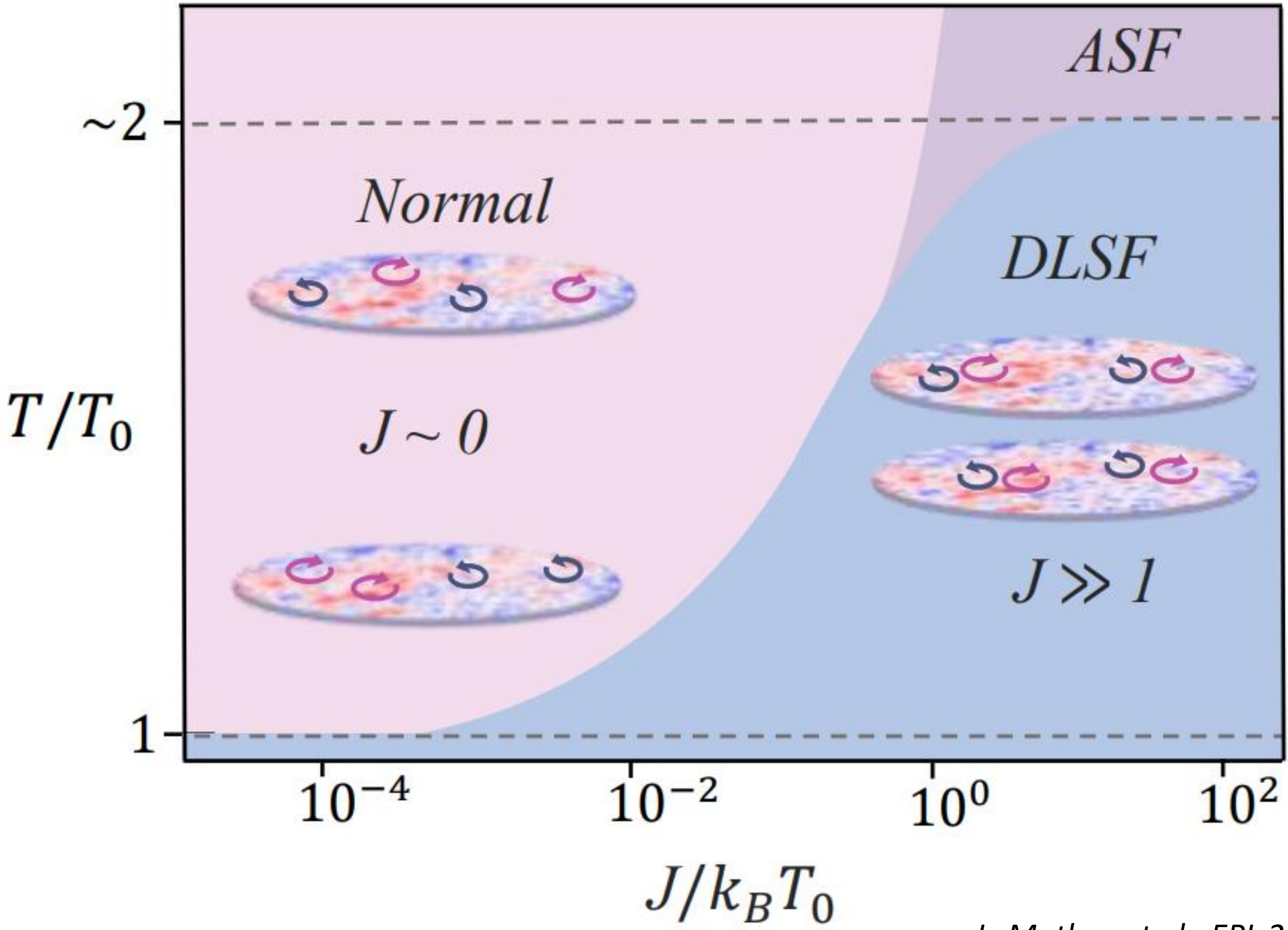
Coupled-Bilayer: 2D Sweep



S. Sunami, V. Singh, E. Rydow, A. Beregi, E. Chang, L. Mathey, C. Foot (2024) arxiv.org/abs/2406.03491

The effect of coupling: Summary

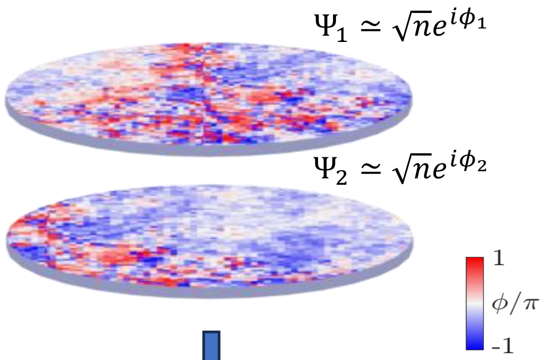
- Experimentally probed common and relative and phase coherence
- Observed tunnelling shifting the transition point of both.
- Detected suppression of vortices.



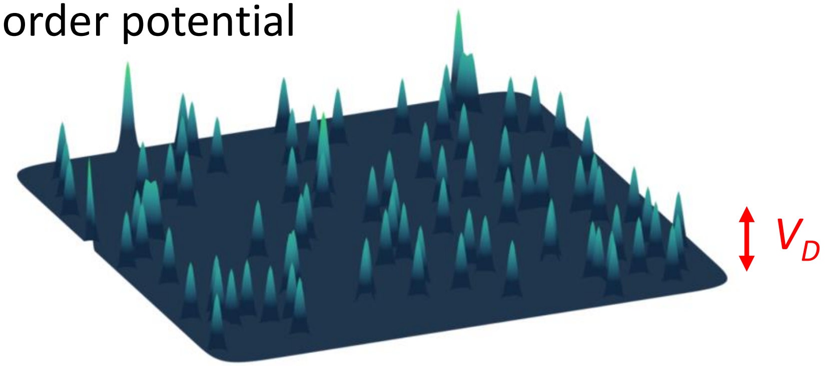
L. Mathey et al., EPL 2007

Ongoing work: disorder-induced superfluid transition

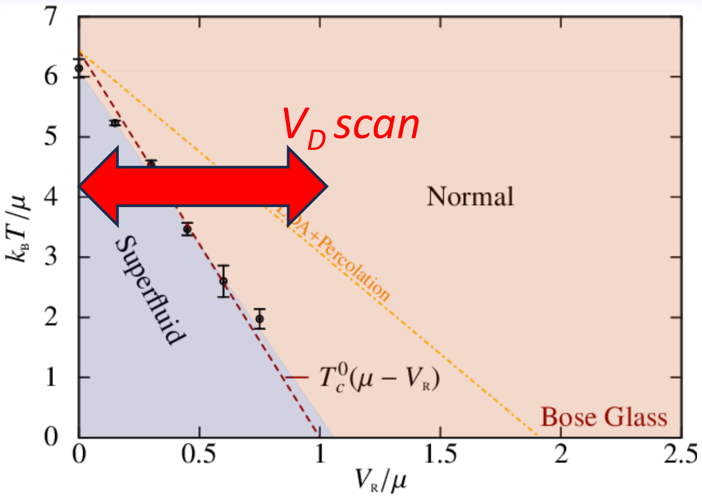
bilayer quantum gas (J=0)



Disorder potential



Point-like disorder potential,
or laser spackle patterns

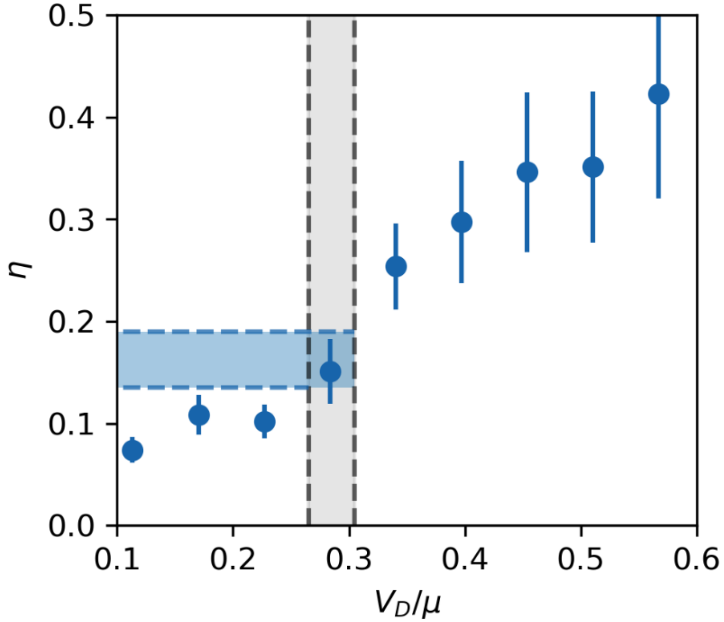


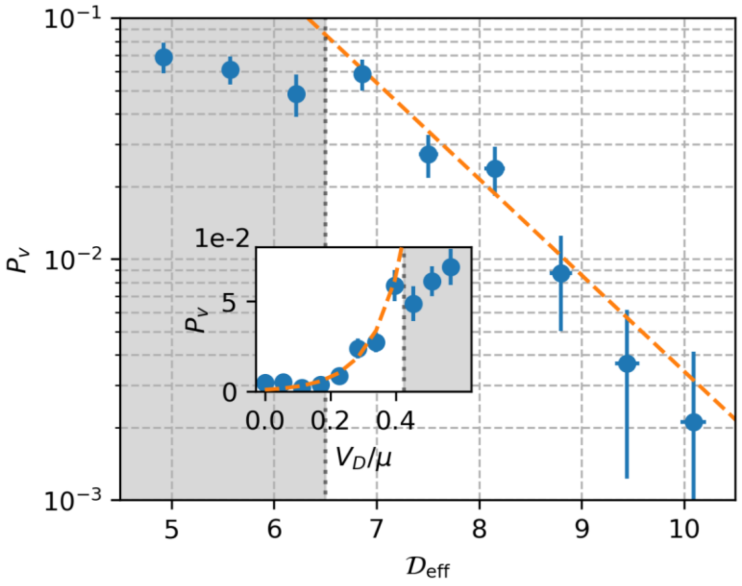
Numerics (QMC):
Carleo , Boéris, Holzmann, Sanchez-Palencia,
PRL 111, 050406 (2013)

c.f. Bourdel (2011, 2012)

Abel Beregi

Experiment:





A. Beregi et al., in preparation

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Oscar (En) Chang


Dr Charu Mishra


Prof. Chris Foot

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