The Bose polaron and the Efimov effect

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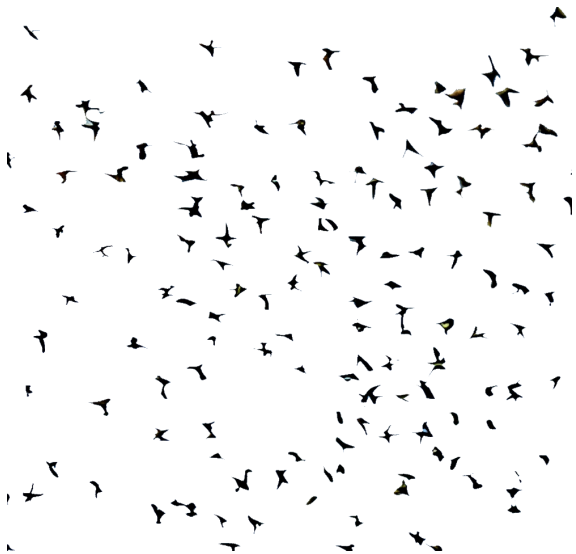
UPC seminar, 24 April 2023

Emergence

Daniel Biber, Germany, 2018 Sony World Photography Awards

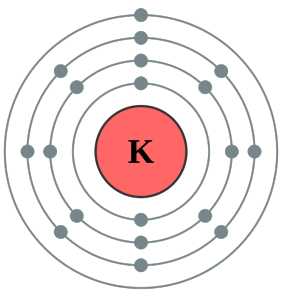
Emergence



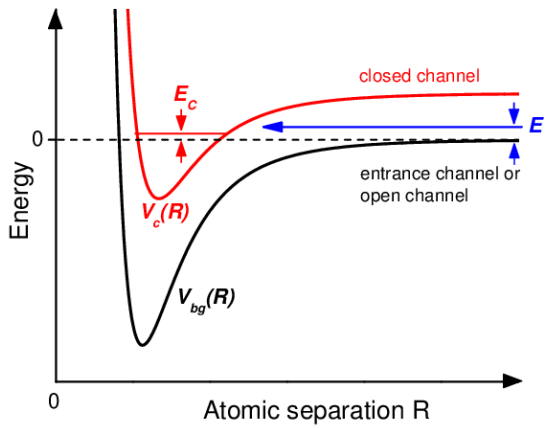
AI generated bird images (https://www.craiyon.com/)

Emergence in ultracold gases

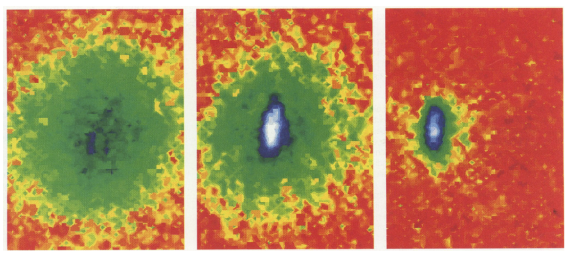
AI generated bird images (https://www.craiyon.com/)



Wikipedia



Cheng Chin et al., Rev. Mod. Phys. 82 (2010)



M. H. Anderson *et al.*, Science 269, (1995)

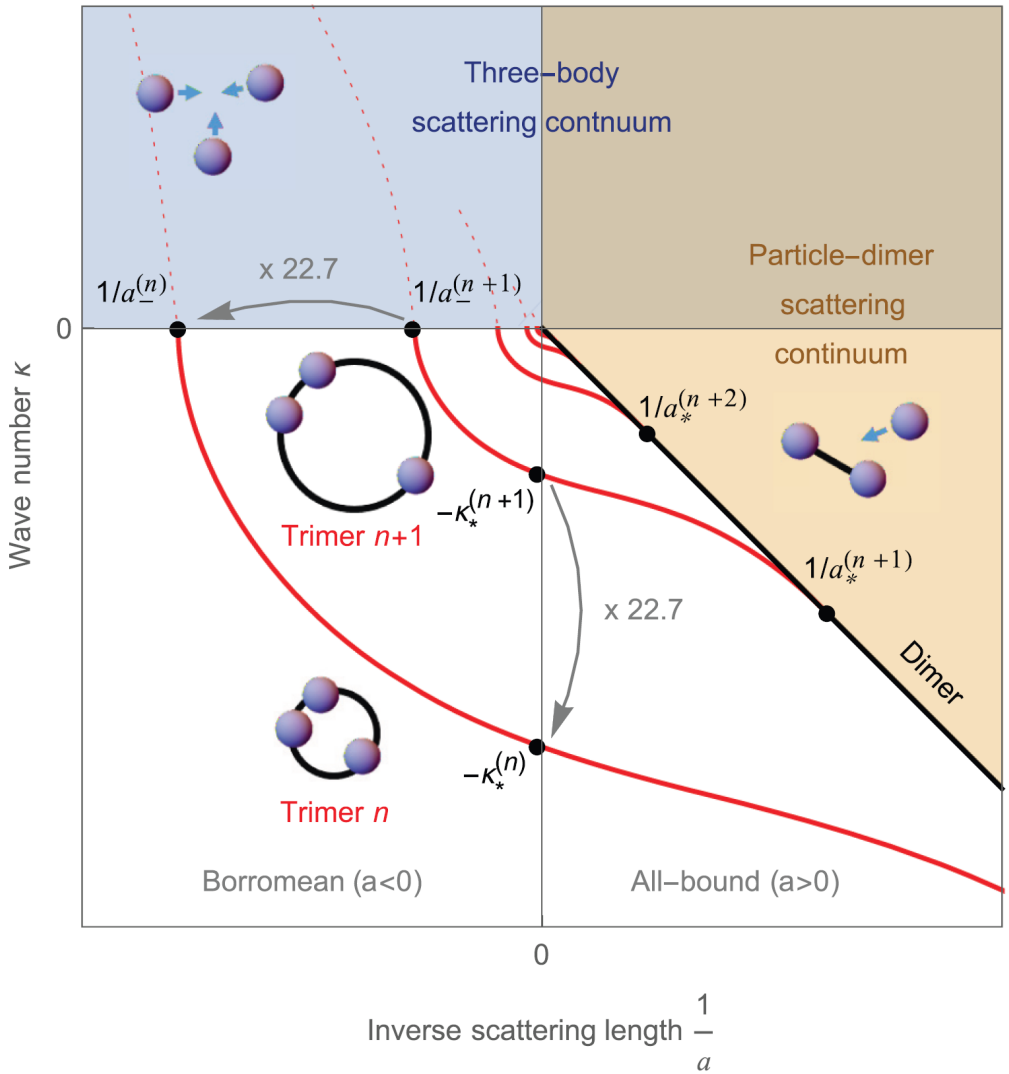
The Efimov effect

The Efimov effect:

The smallest kind of emergence you can think of...

$\rightarrow$ with just three particles!

The Efimov is a **quantum** effect
$\rightarrow$ arises because of the zero-point energy

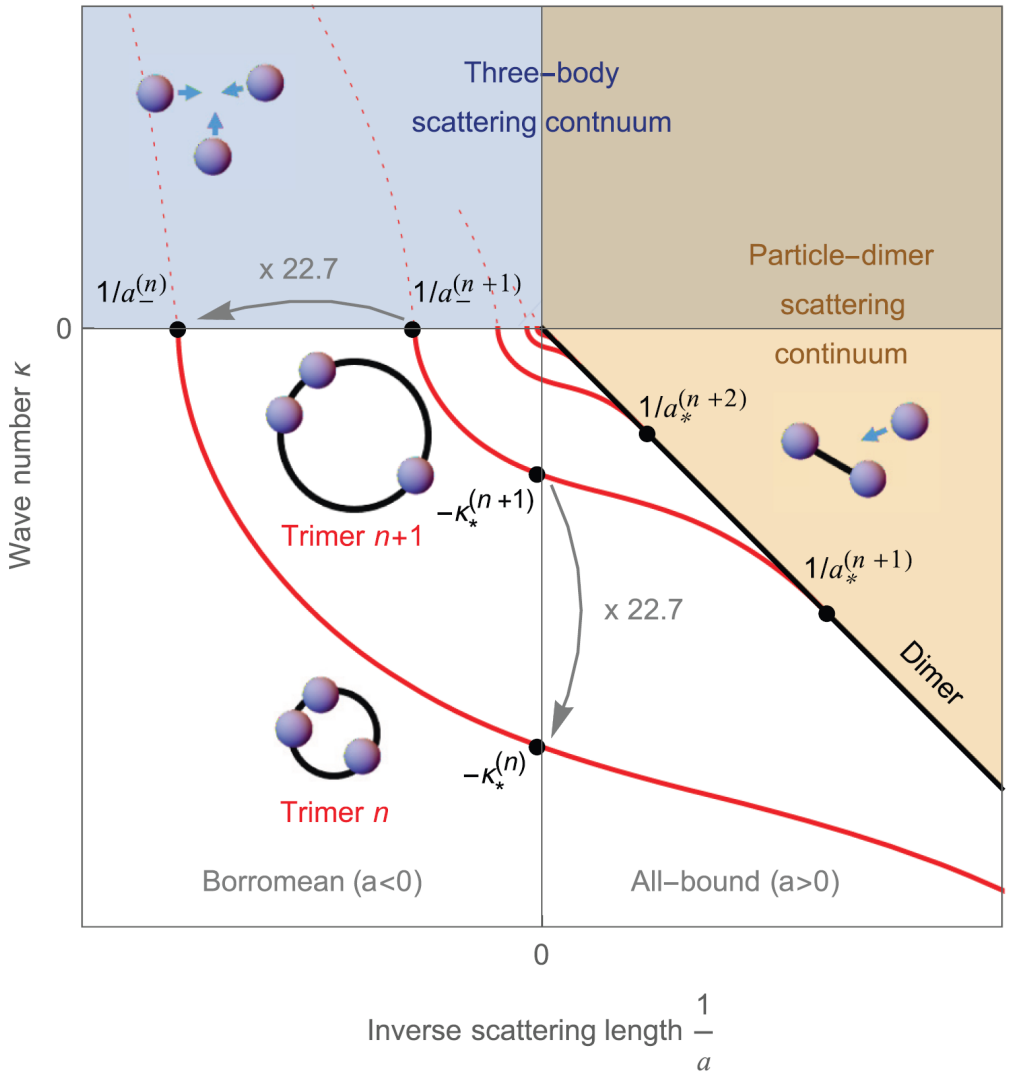


Pascal Naidon and Shimpei Endo, Rep. Prog. Phys. 80 056001 (2017)

The Efimov effect

Characteristics:

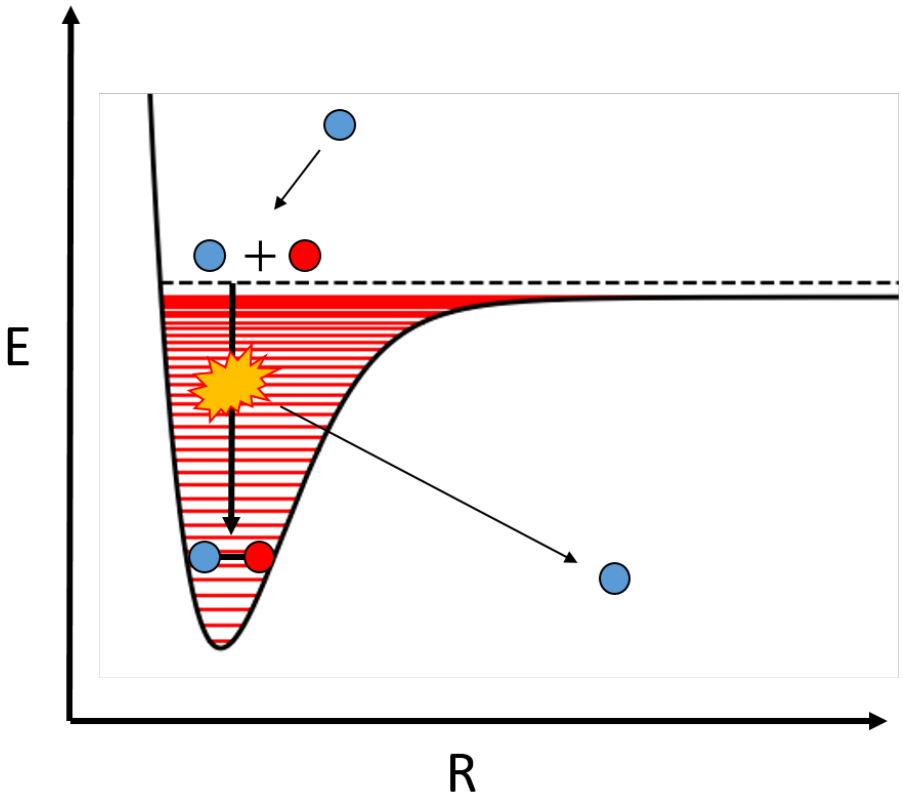
- Contact s-wave interactions between atoms
- "Cooperative binding"
- Effective $-1/R^2$ potential
  $\rightarrow$ Discrete scale invariance
- First Efimov state at a_-
  $\rightarrow$ set by VdWaals length
- Leads to loss due to recombination

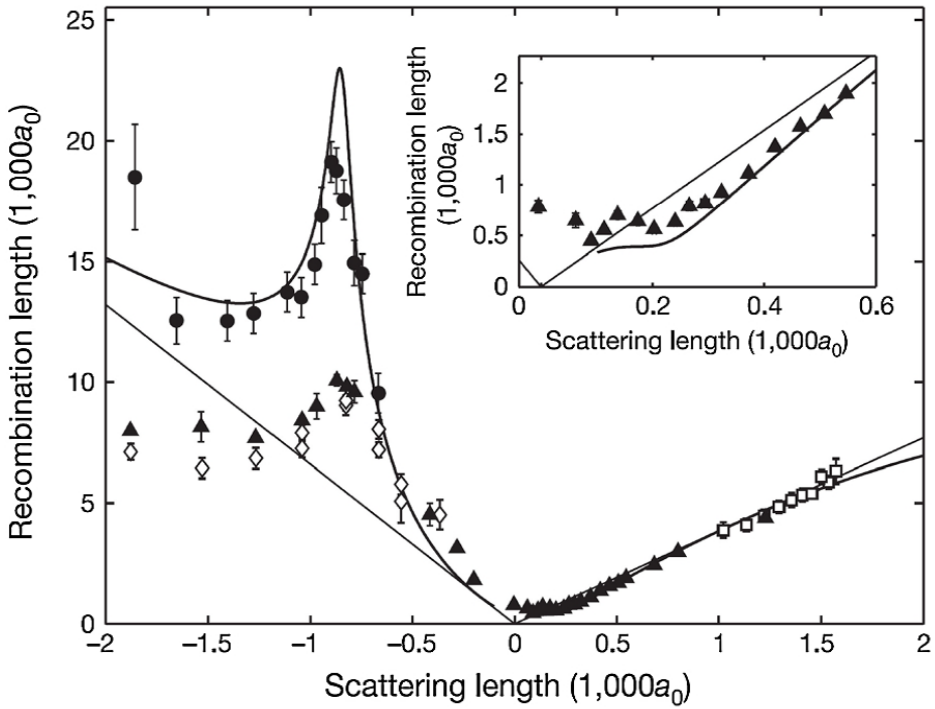


Pascal Naidon and Shimpei Endo, Rep. Prog. Phys. 80 056001 (2017)

Three-body recombination and Efimov resonances

Chemical reactions in cold atomic gases: three-body recombination





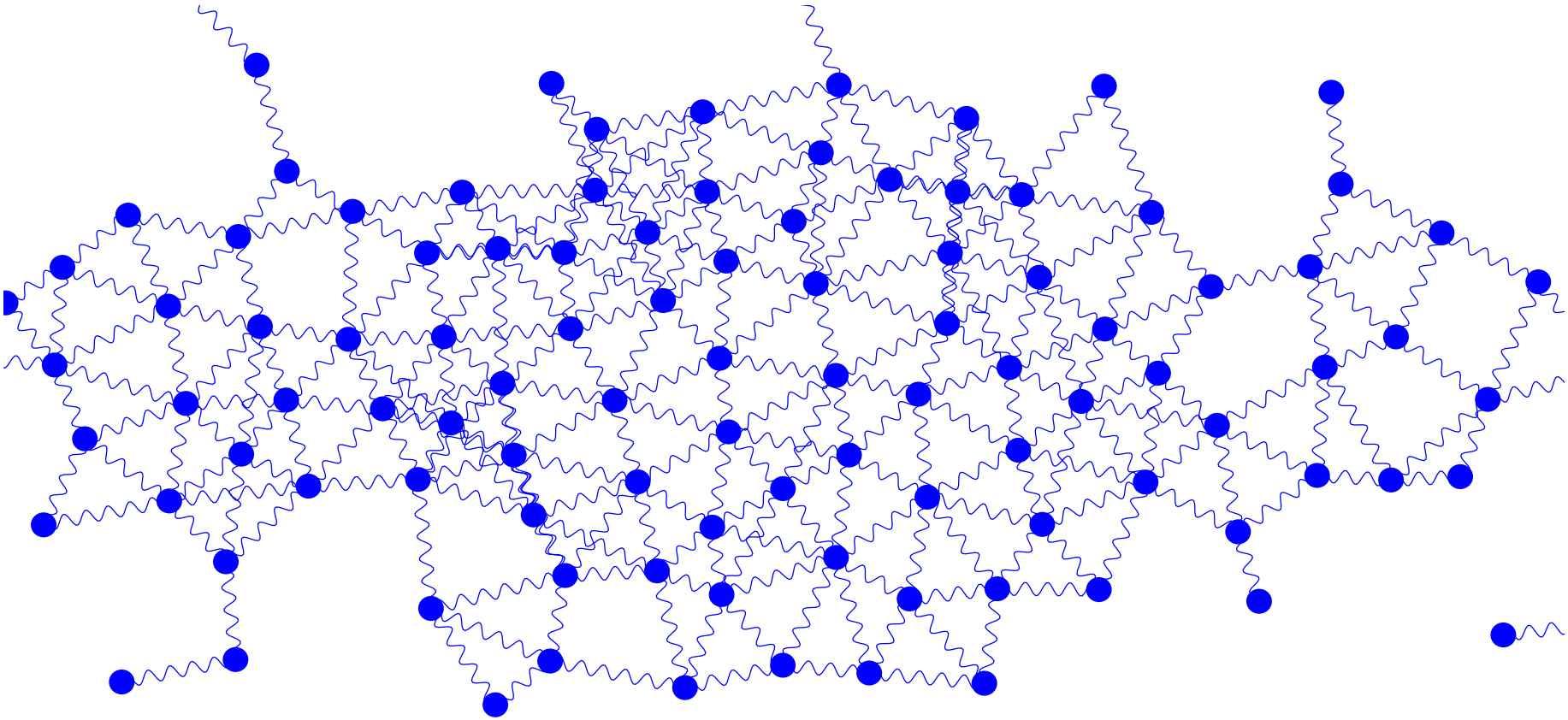
Efimov scattering resonances enhance recombination

T. Kraemer *et al.*, Nature 440, 315 (2006)

The Efimov effect in a medium

How does the Efimov effect manifest itself in a many-body system?

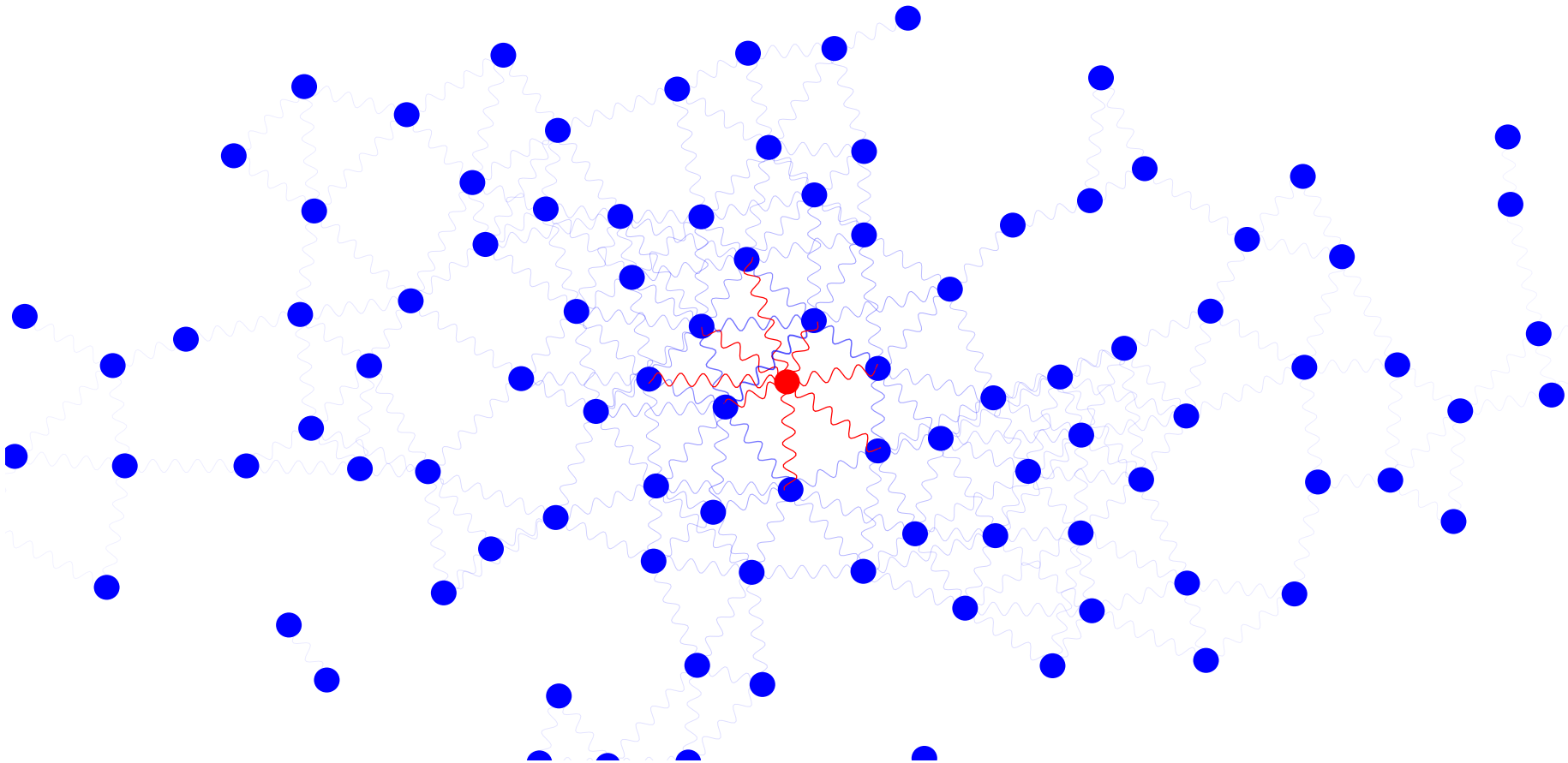
Example: Strongly interacting BEC



The Efimov effect in a medium

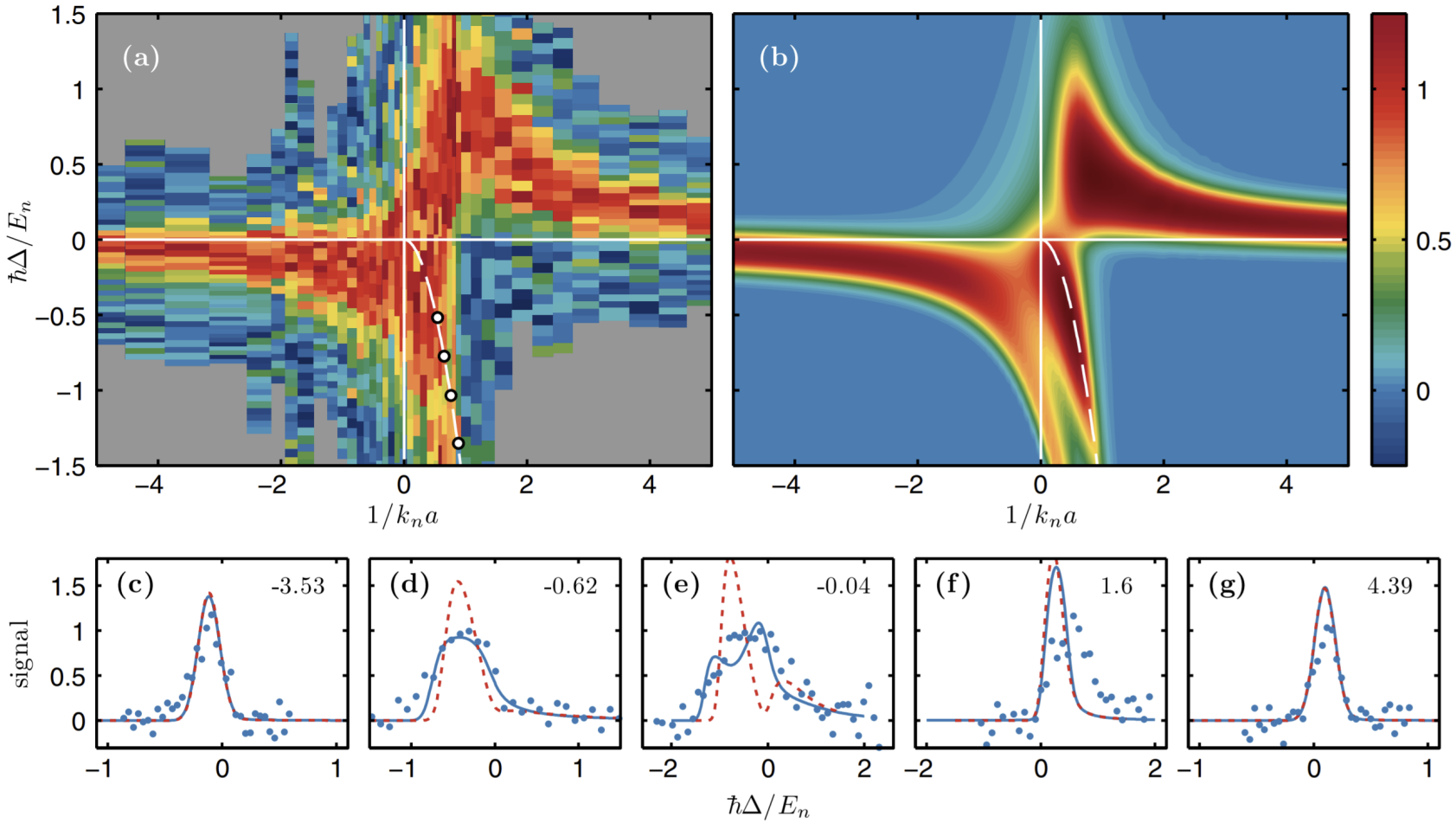
How does the Efimov effect manifest itself in a many-body system?

More tractable: Impurity interacting strongly with weakly interacting gas



Bose polaron: experiments

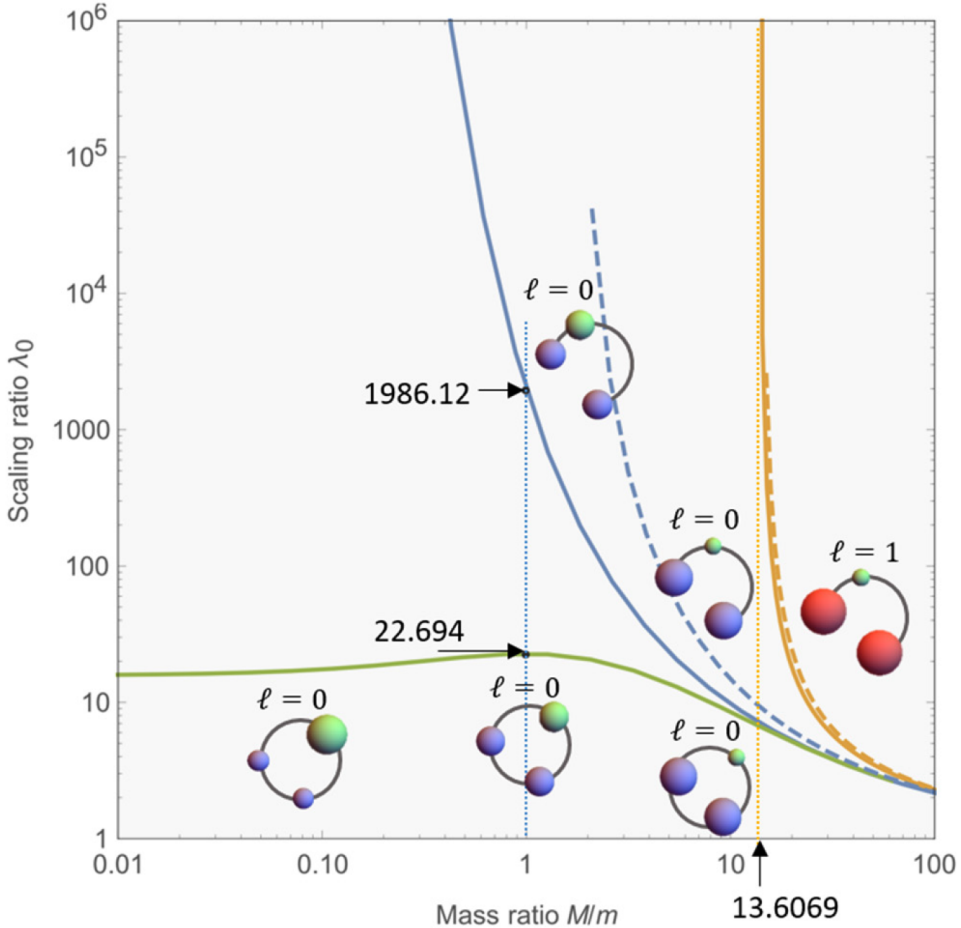
Cold atom experiments: rf injection spectroscopy

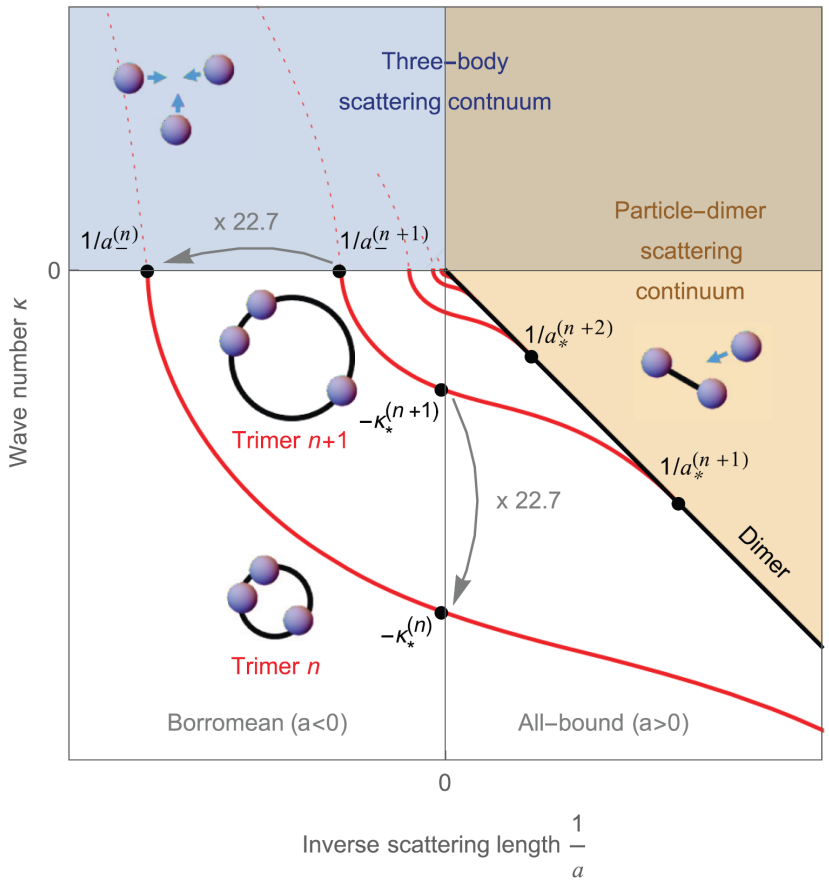


Jørgensen et al., Phys. Rev. Lett. 117 (2016) 055302
Hu, Cornell et al., Phys. Rev. Lett. 117 (2016), 055301

The HETERONUCLEAR Efimov effect

For the heteronuclear Efimov effect, the mass ratio is very important!

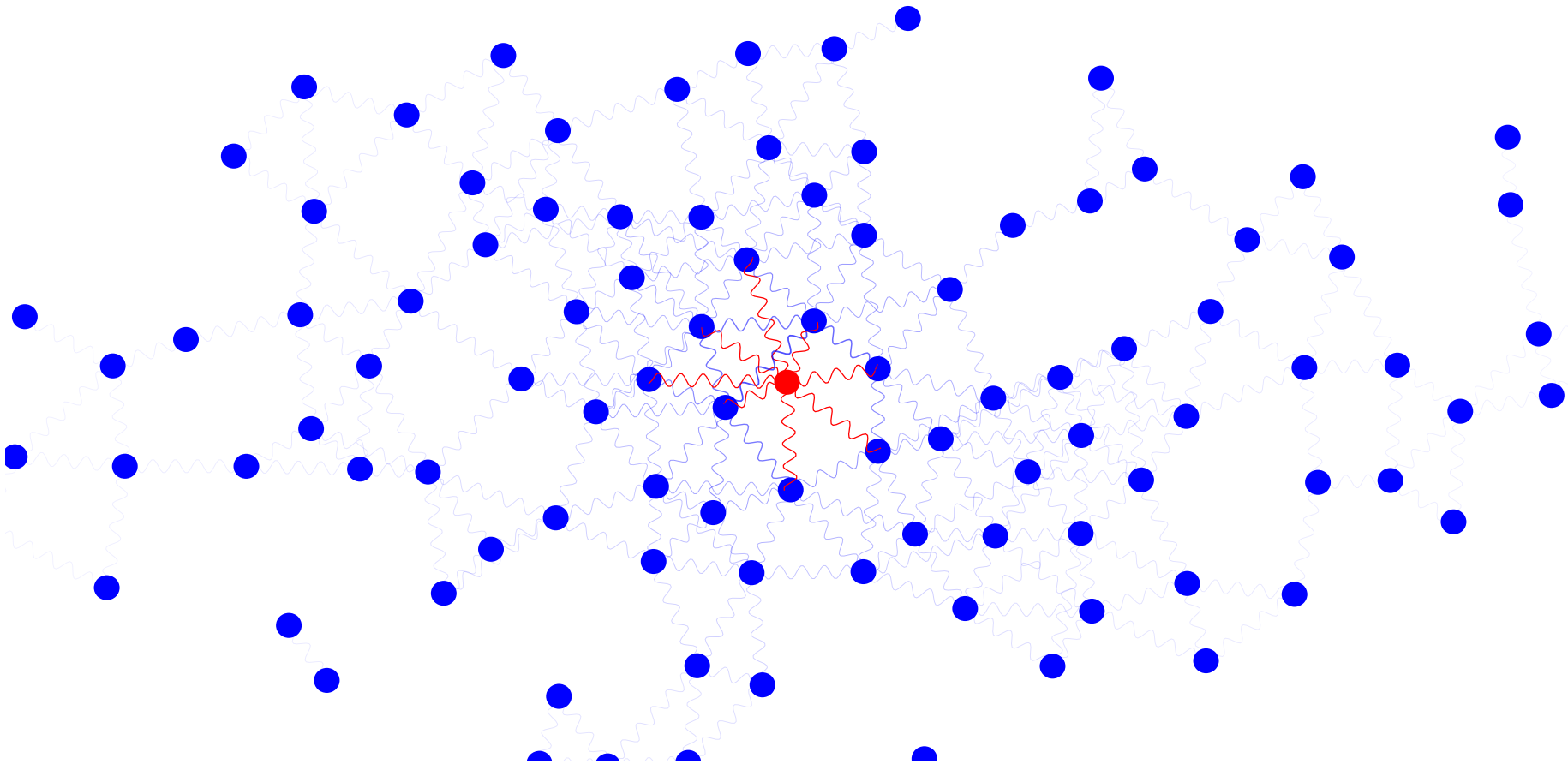




Pascal Naidon and Shimpei Endo, Rep. Prog. Phys. 80 (2017)

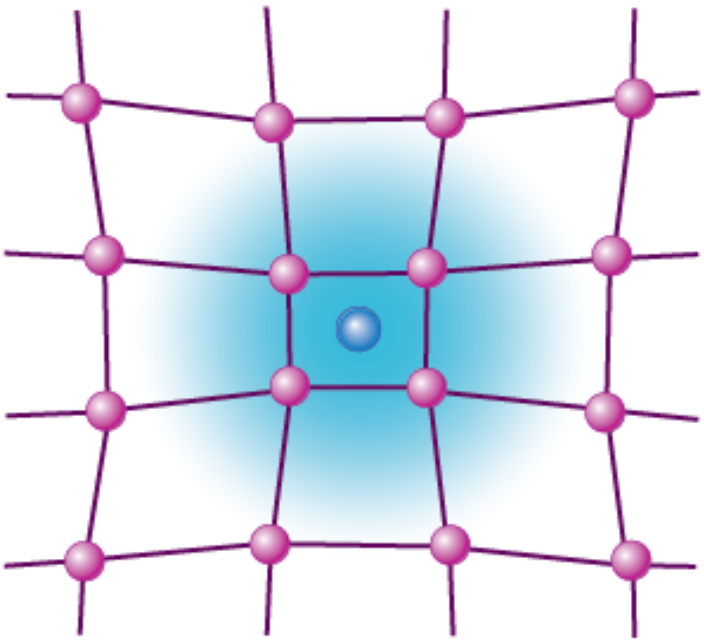
Efimov effect in a medium!

Correlations are localized around the impurity, making it more tractable



Traditional polarons in condensed matter

Electron dressed by phonons



Typically described by Fröhlich model

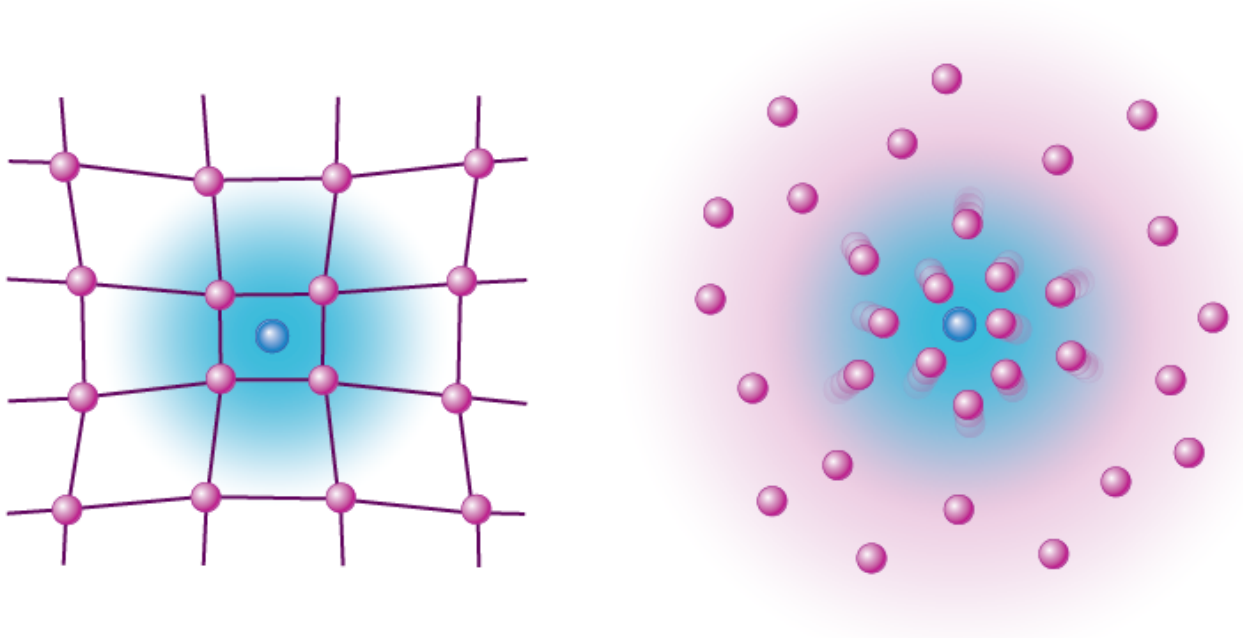
$$\hat{\mathcal{H}}_F = \sum_i \omega_i \hat{b}_i^\dagger \hat{b}_i + \frac{\hat{\boldsymbol{P}}^2}{2M} + \int d\boldsymbol{r}\, V(\boldsymbol{r} - \hat{\boldsymbol{R}})(\hat{b}_{\boldsymbol{r}}^\dagger + \hat{b}_{\boldsymbol{r}})$$

Does not allow bound states!

Adapted from F. Chevy, Physics 9, 86 (2016), picture by APS/Carin Cain

Bose polarons

Bose polaron: at weak coupling similar to condensed matter



Beyond-Fröhlich Hamiltonian: very different at strong coupling

$$\hat{\mathcal{H}} = \sum_i \omega_i \hat{b}_i^\dagger \hat{b}_i + \frac{\hat{\boldsymbol{P}}^2}{2M} + \int d\boldsymbol{r}\, V(\boldsymbol{r} - \hat{\boldsymbol{R}})[n_0 + \sqrt{n_0}(\hat{b}_{\boldsymbol{r}}^\dagger + \hat{b}_{\boldsymbol{r}}) + \hat{b}_{\boldsymbol{r}}^\dagger \hat{b}_{\boldsymbol{r}}]$$

F. Chevy, Physics 9, 86 (2016), picture by APS/Carin Cain

Hamiltonian and Lee-Low-Pines transformation

Hamiltonian (omitting interboson repulsion):

$$\hat{\mathcal{H}}_0 = \int d\boldsymbol{k} \frac{\boldsymbol{k}^2}{2m} \hat{b}^\dagger_{\boldsymbol{k}} \hat{b}_{\boldsymbol{k}} + \frac{\hat{\boldsymbol{P}}^2}{2M} + \int d\boldsymbol{r} V(\boldsymbol{r} - \hat{\boldsymbol{R}})(\hat{b}^\dagger_{\boldsymbol{r}} + \sqrt{n_0})(\hat{b}_{\boldsymbol{r}} + \sqrt{n_0})$$

Lee-Low-Pines transformation: use bosonic momentum as generator to transform to impurity frame:

$$\hat{U}_{LLP} = \exp(-i\hat{\boldsymbol{R}} \int d\boldsymbol{k}\ \boldsymbol{k}\ \hat{b}^\dagger_{\boldsymbol{k}} \hat{b}_{\boldsymbol{k}})$$

Apply to Hamiltonian:

$$\hat{\mathcal{H}} = \hat{U}^\dagger_{LLP} \hat{\mathcal{H}} \hat{U}_{LLP} = \int d\boldsymbol{k}\ \frac{k^2}{2m} \hat{b}^\dagger_{\boldsymbol{k}} \hat{b}_{\boldsymbol{k}} + \frac{(\boldsymbol{P} - \int d\boldsymbol{k}\ \boldsymbol{k}\ \hat{b}^\dagger_{\boldsymbol{k}} \hat{b}_{\boldsymbol{k}})^2}{2M} + \int d\boldsymbol{r} V(\boldsymbol{r})(\hat{b}^\dagger_{\boldsymbol{r}} + \sqrt{n_0})(\hat{b}_{\boldsymbol{r}} + \sqrt{n_0}).$$

Hamiltonian and Lee-Low-Pines transformation

$$\hat{\mathcal{H}} = \hat{U}_{LLP}^{\dagger}\hat{\mathcal{H}}\hat{U}_{LLP} = \int d\boldsymbol{k}\ \frac{k^2}{2m}\hat{b}_{\boldsymbol{k}}^{\dagger}\hat{b}_{\boldsymbol{k}} + \frac{(\boldsymbol{P} - \int d\boldsymbol{k}\ \boldsymbol{k}\ \hat{b}_{\boldsymbol{k}}^{\dagger}\hat{b}_{\boldsymbol{k}})^2}{2M} + \int d\boldsymbol{r} V(\boldsymbol{r})(\hat{b}_{\boldsymbol{r}}^{\dagger} + \sqrt{n_0})(\hat{b}_{\boldsymbol{r}} + \sqrt{n_0}).$$

Set $\boldsymbol{P} = 0$ and normal-order quartic term:

$$\hat{\mathcal{H}} = \int d\boldsymbol{k}\ \frac{k^2}{2\mu_r}\hat{b}_{\boldsymbol{k}}^{\dagger}\hat{b}_{\boldsymbol{k}} + \frac{1}{2M}\int\int d\boldsymbol{k}d\boldsymbol{k}'\ (\boldsymbol{k}\cdot\boldsymbol{k}')\hat{b}_{\boldsymbol{k}}^{\dagger}\hat{b}_{\boldsymbol{k}'}^{\dagger}\hat{b}_{\boldsymbol{k}}\hat{b}_{\boldsymbol{k}'} + \int d\boldsymbol{r} V(\boldsymbol{r})(\hat{b}_{\boldsymbol{r}}^{\dagger} + \sqrt{n_0})(\hat{b}_{\boldsymbol{r}} + \sqrt{n_0}).$$

Impurity degrees of freedom are eliminated at the cost of a nonlocal interaction term.

Variational methods

Double excitation Ansatz:

$$|\psi_{dc}\rangle = \beta_0 + \int d\boldsymbol{k}\beta_{\boldsymbol{k}}\hat{b}^\dagger_{\boldsymbol{k}} + \int\int d\boldsymbol{k}d\boldsymbol{k'}\ \alpha_{\boldsymbol{k},\boldsymbol{k'}}\hat{b}^\dagger_{\boldsymbol{k}}\hat{b}^\dagger_{\boldsymbol{k'}}|BEC\rangle$$

Limits number of excitations

J. Levinsen, M. M. Parish, and G. M. Bruun, Phys. Rev. Lett. 115 (2015)

S. M. Yoshida, S. Endo, J. Levinsen, and M. M. Parish, Phys. Rev. X 8 (2018)

Gaussian state Ansatz

$$|\psi_{GS}\rangle = N\exp(\int_{\boldsymbol{k}}[\phi_{\boldsymbol{k}}\hat{b}^\dagger_{\boldsymbol{k}} - \phi^*_{\boldsymbol{k}}\hat{b}_{\boldsymbol{k}}])\exp(\int_{\boldsymbol{k}}\int_{\boldsymbol{k'}}\frac{\xi_{\boldsymbol{k},\boldsymbol{k'}}}{2}\hat{b}^\dagger_{\boldsymbol{k}}\hat{b}^\dagger_{\boldsymbol{k'}})|BEC\rangle,$$

Limits order of correlations

AC, J.I. Cirac, R.Schmidt, Phys. Rev. Lett. 128 (2022)
AC, J.I. Cirac, R.Schmidt, Phys. Rev. A 105 (2022)

Bose polaron with coherent states

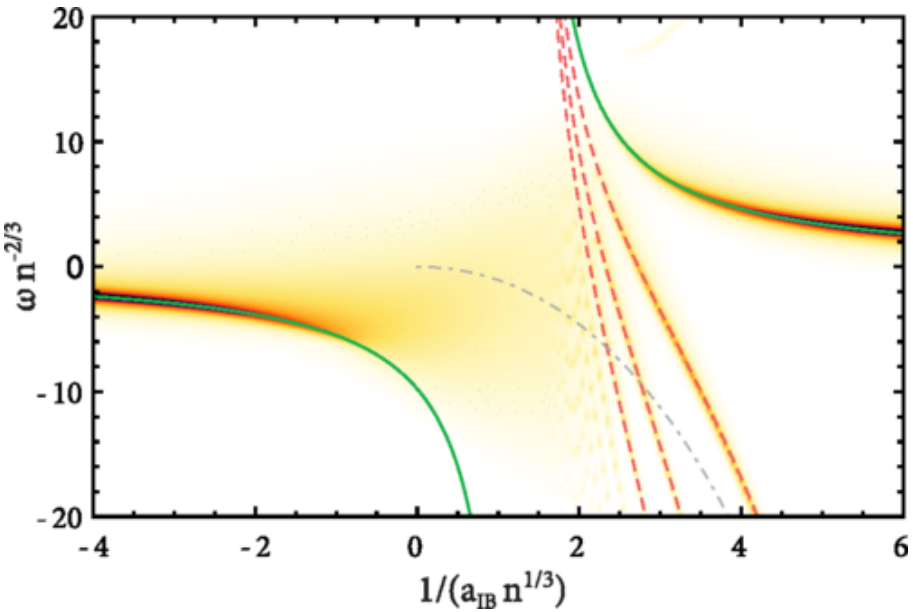
When $\xi = 0$:

$$E_{coh} = \frac{2\pi n_0}{\mu_r (a^{-1} - a_{shift}^{-1})}$$

Bound state appears from continuum
$\rightarrow$ Energy & particle number diverge
$\rightarrow$ Shift from unitarity due to mean-field repulsion

Self-stabilization required by including interboson repulsion fully

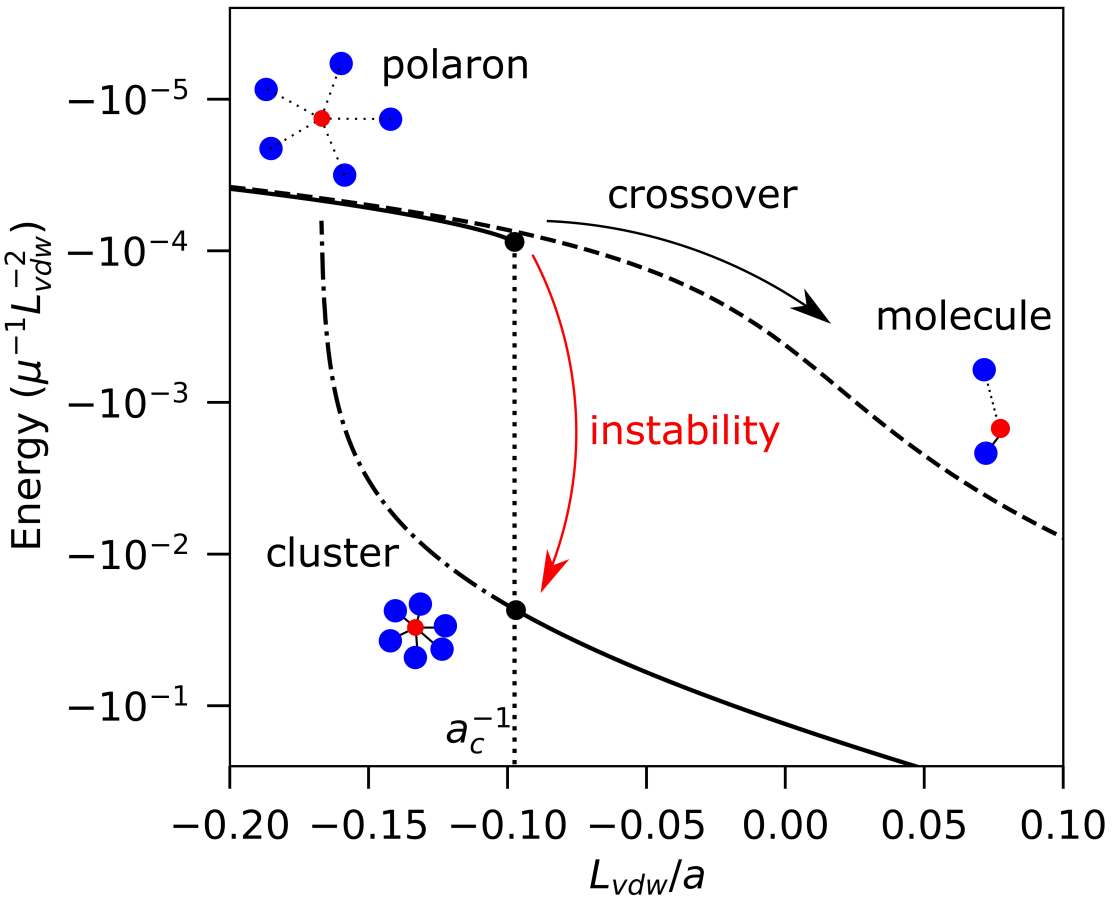
N.E. Guenther *et al.* PRA 103, 013317 (2021)
R. Schmidt, T. Enss, SciPost Physics 13.3 (2022)



Y. E. Shchadilova, *et al.* PRL 117, 113002 (2016)

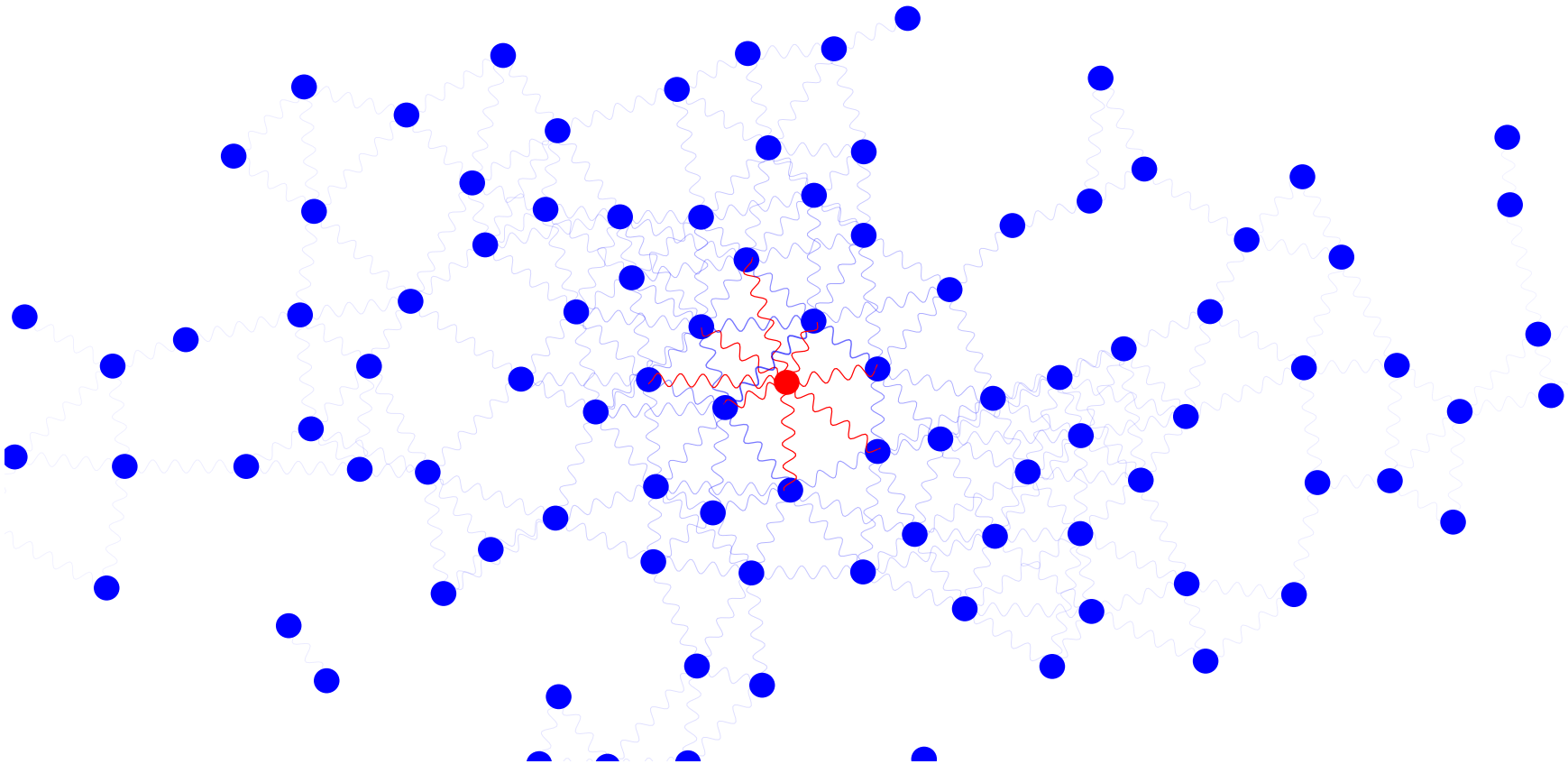
Qualitative picture

Variational methods predict qualitatively different behaviour!



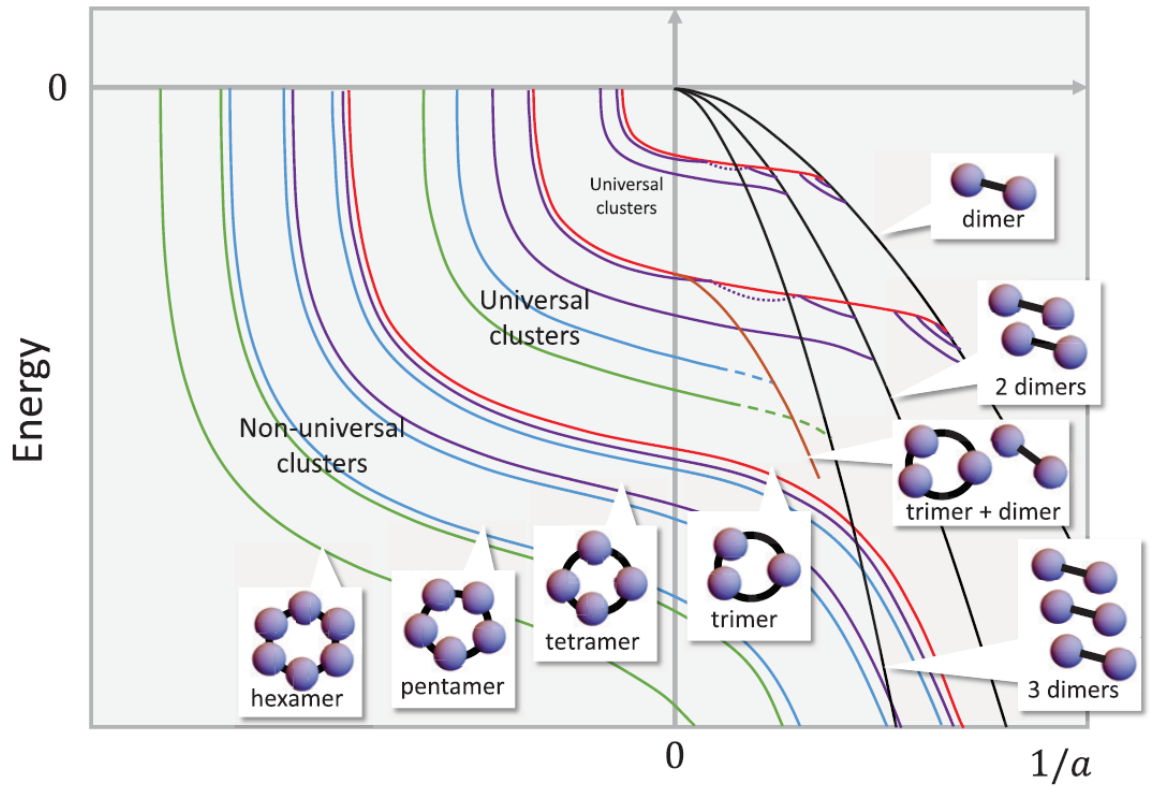
AC, J.I. Cirac, R.Schmidt, *in preparation*

Mediated interactions



The Efimov effect beyond three particles

Homonuclear case:



P. Naidon and S. Endo, Rep. Prog. Phys. 80 (2017)

The Efimov effect beyond three particles

Does the cooperative binding persist beyond three particles?

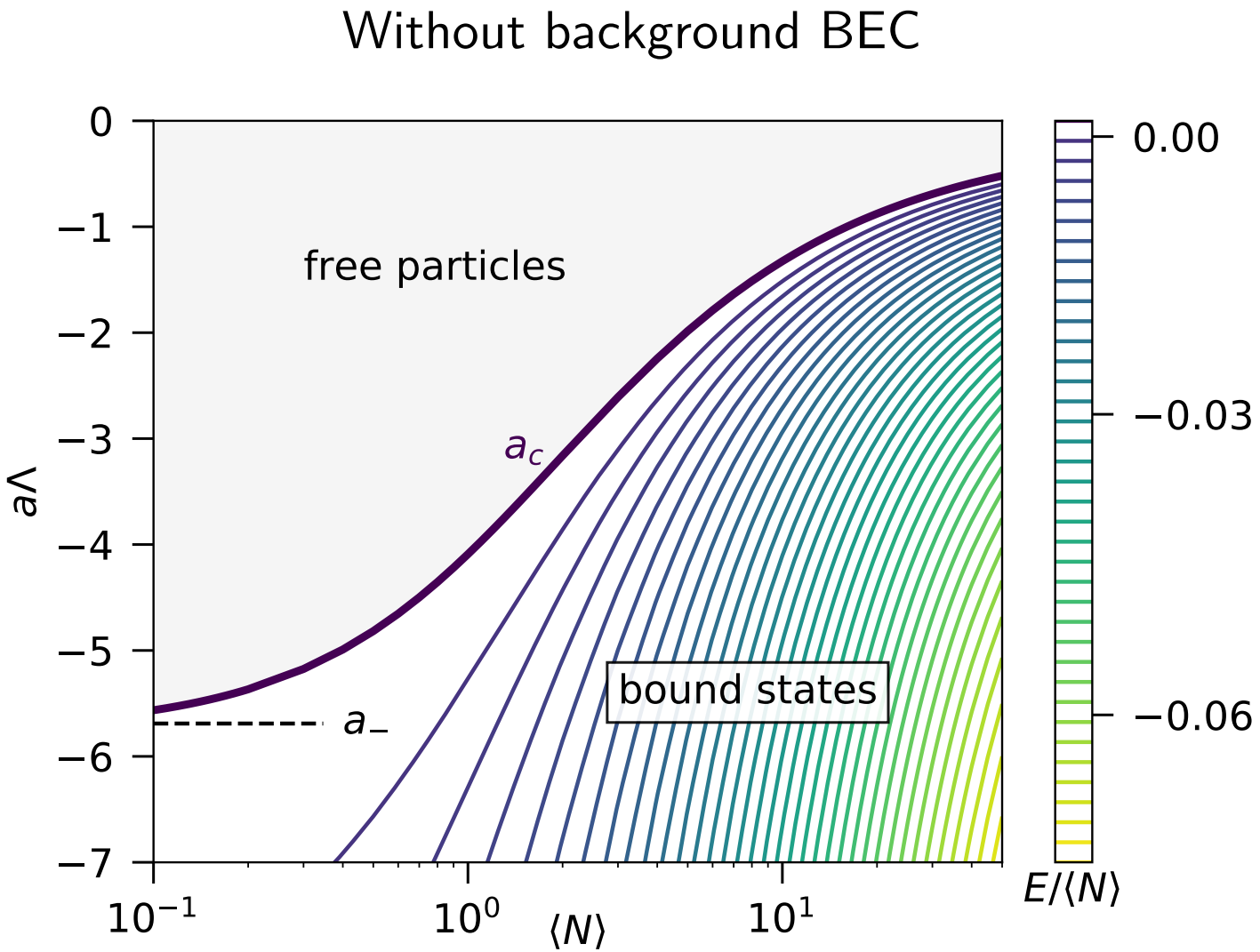
Trick: don't solve the N-body problem, but the $\langle \hat{N} \rangle$-body problem with pairwise correlations.

Gaussian State:

$$|GS\rangle = \exp(\frac{1}{2}\hat{b}^\dagger \xi \hat{b}^\dagger)|0\rangle = c^{(0)}|0\rangle + \sum_{i2} c^{(2)}_{i2}|2\rangle_{i2} + \sum_{i4} c^{(4)}_{i4}|4\rangle_{i4} \ldots$$

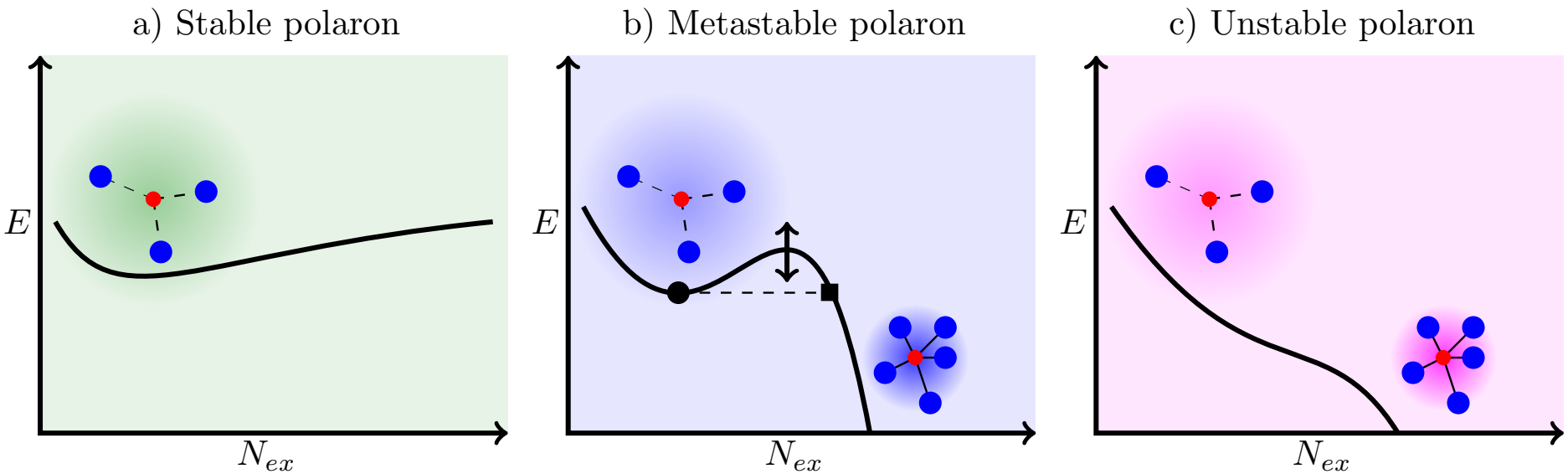
Fix $\langle \hat{N} \rangle$ of Gaussian State, by including a dynamically changing chemical potential.

Cooperative binding



AC, JI Cirac, R Schmidt, Phys. Rev. Lett. 128, 183401 (2022)
AC, JI Cirac, R Schmidt, Phys. Rev. A 105, 053302 (2022)

Results: polaronic instability



AC, JI Cirac, R Schmidt, Phys. Rev. A 105, 053302 (2022)

Polaronic instability: shifted Efimov resonance

Variational Ansatz:

$$|\psi\rangle = N \exp\Big(\int_{\boldsymbol{k}} [\phi_{\boldsymbol{k}} \hat{b}^\dagger_{\boldsymbol{k}} - \phi^*_{\boldsymbol{k}} \hat{b}_{\boldsymbol{k}}]\Big) \exp\Big(\frac{1}{2} \int_{\boldsymbol{k}} \int_{\boldsymbol{k}'} \hat{b}^\dagger_{\boldsymbol{k}} \xi_{\boldsymbol{k},\boldsymbol{k}'} \hat{b}^\dagger_{\boldsymbol{k}'}\Big) |BEC\rangle,$$

Instability occurs when small perturbation can lower energy.

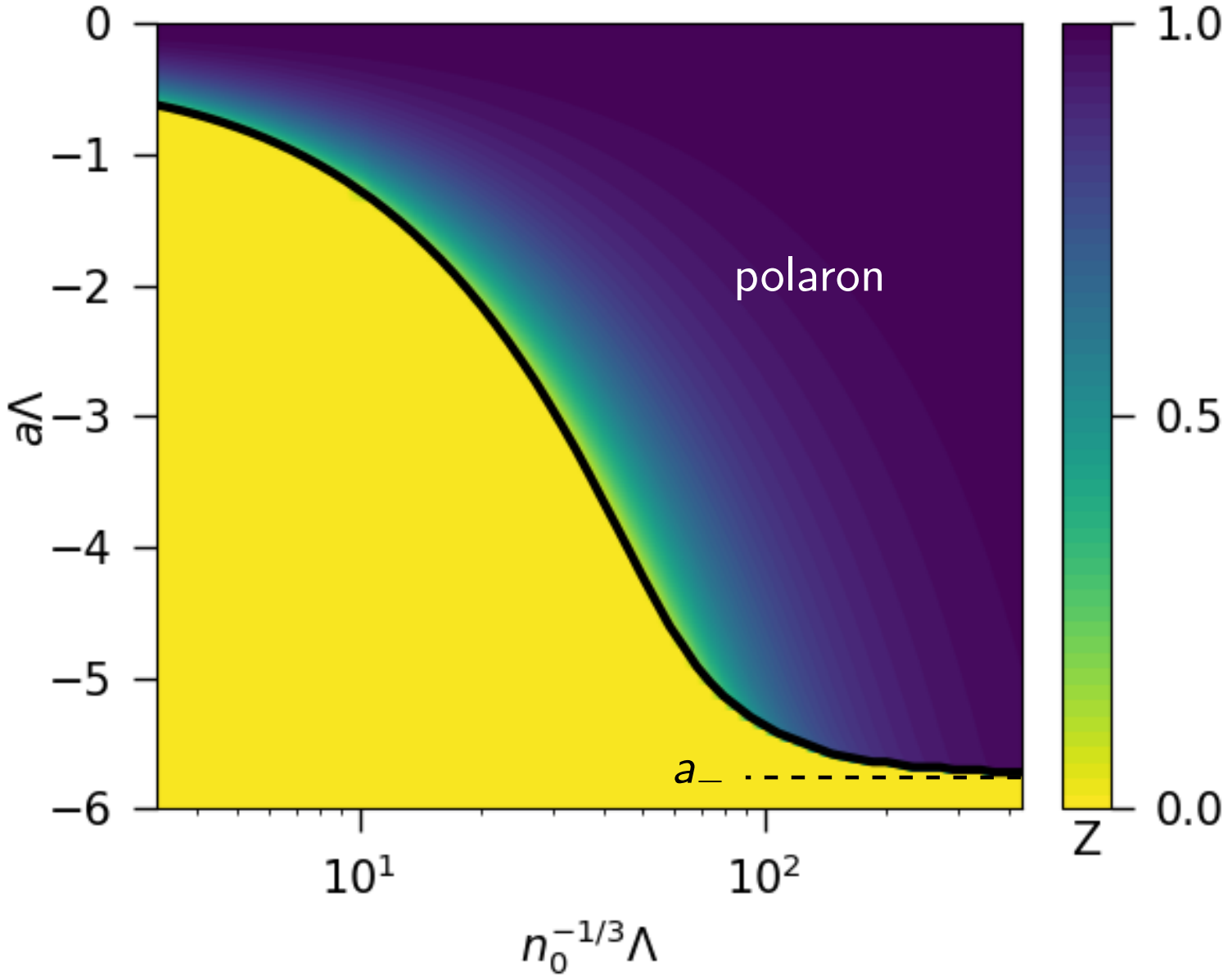
Tangent vectors on variational manifold:

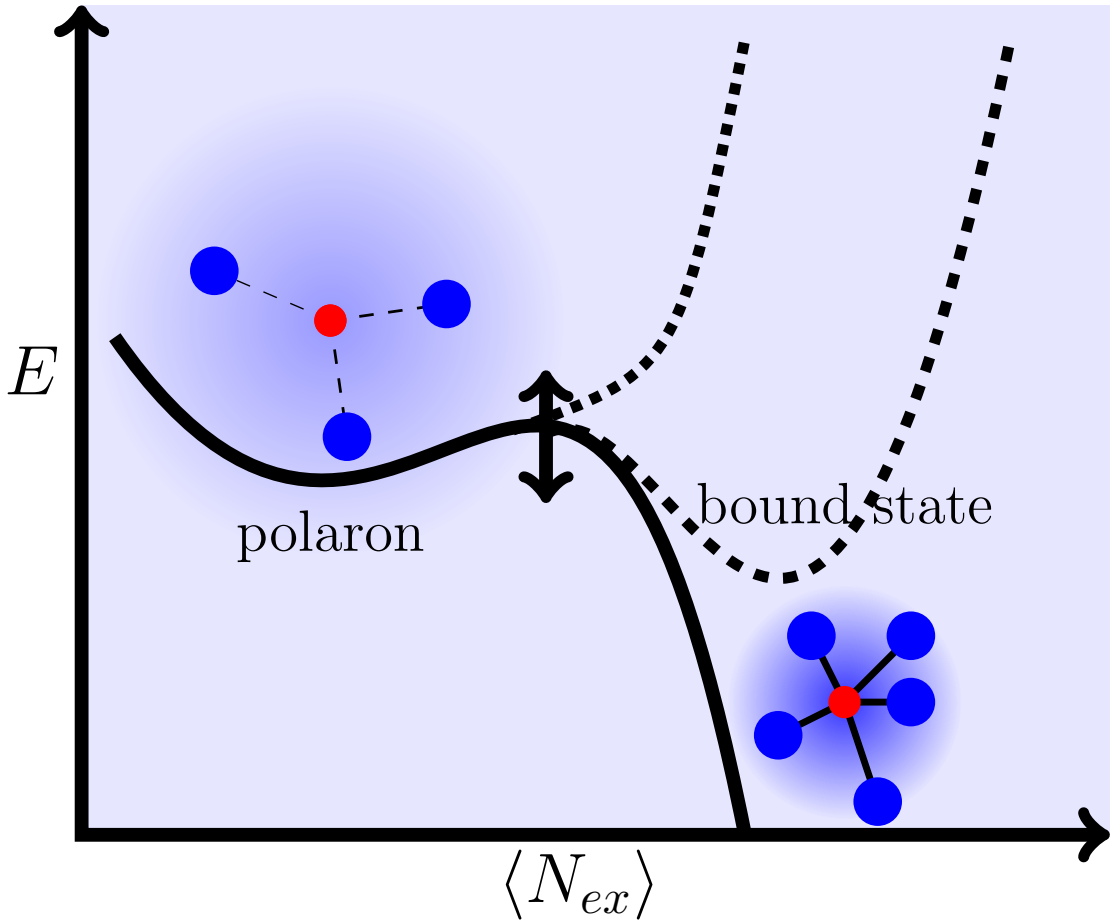
$$\frac{\partial |\psi\rangle}{\partial \phi_{\boldsymbol{k}}} \approx c \hat{b}^\dagger_{\boldsymbol{k}} |\psi\rangle$$

$$\frac{\partial |\psi\rangle}{\partial \xi_{\boldsymbol{k},\boldsymbol{k}'}} \approx c \hat{b}^\dagger_{\boldsymbol{k}} \hat{b}^\dagger_{\boldsymbol{k}'} |\psi\rangle$$

$\rightarrow$ instability when adding one or two particles to polaron can lead to bound state formation!

Polaronic instability: shifted Efimov resonance



interparticle distance

Including repulsion