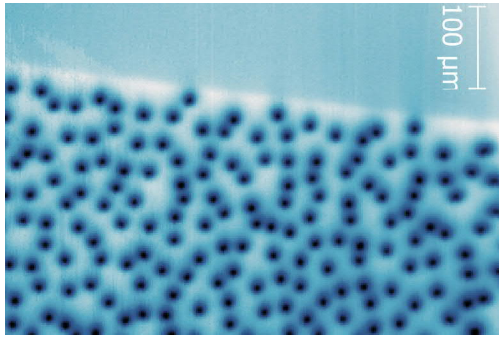
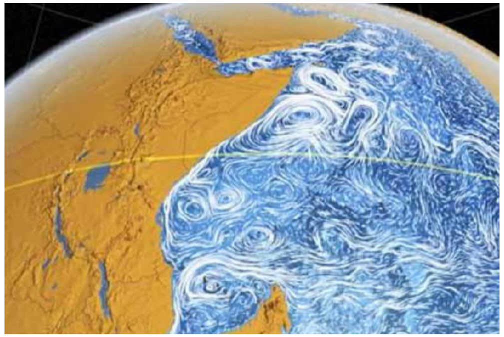
Andrea Richaud

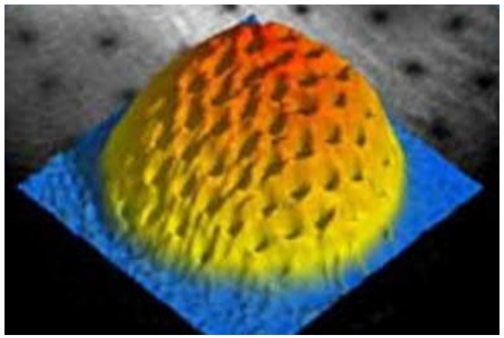
Massive vortices in a binary mixture of Bose-Einstein condensates

Barcelona, May 6th, 2022

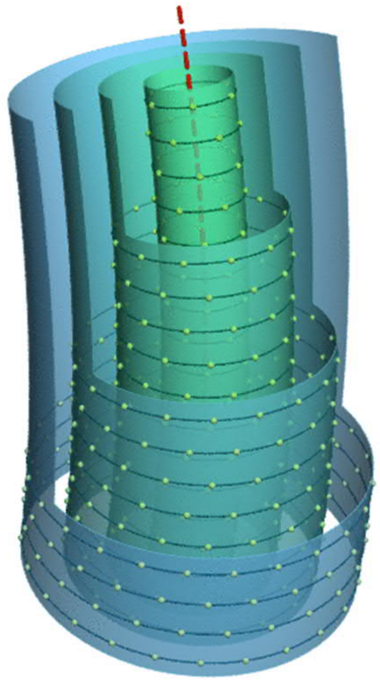
Vortices are everywhere







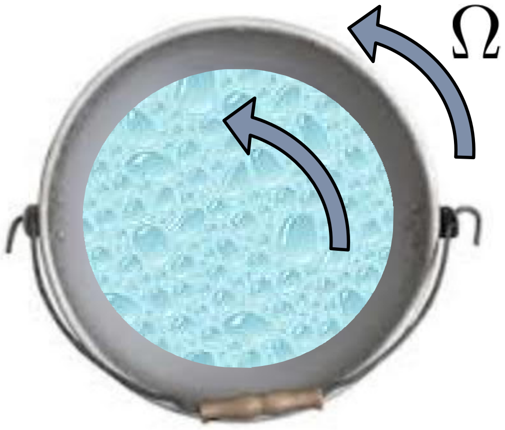
What is a vortex?



A **vortex** is a region in a fluid in which the flow revolves around an axis line.

Rotating buckets of normal/superfluids

A quantum fluid **CANNOT rotate around its own axis** as if it was a rigid body!



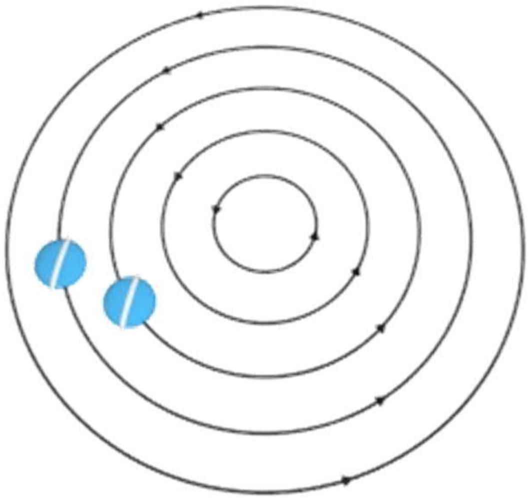
Normal fluid

Quantum fluid

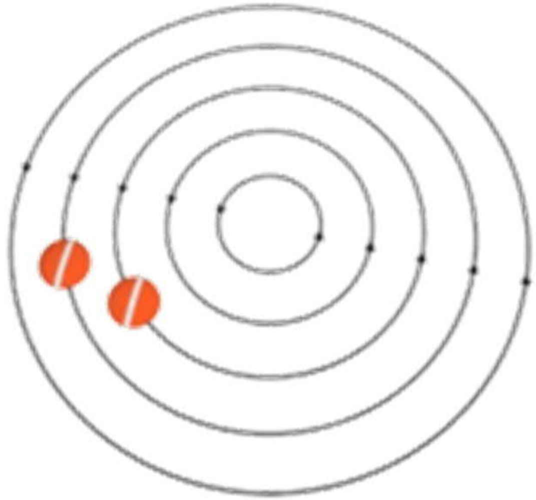
A.J. Leggett, Rev. Mod. Phys. **73**, 307 (2001)

Rotational vs Irrotational vortices

Rotational vortex

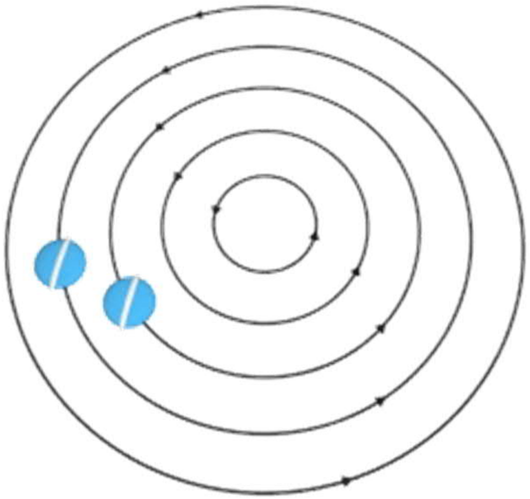


Irrotational vortex



Rotational vs Irrotational vortices

Rotational vortex



- What you get if you spin a **water bucket**!
- **Rigid-body**-like rotation:

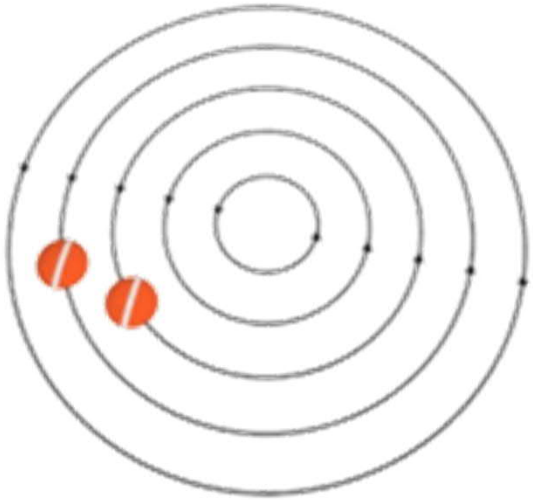
$$v = \Omega r$$

- A tiny rough tracer ball carried by the flow rotates around its center.
- **Vorticity** is the same everywhere:

$$\vec{w} := \mathrm{curl}(\vec{v}) = 2\vec{\Omega}$$

Rotational vs Irrotational vortices

Irrotational vortex



- What you get if you spin a **bucket of superfluid**!
- Velocity is inversely proportional to distance from axis:

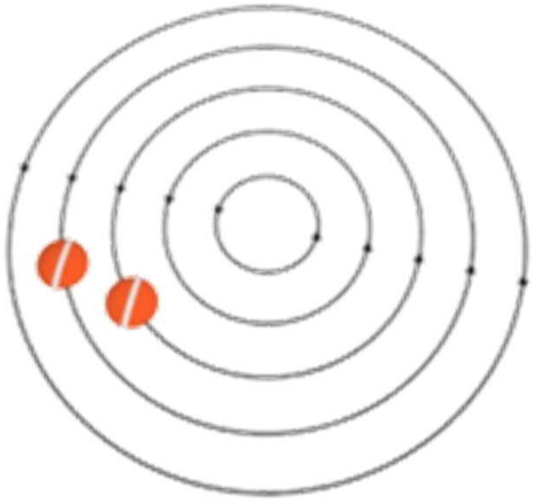
$$v \sim r^{-1}$$

- A tiny rough tracer ball carried by the flow keeps its orientation constant while orbiting.
- **Vorticity** vanishes everywhere but at the origin:

$$\vec{w} := \mathrm{curl}(\vec{v}) \propto \delta^2(\vec{r} - \vec{r}_0)$$

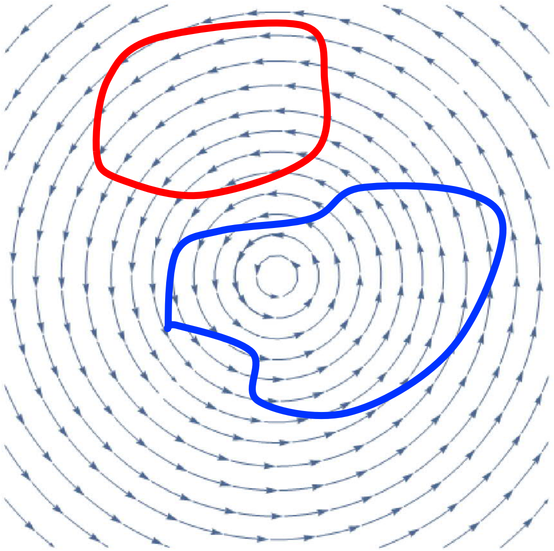
Rotational vs Irrotational vortices

Irrotational vortex



Quantum fluids cannot rotate as rigid bodies and angular momentum can enter only in the form of **irrotational vortices**.

Vortex as a topological excitation



$$\vec{v}(\vec{r}) = \frac{k}{2\pi}\hat{e}_3 \wedge \frac{\vec{r}}{r^2}$$

Consider the circulation of the velocity vector field along a closed path γ

$$\oint_\gamma \vec{v}\cdot \mathrm{d}\vec{r} = 0$$

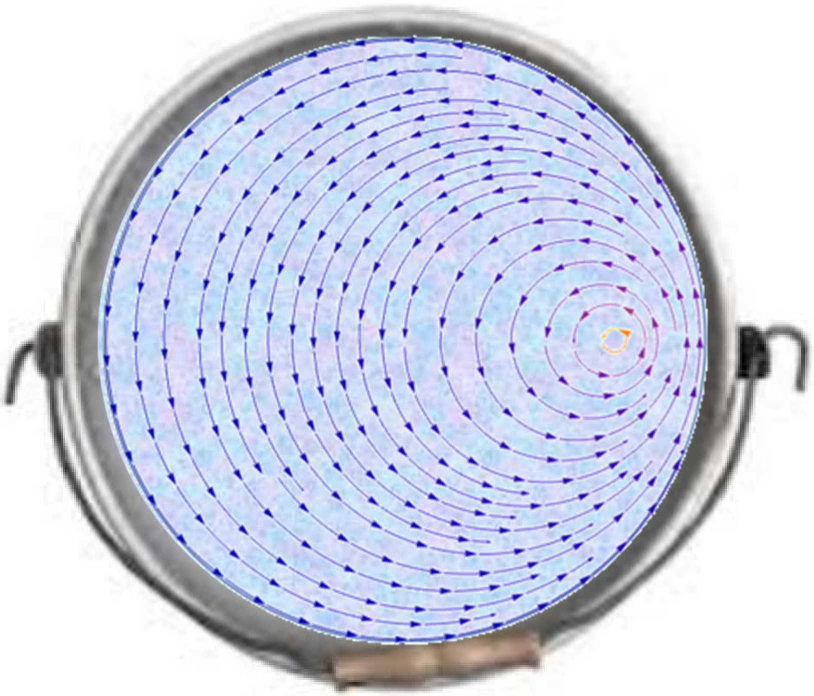
if the path **does NOT** encircle the singularity

$$\oint_\gamma \vec{v}\cdot \mathrm{d}\vec{r} = \boxed{n\frac{h}{m}}$$

if the path **does** encircle the singularity

This topological label is the vortex strength, where $n \in \mathbb{Z}$.

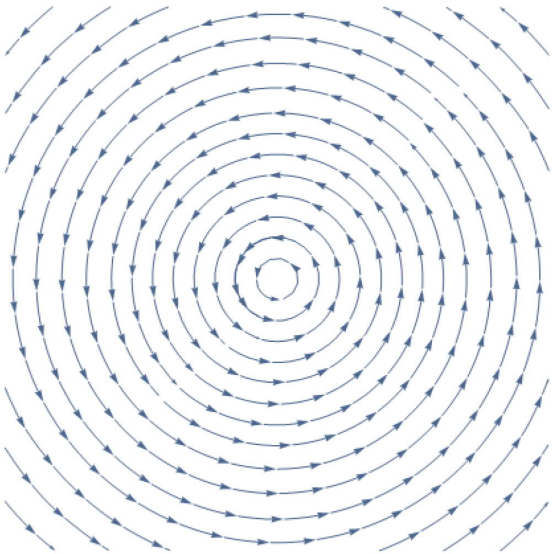
Point-like vortex dynamics



How does a superfluid vortex move?

<u>Assumptions:</u>
1) Vortex size << box size;
2) Vortex size << inter-vortex distance.

One solitary vortex does not move



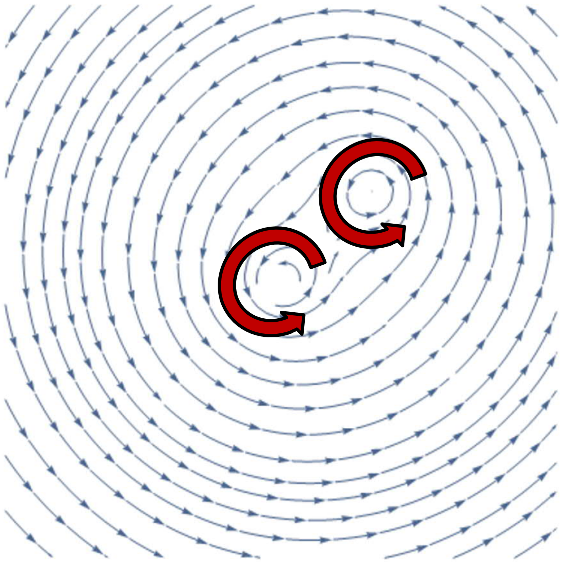
If you have just one vortex in an ideal unbounded sample

$\Downarrow$

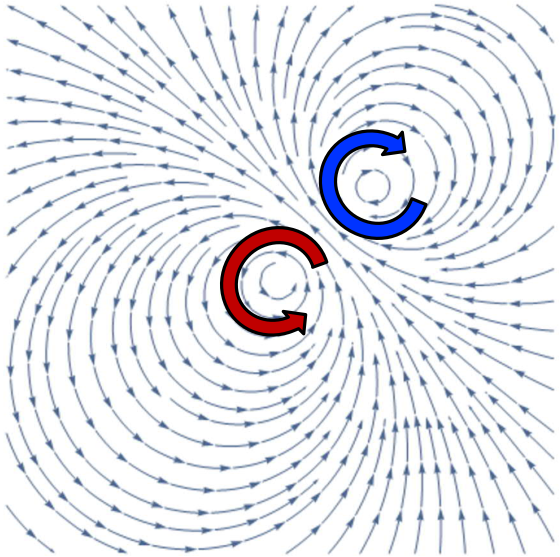
It stays at rest.

Two-vortex systems in unbounded domain

<u>Superposition</u> of the velocity fields generated by each vortex



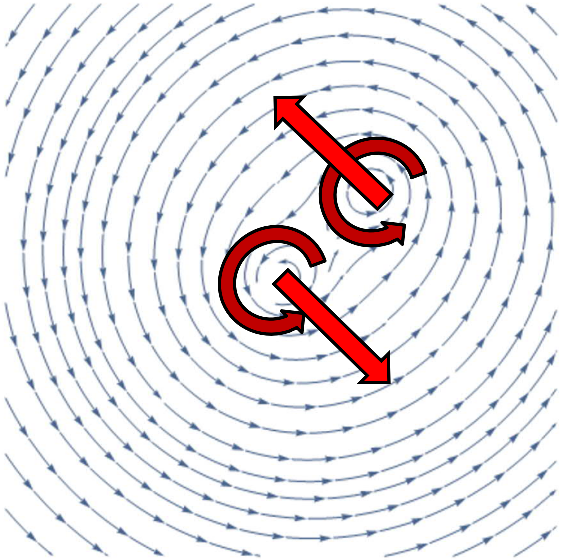
Two **CO-rotating** vortices



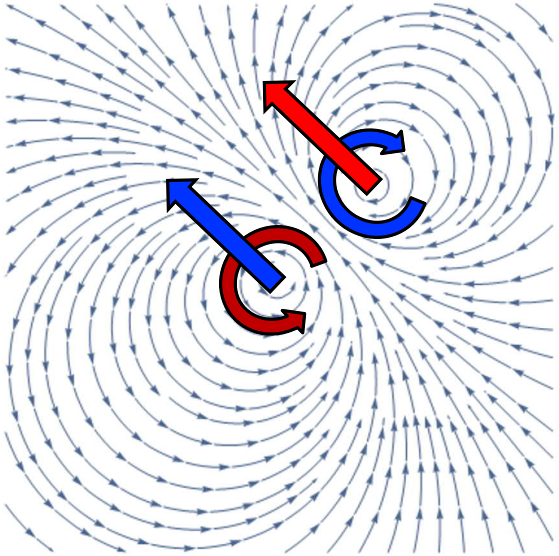
Two **COUNTER-rotating** vortices

Two-vortex systems in unbounded domain

Superposition of the velocity fields generated by each vortex

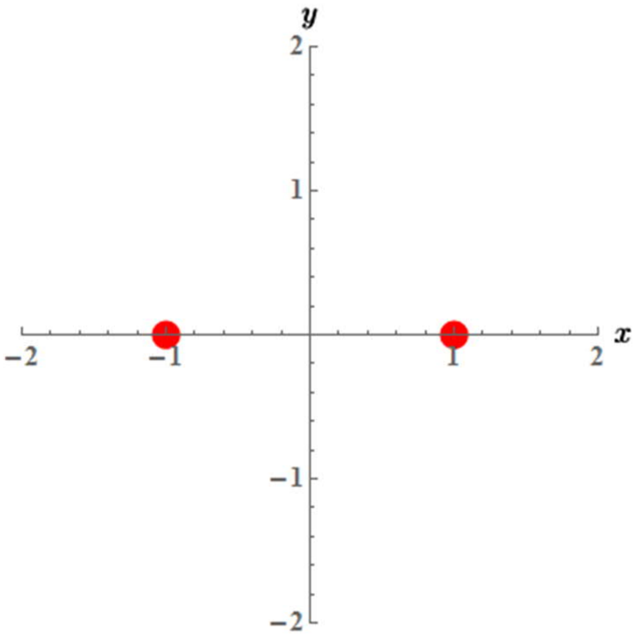


Precession

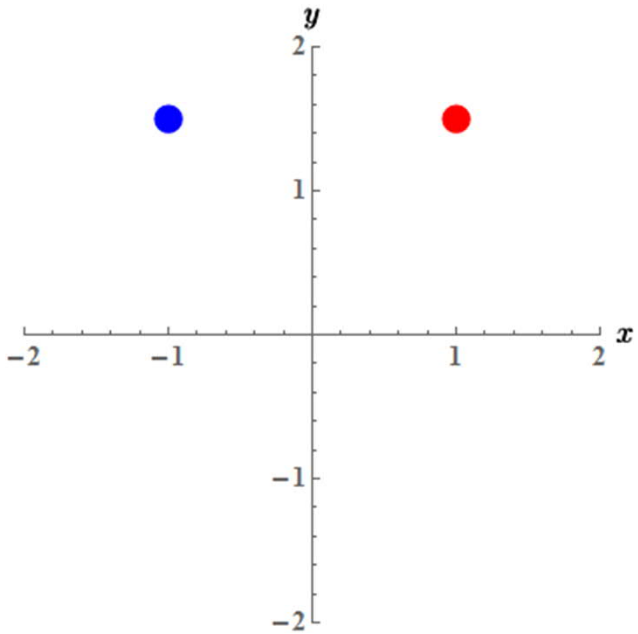


Translation

Two-vortex systems in unbounded domain

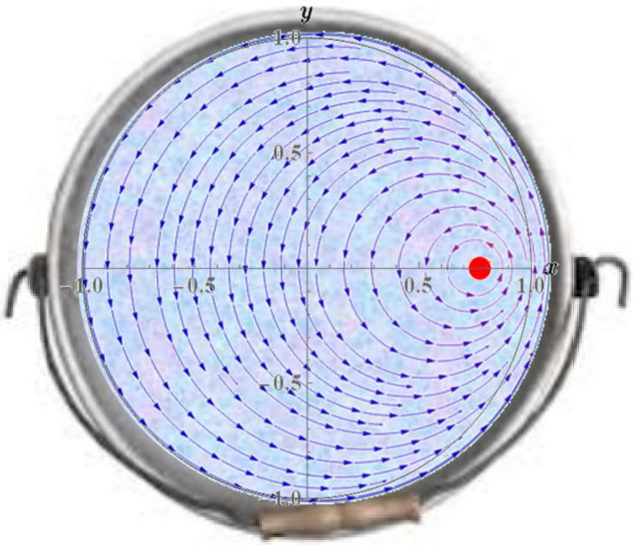


Two **CO-rotating** vortices
Precession

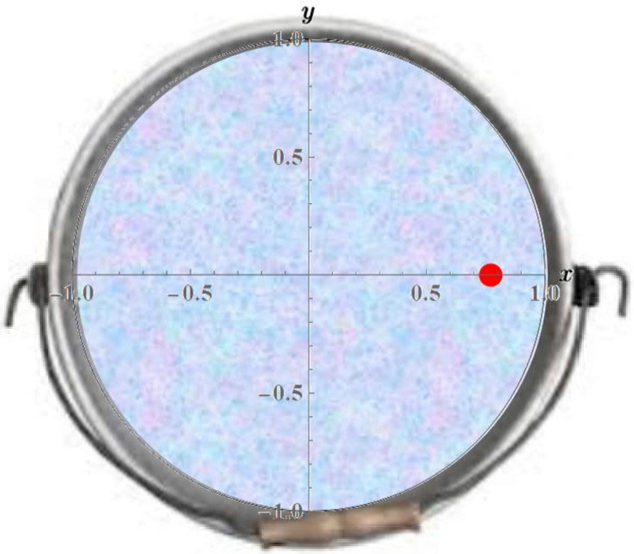


Two **COUNTER-rotating** vortices
Translation

Single vortex in a circular trap



Single vortex in a circular trap

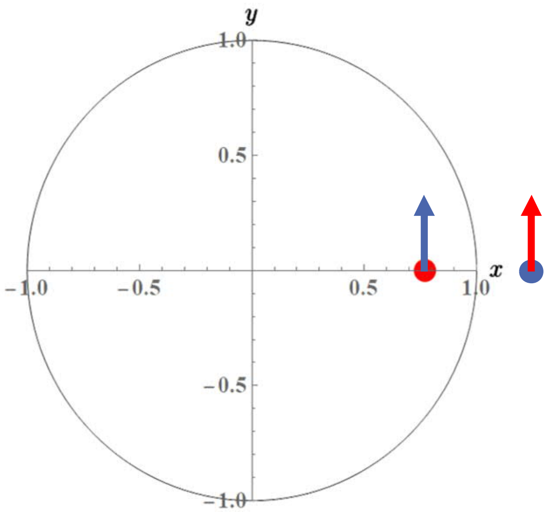


Uniform circular motion:

$$\dot{\theta}_0 = \frac{\hbar}{m} \frac{1}{R^2 - r_0^2}$$

J. Kim and A. L. Fetter, Phys. Rev A **70**, 043624 (2004)

Single vortex in a circular trap



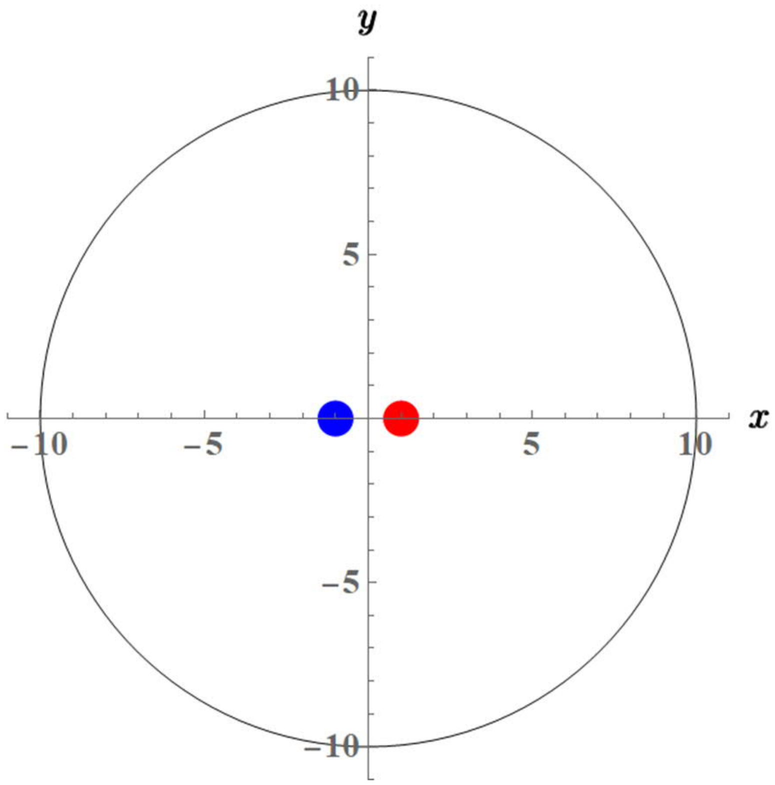
Presence of an "image vortex" at:

$$\begin{cases} x_0' = \dfrac{R^2}{x_0^2 + y_0^2} x_0 \\ y_0' = \dfrac{R^2}{x_0^2 + y_0^2} y_0 \end{cases}$$

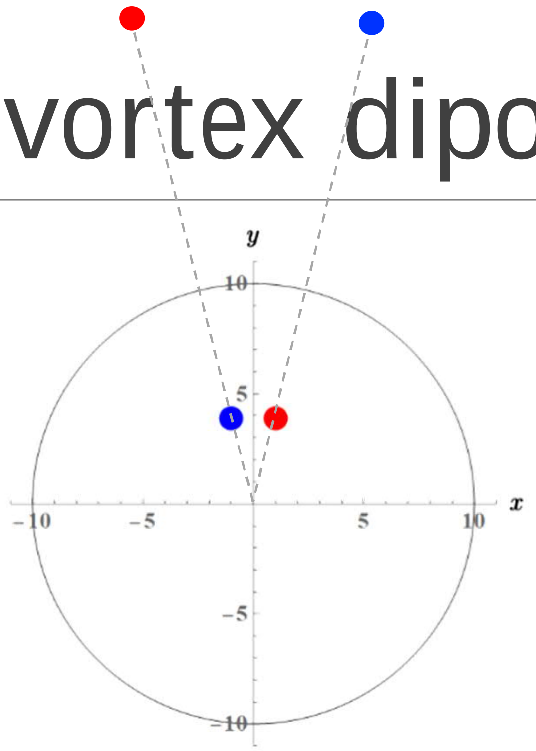
The real vortex orbits because it is effectively dragged by its image vortex.

J. Kim and A. L. Fetter, Phys. Rev. A **70**, 043624 (2004)

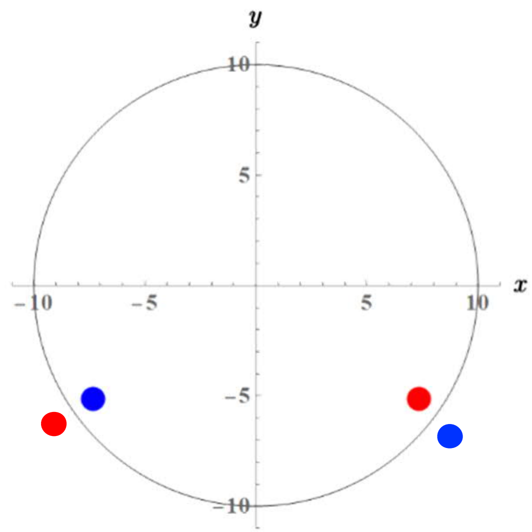
A vortex dipole in a circular trap



A vortex dipole in a circular trap

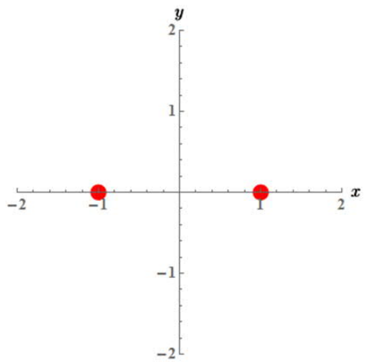


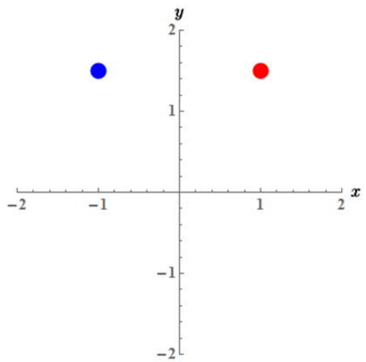
When vortices are far from the boundary, the interaction between real vortices prevail.



When vortices are close to the boundary, the interaction between a vortex and its image prevails.

Point-like Vortex Dynamics





The dynamics of M vortices in an ideal unbounded fluid is ruled by:

$$\boxed{H_\infty(\vec{r}_1,\ldots,\vec{r}_M) = -\frac{\rho_*}{4\pi}\sum_{i=1}^{M}\sum_{j\neq i} k_i k_j \ln\left(\frac{|\vec{r}_i-\vec{r}_j|}{\lambda}\right)}$$

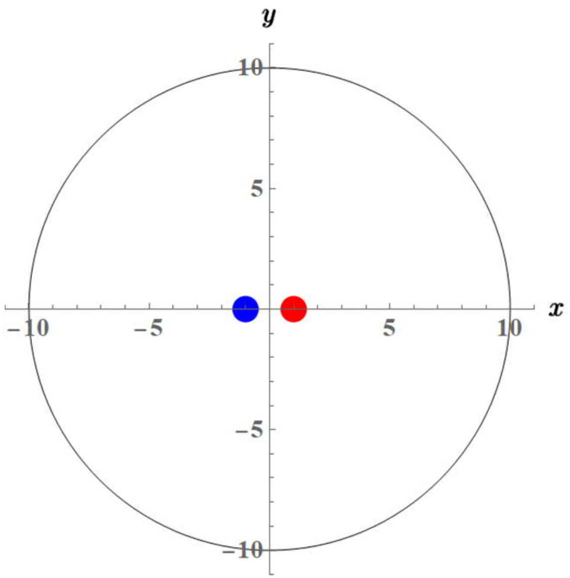
$$\{A,B\} = \sum_i \frac{1}{\rho_* k_i}\left[\frac{\partial A}{\partial x_i}\frac{\partial B}{\partial y_i} - \frac{\partial B}{\partial x_i}\frac{\partial A}{\partial y_i}\right]$$

$$\dot{x}_j = \{x_j, H\} = +\frac{1}{\rho^* k_j}\frac{\partial H}{\partial y_j},$$

$$\dot{y}_j = \{y_j, H\} = -\frac{1}{\rho^* k_j}\frac{\partial H}{\partial x_j}$$

The components of the position of each vortex constitute **canonically conjugate variables**.

Walls affect the trajectory of vortices



You can incorporate the presence of **walls** by adding some extra terms to the Hamiltonian:

$$H = \frac{\rho_*}{4\pi}\left[k_1 k_2 \log \frac{\left(R^2 - x_1 x_2 - y_1 y_2\right)^2 + (x_2 y_1 - x_1 y_2)^2}{R^2((x_1 - x_2)^2 + (y_1 - y_2)^2)} + \right.$$

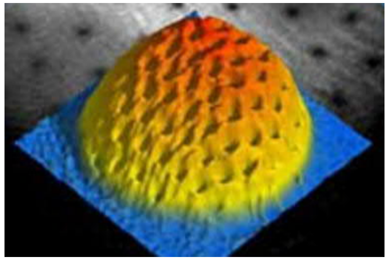
$$\left. + k_1^2 \log \frac{R^2 - x_1^2 - y_1^2}{R^2} + k_2^2 \log \frac{R^2 - x_2^2 - y_2^2}{R^2}\right]$$

$$\dot{x}_j = \{x_j, H\} = +\frac{1}{\rho^* k_j}\frac{\partial H}{\partial y_j},$$

$$\dot{y}_j = \{y_j, H\} = -\frac{1}{\rho^* k_j}\frac{\partial H}{\partial x_j}$$

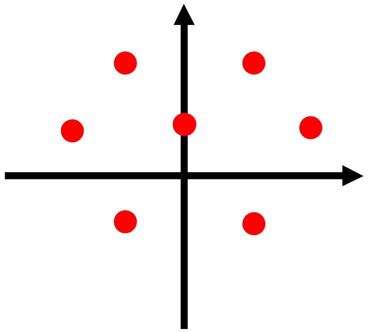
Derivation of the Point-Vortex model

How to go from the Gross-Piataevskii (GP) framework to an effective point-like model?



$$H = \int \left[-\frac{\hbar^2}{2m} \psi^* \Delta \psi + V(\vec{r})|\psi|^2 + \frac{g}{2}|\psi|^4 \right] \mathrm{d}^2 r$$

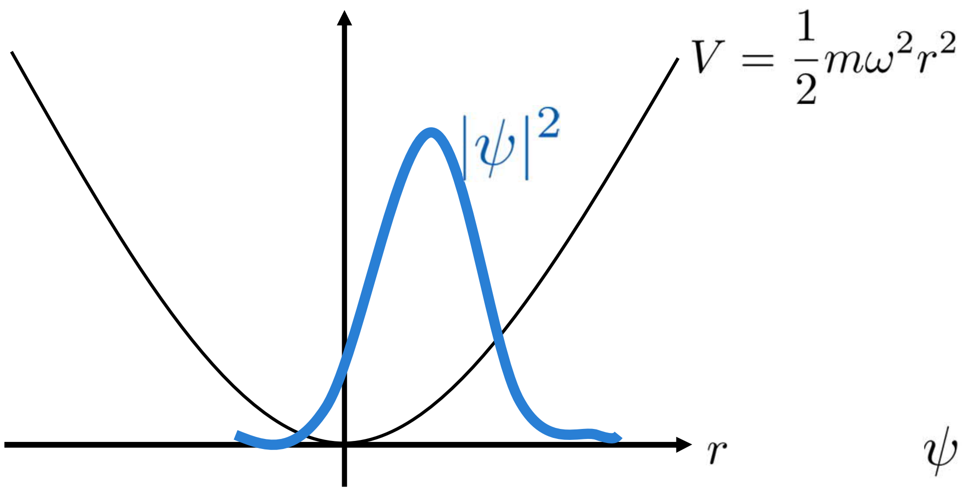
?



$$H_\infty(\vec{r}_1, \ldots, \vec{r}_M) = -\frac{\rho_*}{4\pi} \sum_{i=1}^{M} \sum_{j \neq i} k_i k_j \ln\left(\frac{|\vec{r}_i - \vec{r}_j|}{\lambda} \right)$$

Time-dependent variational Lagrangian (TDVL) approach

V. Perez-Garcia et al., Phys. Rev. Lett 77, 5320 (1996)

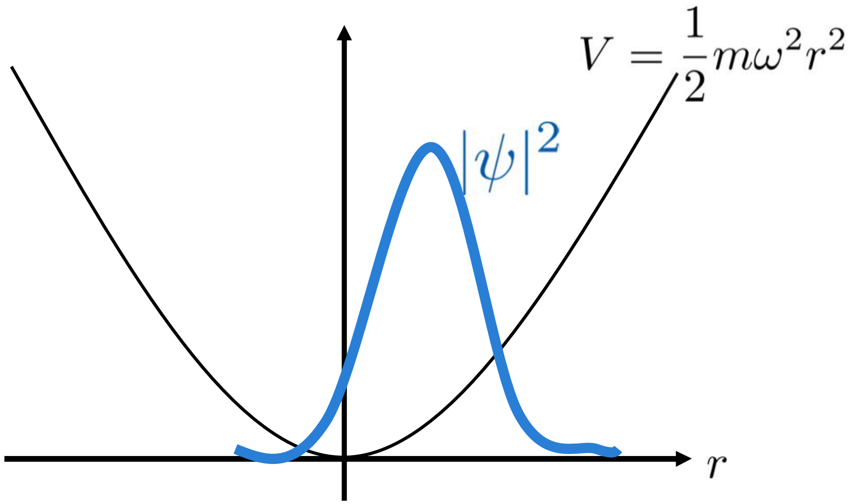


Instead of focusing on the full field $\psi(x,t)$, one can keep only some notable time-dependent variational parameters:

$$\psi(x,t) = A(t) e^{-\frac{[x - x_0(t)]^2}{2\sigma^2(t)} + i x \alpha(t) + i x^2 \beta(t)}$$

Time-dependent variational Lagrangian (TDVL) approach

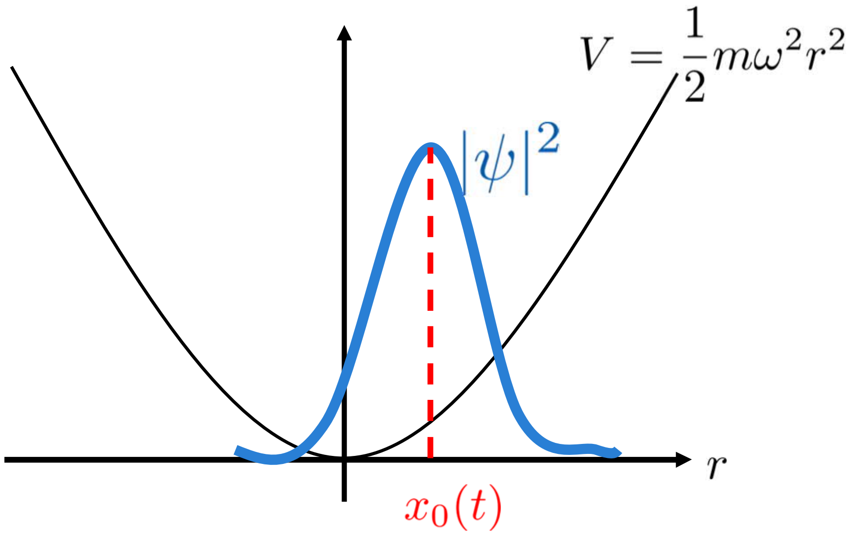
V. Perez-Garcia et al., Phys. Rev. Lett **77**, 5320 (1996)



$$\psi(x,t) = A(t)e^{-\frac{[x-x_0(t)]^2}{2\sigma^2(t)} + ix\alpha(t) + ix^2\beta(t)}$$

Time-dependent variational Lagrangian (TDVL) approach

V. Perez-Garcia et al., Phys. Rev. Lett **77**, 5320 (1996)

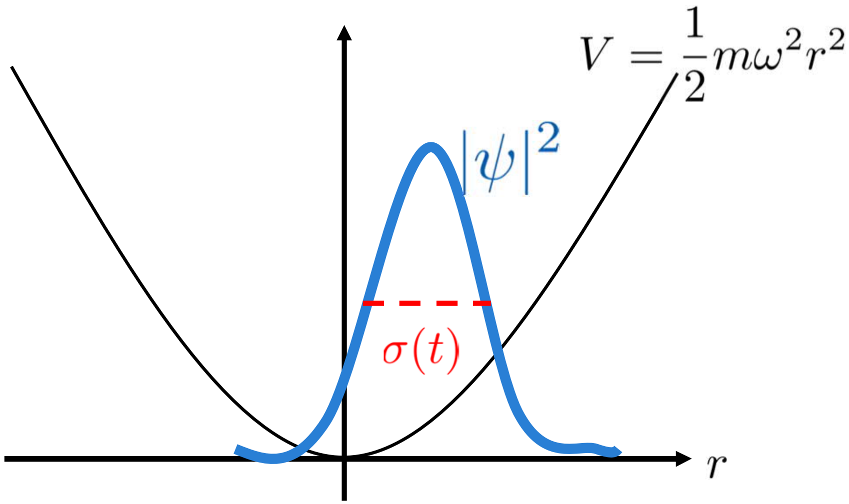


$$\psi(x,t) = A(t)e^{-\frac{[x - x_0(t)]^2}{2\sigma^2(t)} + ix\alpha(t) + ix^2\beta(t)}$$

Center of the Gaussian

Time-dependent variational Lagrangian (TDVL) approach

V. Perez-Garcia et al., Phys. Rev. Lett 77, 5320 (1996)

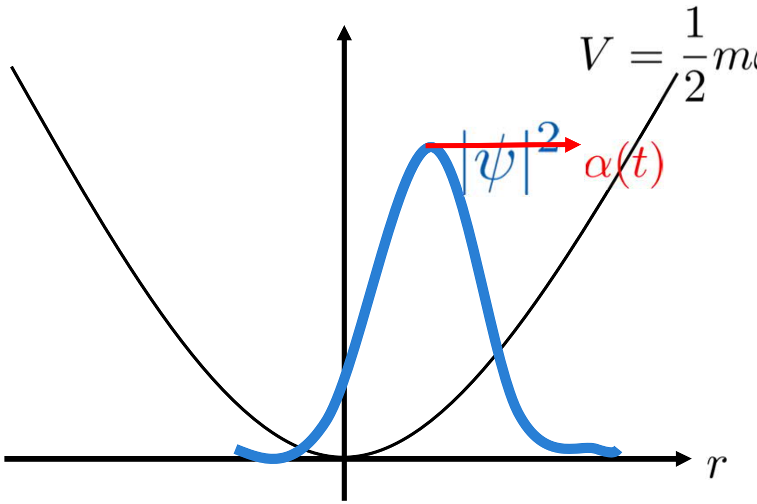


$$\psi(x,t) = A(t) e^{-\frac{[x - x_0(t)]^2}{2\sigma^2(t)} + ix\alpha(t) + ix^2\beta(t)}$$

Width of the Gaussian

Time-dependent variational Lagrangian (TDVL) approach

V. Perez-Garcia et al., Phys. Rev. Lett **77**, 5320 (1996)

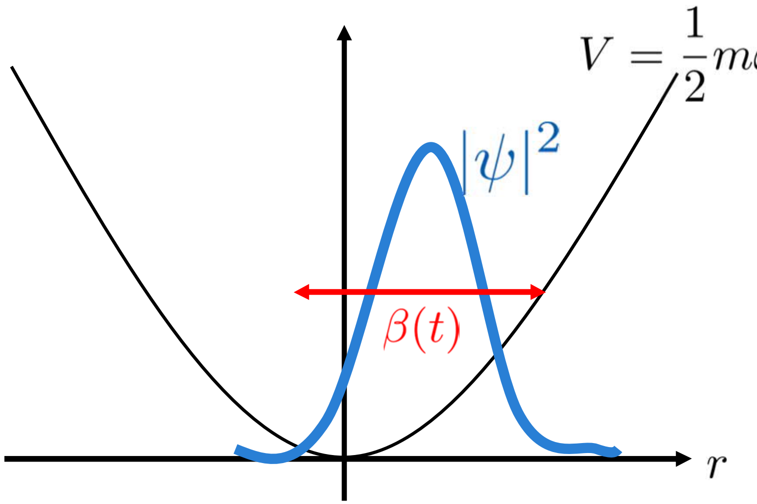


$$\psi(x,t) = A(t) e^{-\frac{[x - x_0(t)]^2}{2\sigma^2(t)} + ix\alpha(t) + ix^2\beta(t)}$$

"Translational Velocity" of the Gaussian

Time-dependent variational Lagrangian (TDVL) approach

V. Perez-Garcia et al., Phys. Rev. Lett **77**, 5320 (1996)

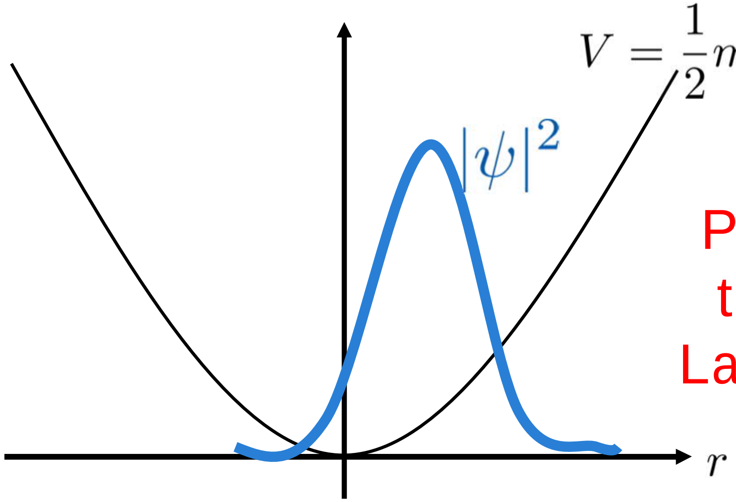


$$\psi(x,t) = A(t) e^{-\frac{[x - x_0(t)]^2}{2\sigma^2(t)} + ix\alpha(t) + ix^2\beta(t)}$$

"Expansion/contraction Velocity" of the Gaussian

Time-dependent variational Lagrangian (TDVL) approach

V. Perez-Garcia et al., Phys. Rev. Lett 77, 5320 (1996)



$$\psi(x,t) = A(t)e^{-\frac{[x-x_0(t)]^2}{2\sigma^2(t)}+ix\alpha(t)+ix^2\beta(t)}$$

Plug into the field Lagrangian

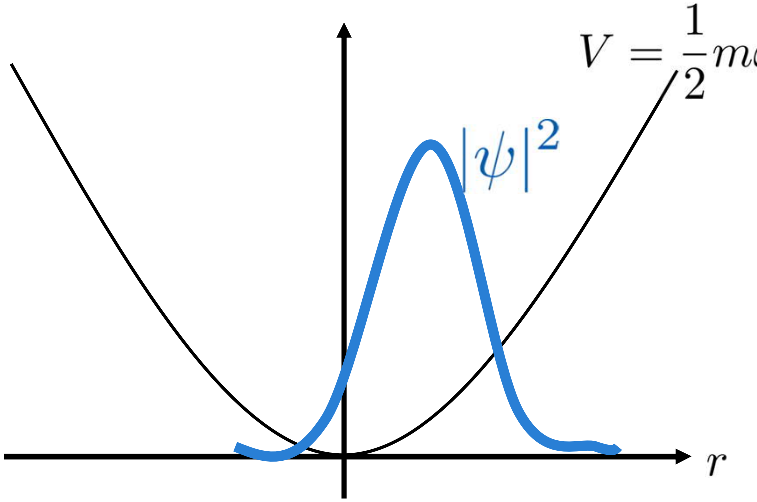
$$\mathcal{L}[\psi] = \mathcal{T}[\psi] - \mathcal{E}[\psi],$$

$$\mathcal{T}[\psi] = \frac{i\hbar}{2}\int\left(\psi^*\frac{\partial\psi}{\partial t} - \frac{\partial\psi^*}{\partial t}\psi\right)d\ r$$

$$\mathcal{E}[\psi] = \int\left(\frac{\hbar^2}{2m}|\nabla\psi|^2 + V_{\mathrm{tr}}|\psi|^2 + \frac{g}{2}|\psi|^4\right)d\ r.$$

Time-dependent variational Lagrangian (TDVL) approach

V. Perez-Garcia et al., Phys. Rev. Lett 77, 5320 (1996)



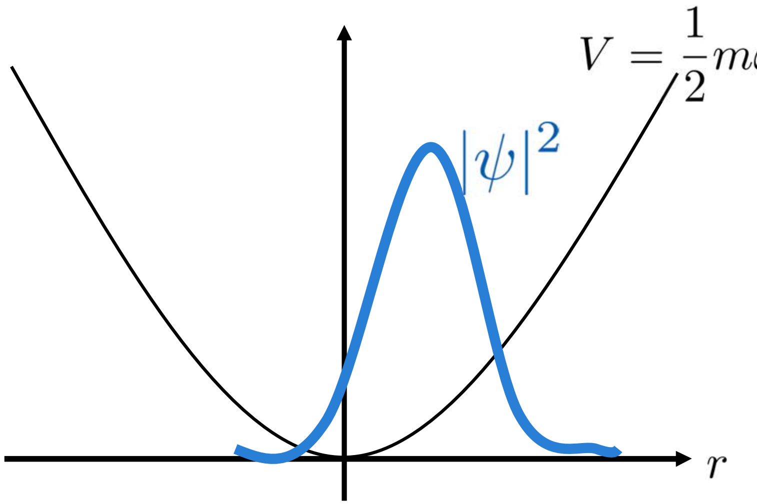
$$\mathcal{L}[\psi] = \mathcal{T}[\psi] - \mathcal{E}[\psi],$$

Integrate the spatial variable of the field and remain with an effective classical Lagrangian depending only on the chosen time-dependent variational parameters:

$$L = L\left(x_0(t), \sigma(t), \alpha(t), \dot{\alpha}(t), \beta(t), \dot{\beta}(t)\right)$$

Time-dependent variational Lagrangian (TDVL) approach

V. Perez-Garcia et al., Phys. Rev. Lett 77, 5320 (1996)



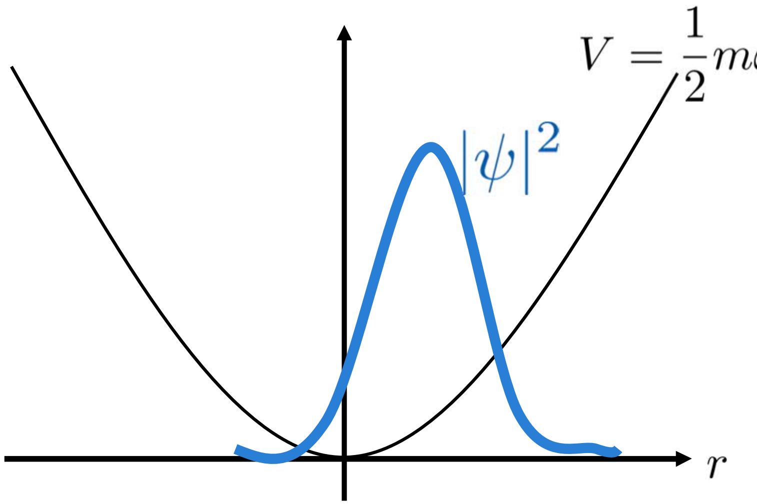
$$L = L\left(x_0(t), \sigma(t), \alpha(t), \dot{\alpha}(t), \beta(t), \dot{\beta}(t)\right)$$

Write the Euler-Lagrange equations:

$$\begin{cases} \ddot{x}_0 + \omega^2 x_0 = 0 \\ \ddot{\sigma} + \omega^2 \sigma = \dfrac{\hbar^2 + cg}{m^2 \sigma^3} \end{cases}$$

Time-dependent variational Lagrangian (TDVL) approach

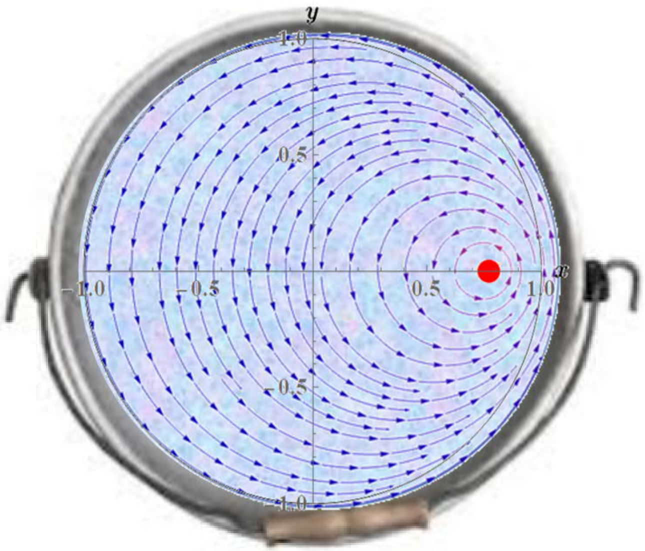
V. Perez-Garcia et al., Phys. Rev. Lett 77, 5320 (1996)

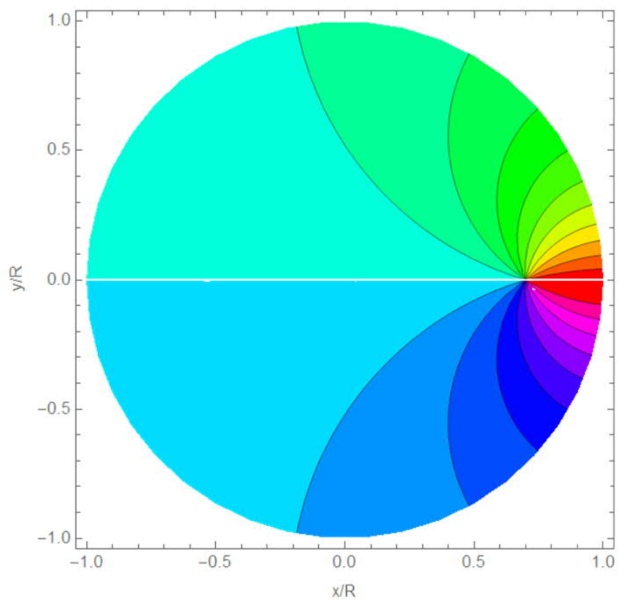


$$\begin{cases} \ddot{x}_0 + \omega^2 x_0 = 0 \\ \ddot{\sigma} + \omega^2 \sigma = \dfrac{\hbar^2 + cg}{m^2 \sigma^3} \end{cases}$$

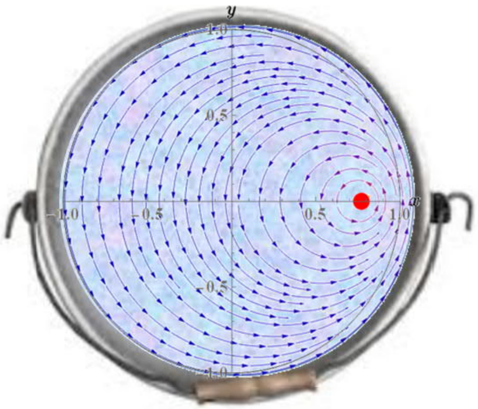
One is thus provided with the (approximated) dynamics of the real wavefunction!

TDVL for a single vortex in a bucket





TDVL for a single vortex in a bucket



Instead of focusing on the full field $\psi(x,t)$, one can keep only some notable time-dependent variational parameters:

$$\psi(x,y,t) = \rho(x,y) e^{i\left[+1 \arctan\left(\frac{y-y_0(t)}{x-x_0(t)}\right) - 1 \arctan\left(\frac{y-y_0'(t)}{x-x_0'(t)}\right)\right]}$$

$$\rho(x,y) = \begin{cases} \rho_0 & \text{for } x^2+y^2 \le R^2, \\ 0 & \text{for } x^2+y^2 > R^2 \end{cases}$$

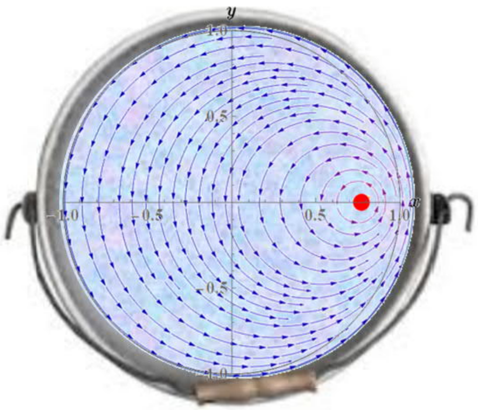
$$x_0' = \frac{R^2}{x_0^2+y_0^2} x_0$$

$$y_0' = \frac{R^2}{x_0^2+y_0^2} y_0$$

Position of the image vortex

J. Kim and A. L. Fetter, Phys. Rev. A **70**, 043624 (2004)

TDVL for a single vortex in a bucket



$$\psi(x,y,t) = \rho(x,y) e^{i\left[+1\arctan\left(\frac{y-y_0(t)}{x-x_0(t)}\right) - 1\arctan\left(\frac{y-y_0'(t)}{x-x_0'(t)}\right)\right]}$$

Plug into the field Lagrangian

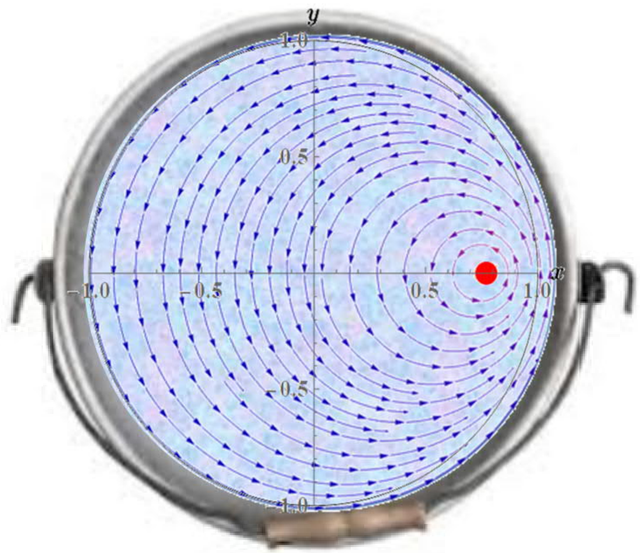
$$\mathcal{L}[\psi] = \mathcal{T}[\psi] - \mathcal{E}[\psi],$$

$$\mathcal{T}[\psi] = \frac{i\hbar}{2}\int\left(\psi^*\frac{\partial\psi}{\partial t} - \frac{\partial\psi^*}{\partial t}\psi\right)d^2r$$

$$\mathcal{E}[\psi] = \int\left(\frac{\hbar^2}{2m}|\nabla\psi|^2 + V_{\mathrm{tr}}|\psi|^2 + \frac{g}{2}|\psi|^4\right)d^2r.$$

J. Kim and A. L. Fetter, Phys. Rev. A **70**, 043624 (2004)

TDVL for a single vortex in a bucket



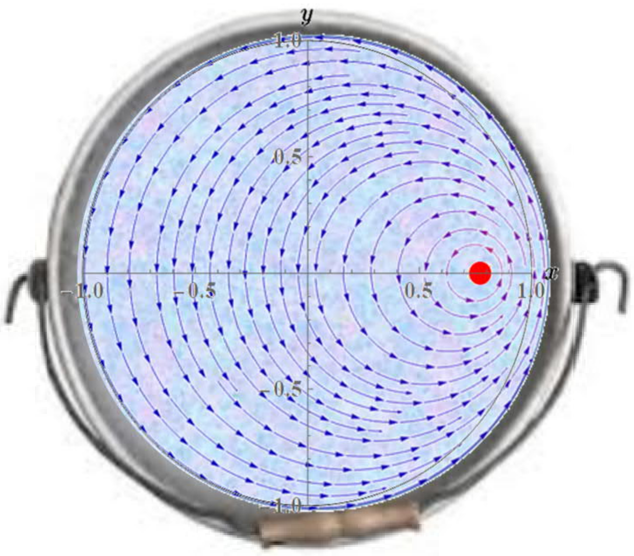
$$\mathcal{L}[\psi] = \mathcal{T}[\psi] - \mathcal{E}[\psi],$$

Integrate the spatial variable of the field and remain with an effective classical Lagrangian depending only on the chosen time-dependent variational parameters:

$$L = L(r_0, \theta_0, \dot{\theta}_0)$$

J. Kim and A. L. Fetter, Phys. Rev. A **70**, 043624 (2004)

TDVL for a single vortex in a bucket



$$L = L(r_0, \theta_0, \dot{\theta}_0)$$

Compute the two Euler-Lagrange equations:

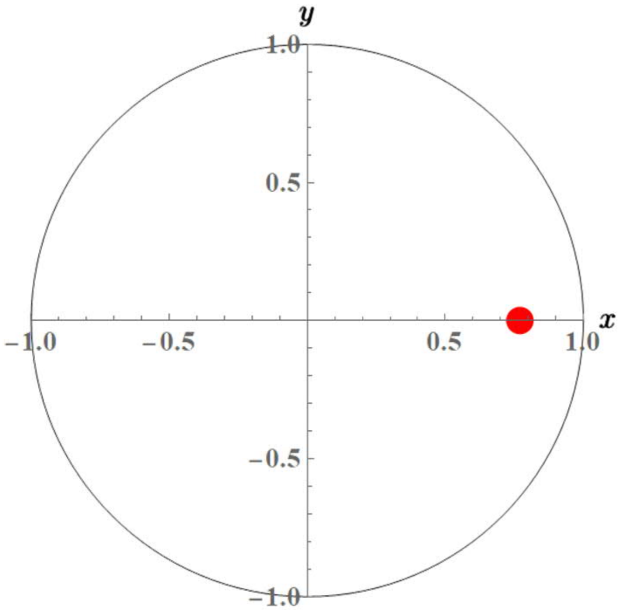
$$\dot{r}_0 = 0$$

$$\dot{\theta}_0 = \frac{\hbar}{m(R^2 - r_0^2)}$$

Signature of a uniform circular motion!

J. Kim and A. L. Fetter, Phys. Rev. A **70**, 043624 (2004)

TDVL for a single vortex in a bucket



$$L = L(r_0, \theta_0, \dot{\theta}_0)$$

Compute the two Euler-Lagrange equations:

$$\dot{r}_0 = 0$$

$$\dot{\theta}_0 = \frac{\hbar}{m(R^2 - r_0^2)}$$

Signature of a uniform circular motion!

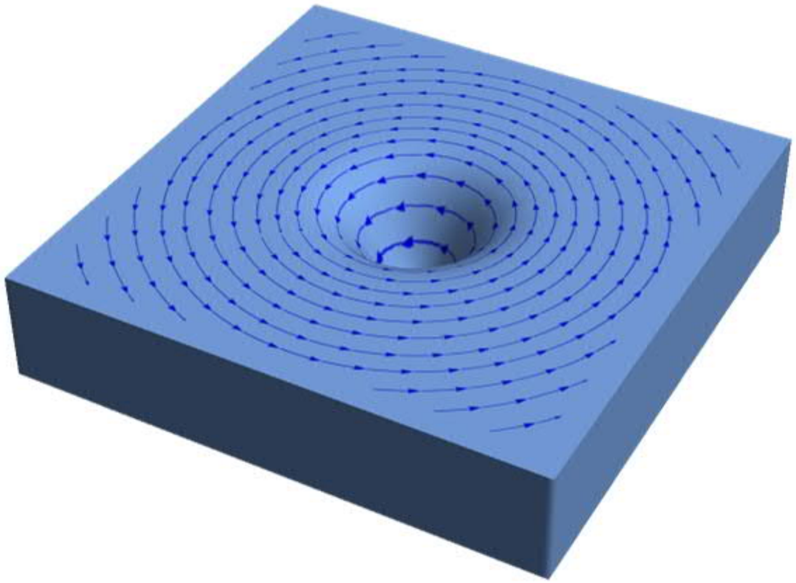
J. Kim and A. L. Fetter, Phys. Rev. A **70**, 043624 (2004)

From massless to massive vortices

[A. Richaud, V. Penna, R. Mayol, M. Guilleumas, Phys. Rev. A **101**, 013630 (2020)]

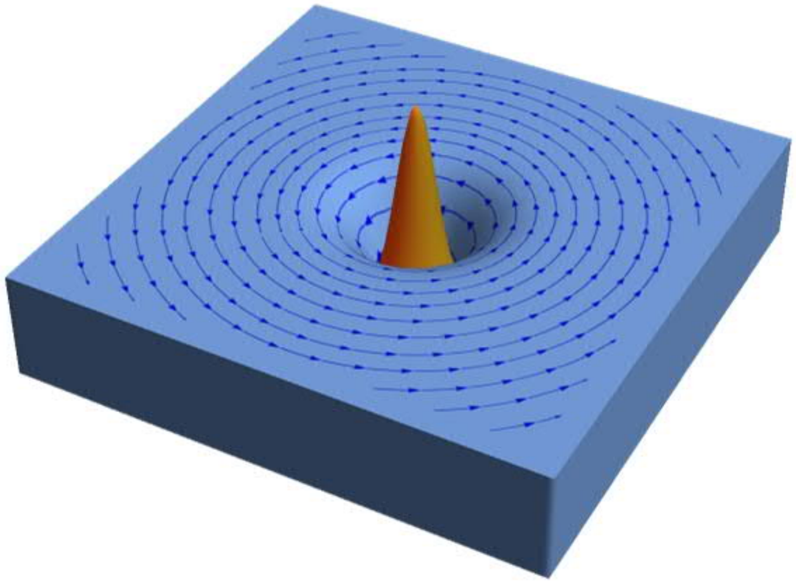
[A. Richaud, V. Penna, A. L. Fetter, Phys. Rev. A **103**, 023311 (2021)]

Vortices: just empty holes?



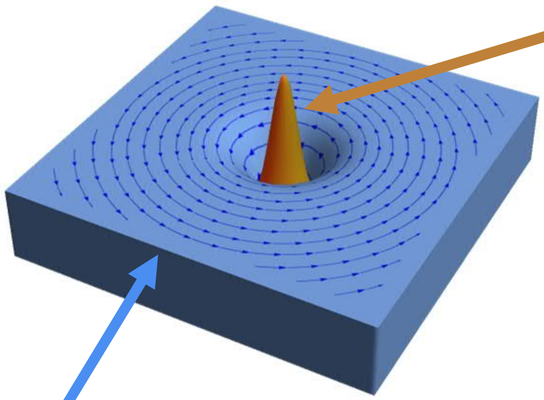
Traditionally, the core is represented as a funnel-like **<u>hole</u>** around which the superfluid exhibits a swirling flow, a sort of *tornado* in the corresponding wavefuction.

Vortices with filled massive cores



Actually, the vortex core turns out to be commonly filled by particles!

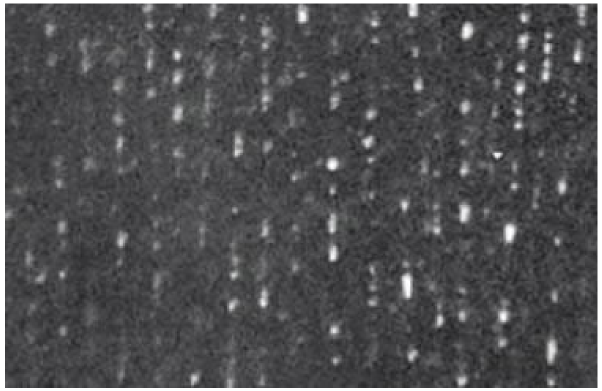
Vortices with filled massive cores



Superfluid

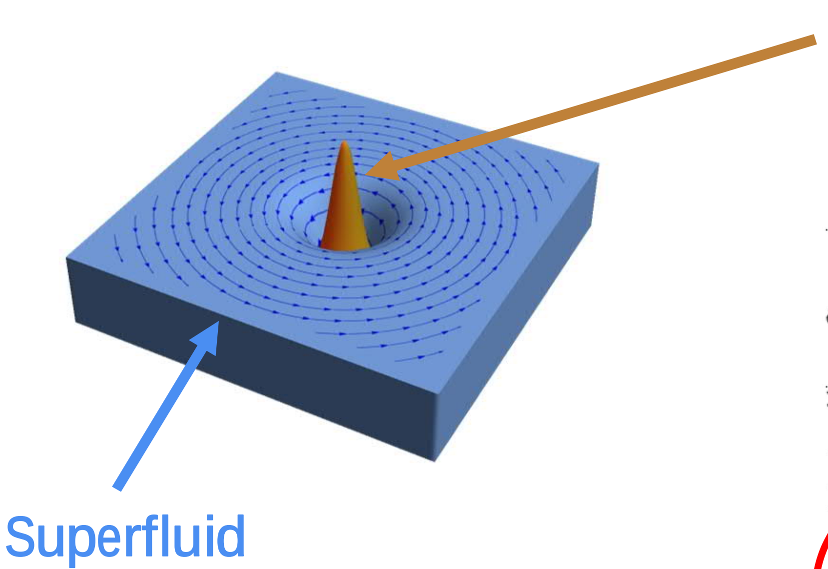
Tracers particles

Experimentalists use particles as "vorticity tracers", e.g. in liquid helium.

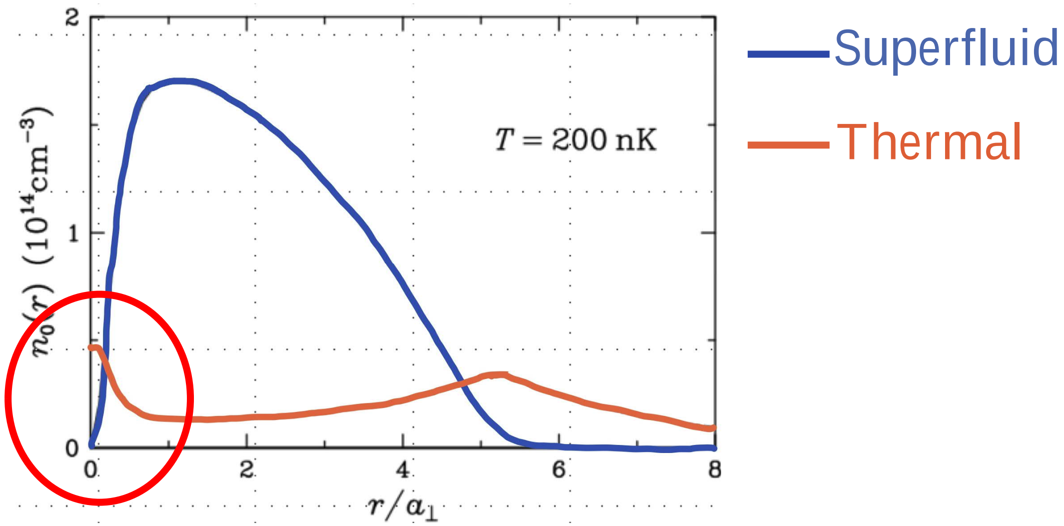


G. P. Bewley et al., Nature **441**, 588 (2006)

Vortices with filled massive cores

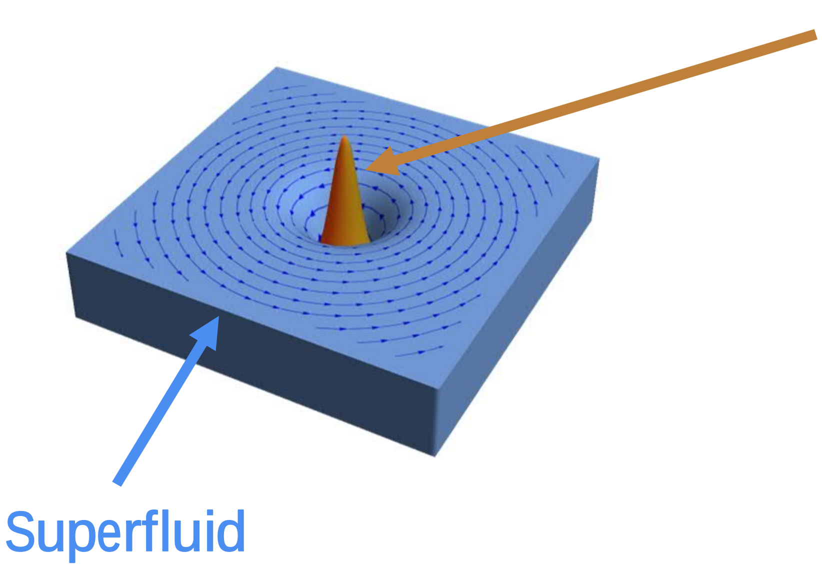


Atoms which do not belong to the superfluid fraction.



A. Griffin, T. Nikuni, E. Zaremba, *Bose-Condensed Gases at Finite Temperature*, Chap. 9, Cambridge University Press (2009)

Vortices with filled massive cores



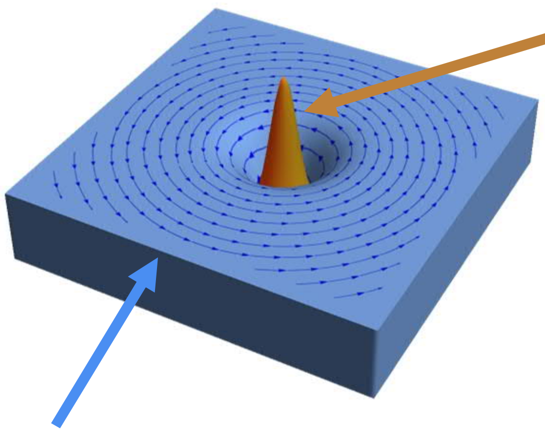
In Fermionic superfluids, due to pair-breaking excitations, vortices' cores are filled up with quasiparticle bound states even at zero temperature.

N. B. Kopnin et al., Phys. Rev. B **44**, 9667 (1991)

W. J. Kwon et al., Nature **600**, 64 (2021)

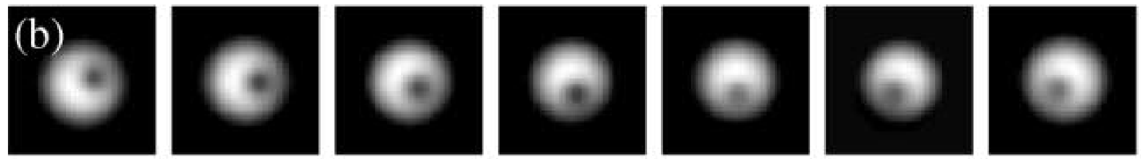
Vortices with filled massive cores

A second (minority) component



Superfluid

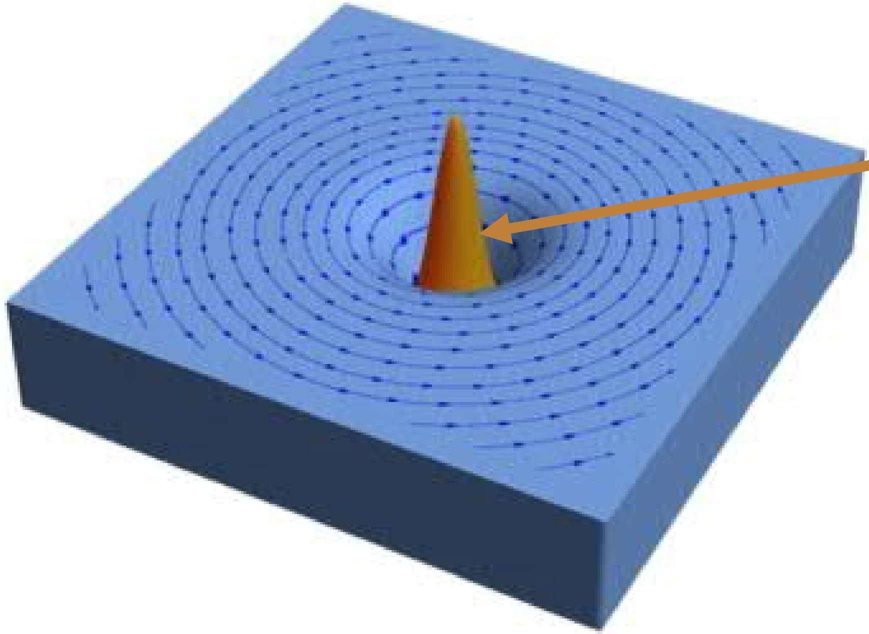
One of the first vortices ever observed in a BEC, had a core filled by another component!



The two components were two different internal states of ^{87}Rb.

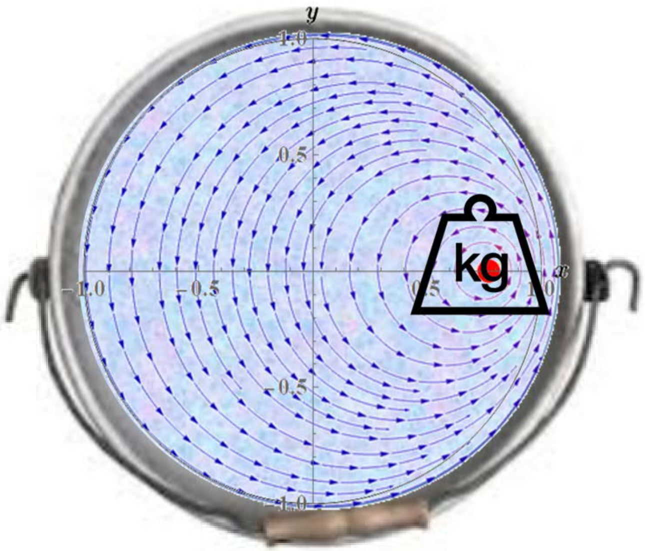
B. P. Anderson et al., Phys. Rev. Lett. **85**, 2857 (2000)

Vortices with filled massive cores



- Tracers particles
- Thermal atoms
- Quasiparticle bound states
- Another (minority) BEC

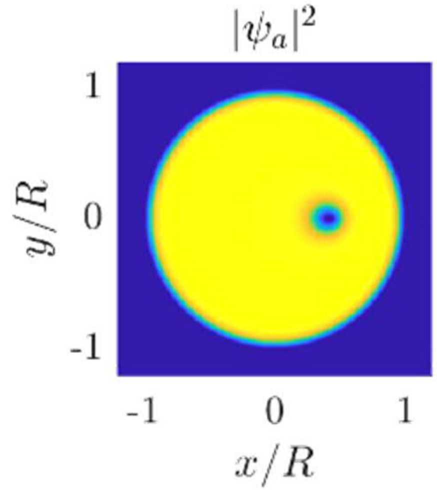
Massive Point Vortex Model

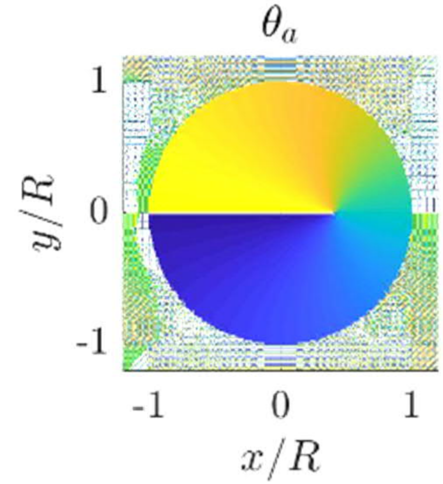


A. Richaud, V. Penna, R. Mayol, M. Guilleumas, Phys. Rev. A **101**, 013630 (2020)

A. Richaud, V. Penna, A. L. Fetter, Phys. Rev. A **103**, 023311 (2021)

Massive Point Vortex Model





$$\psi_a(x,y,t) = \rho_a(x,y)\, e^{i\left[+1\arctan\left(\frac{y-y_0(t)}{x-x_0(t)}\right) - 1\arctan\left(\frac{y-y_0'(t)}{x-x_0'(t)}\right)\right]}$$

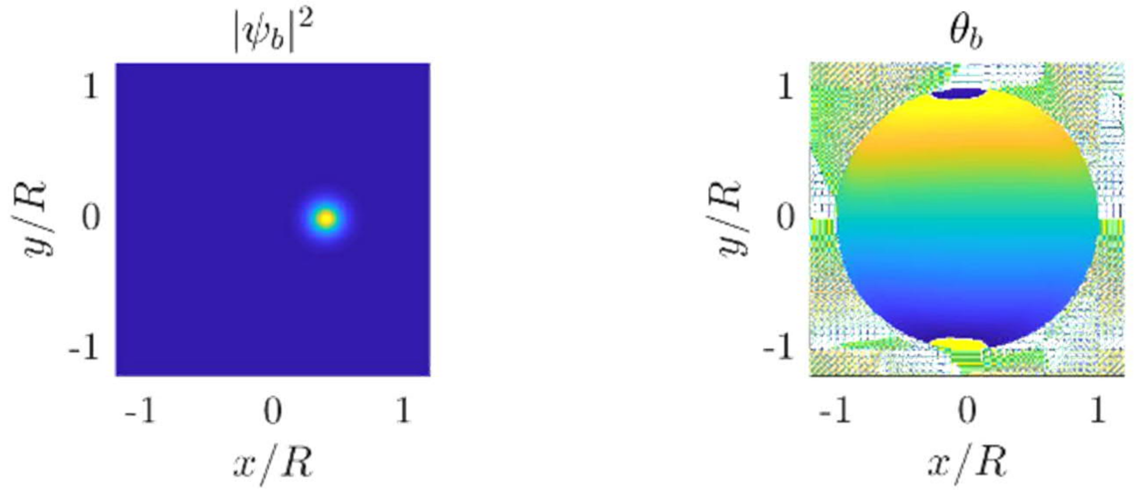
$$\rho_a(x,y) = \begin{cases} \rho_0 & \text{for } x^2+y^2 \le R^2, \\ 0 & \text{for } x^2+y^2 > R^2 \end{cases}$$

Position of the image vortex

$$x_0' = \frac{R^2}{x_0^2+y_0^2} x_0$$

$$y_0' = \frac{R^2}{x_0^2+y_0^2} y_0$$

Massive Point Vortex Model



$$\psi_b(x,y,t) = \sqrt{\frac{N_b}{\pi\sigma^2}}\, e^{-\frac{|\vec{r}-\vec{r}_0(t)|^2}{2\sigma^2}}\, \underbrace{e^{i\vec{r}\cdot\vec{\alpha}(t)}}$$

This term allows the Gaussian peak to have a non-zero velocity

Massive Point Vortex Model

$$\psi_a(x,y,t)=\rho_a(x,y)e^{i\left[+1\arctan\left(\frac{y-\boxed{y_0(t)}}{x-\boxed{x_0(t)}}\right)-1\arctan\left(\frac{y-y_0'(t)}{x-x_0'(t)}\right)\right]}$$

$$\psi_b(x,y,t)=\sqrt{\frac{N_b}{\pi\sigma^2}}e^{-\frac{|\vec{r}-\boxed{\vec{r}_0(t)}|^2}{2\sigma^2}}e^{i\vec{r}\cdot\vec{\alpha}(t)}$$

Notice that, in our time-dependent variational ansatz, the center of species-a vortex is assumed to always coincide with the center of species-b Gaussian peak:

$$\vec{r}_0(t):=(x_0(t)\,;\,y_0(t))$$

This is motivated by the assumed strong immiscibility of the two components!

Massive Point Vortex Model

$$\psi_a(x,y,t) = \rho_a(x,y) e^{i\left[+1 \arctan\left(\frac{y-y_0(t)}{x-x_0(t)}\right) - 1 \arctan\left(\frac{y-y_0'(t)}{x-x_0'(t)}\right)\right]}$$

$$\psi_b(x,y,t) = \sqrt{\frac{N_b}{\pi\sigma^2}} e^{-\frac{|\vec{r}-\vec{r}_0(t)|^2}{2\sigma^2}} e^{i\vec{r}\cdot\vec{\alpha}(t)}$$

Plug into the field Lagrangian $\mathcal{L}[\psi_a, \psi_b] = \mathcal{L}_a[\psi_a] + \mathcal{L}_b[\psi_b]$

Massive Point Vortex Model

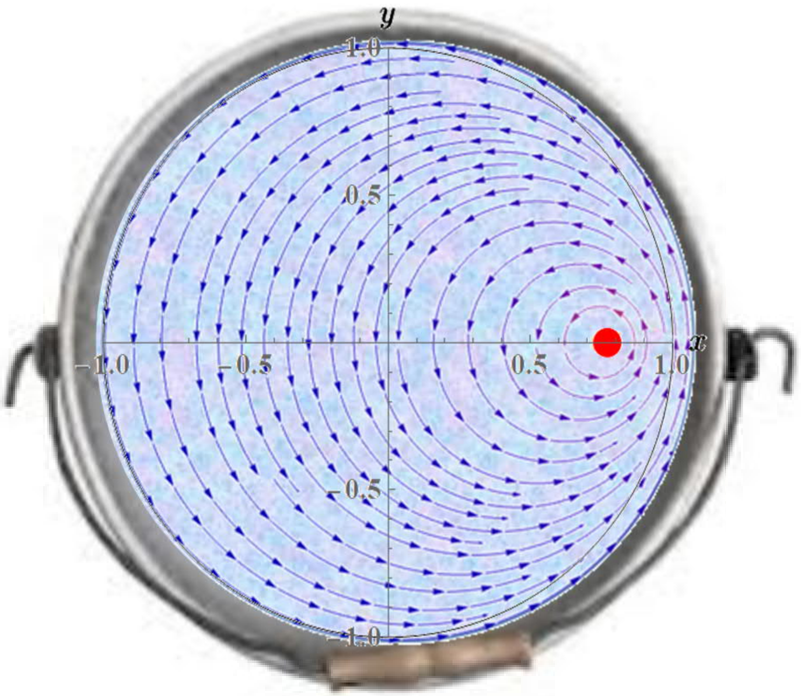
$$\mathcal{L}[\psi_a,\, \psi_b] = \mathcal{L}_a[\psi_a] + \mathcal{L}_b[\psi_b]$$

Integrate the spatial variables of the field and remain with an effective classical Lagrangian depending only on the chosen time-dependent variational parameters:

$$L = L(r_0, \boxed{\dot{r}_0}, \theta_0, \dot{\theta}_0)$$

Notice that, as opposed to the massless case, now the effective point-like Lagrangian depends also on the radial velocity.

Massive Point Vortex Model



$$L = L(r_0, \dot{r}_0, \theta_0, \dot{\theta}_0)$$

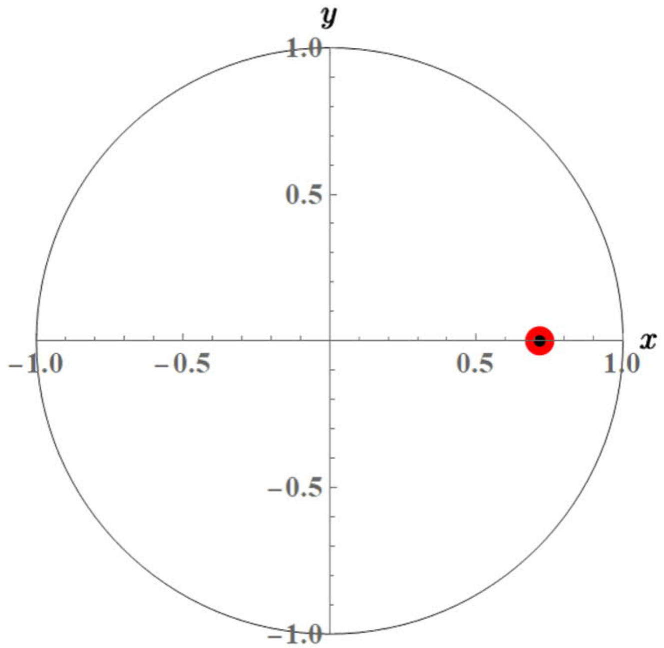
Compute the two Euler-Lagrange equations:

$$2m\pi\ddot{r}_0 = -r_0\left[\frac{k^2\rho_*}{r_0^2 - R^2} + 2\pi\dot{\theta}_0(k\rho_* - m\dot{\theta}_0)\right]$$

$$mr_0\ddot{\theta}_0 = \dot{r}_0(k\rho_* - 2m\dot{\theta}_0)$$

These equations tell us that the motion is not simply a uniform circular one!

Massive Point Vortex Model



$$L = L(r_0, \dot{r}_0, \theta_0, \dot{\theta}_0)$$

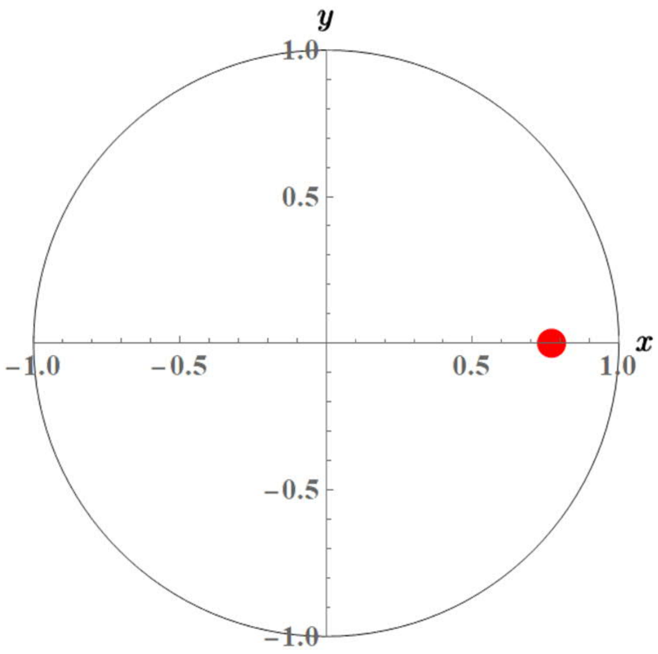
Compute the two Euler-Lagrange equations:

$$2m\pi\ddot{r}_0 = -r_0\left[\frac{k^2\rho_*}{r_0^2 - R^2} + 2\pi\dot{\theta}_0(k\rho_* - m\dot{\theta}_0)\right]$$

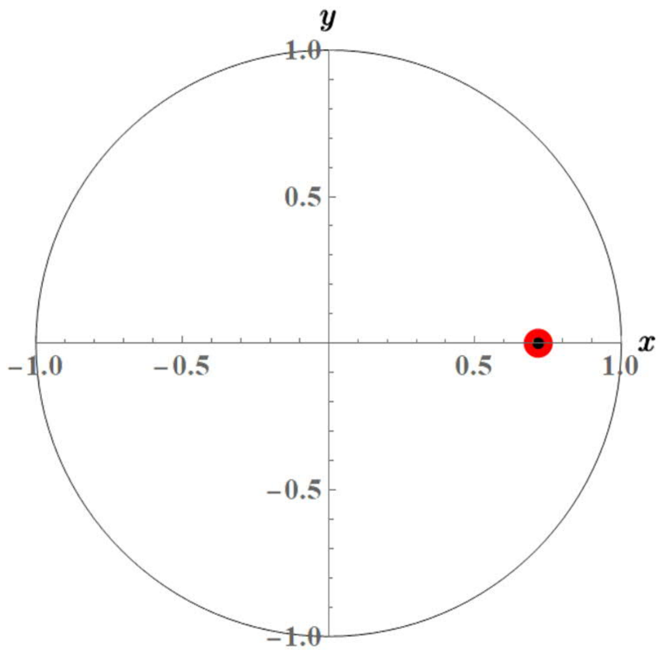
$$mr_0\ddot{\theta}_0 = \dot{r}_0(k\rho_* - 2m\dot{\theta}_0)$$

These equations tell us that the motion is not simply a uniform circular one!

Massless vs Massive Vortices



Massless → Only uniform circular orbits



Massive → Radial oscillations superimposed to circular orbits.

Gross-Pitaevskii simulations

$$\Psi^T = (\psi_a,\, \psi_b)$$

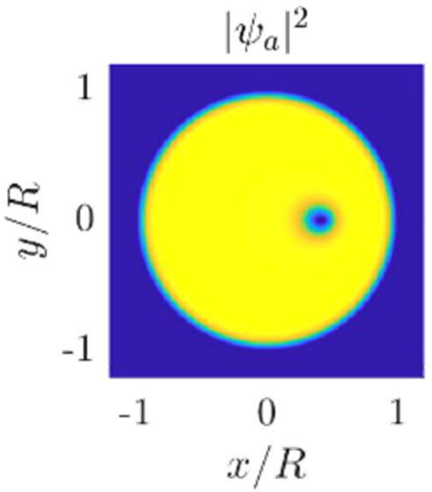
$$i\hbar \frac{\partial \Psi}{\partial t} = \mathcal{H}\Psi$$

$$\mathcal{H} = \begin{pmatrix} H_a & 0 \\ 0 & H_b \end{pmatrix}$$

$$H_a = -\frac{\hbar^2 \nabla^2}{2m_a} + V_{\mathrm{tr}}^a + \frac{g_a N_a}{d_z}|\psi_a|^2 + \frac{g_{ab} N_b}{d_z}|\psi_b|^2$$

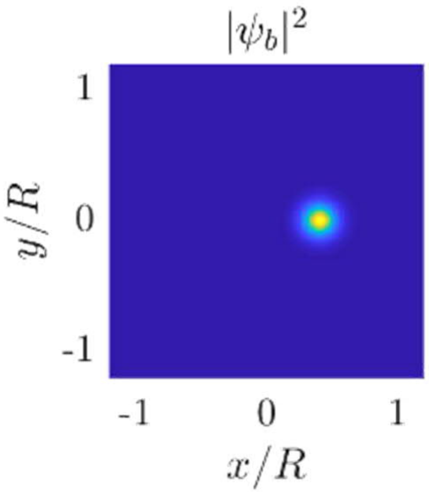
$$H_b = -\frac{\hbar^2 \nabla^2}{2m_b} + V_{\mathrm{tr}}^b + \frac{g_{ab} N_a}{d_z}|\psi_a|^2 + \frac{g_b N_b}{d_z}|\psi_b|^2$$

Assumed parameters



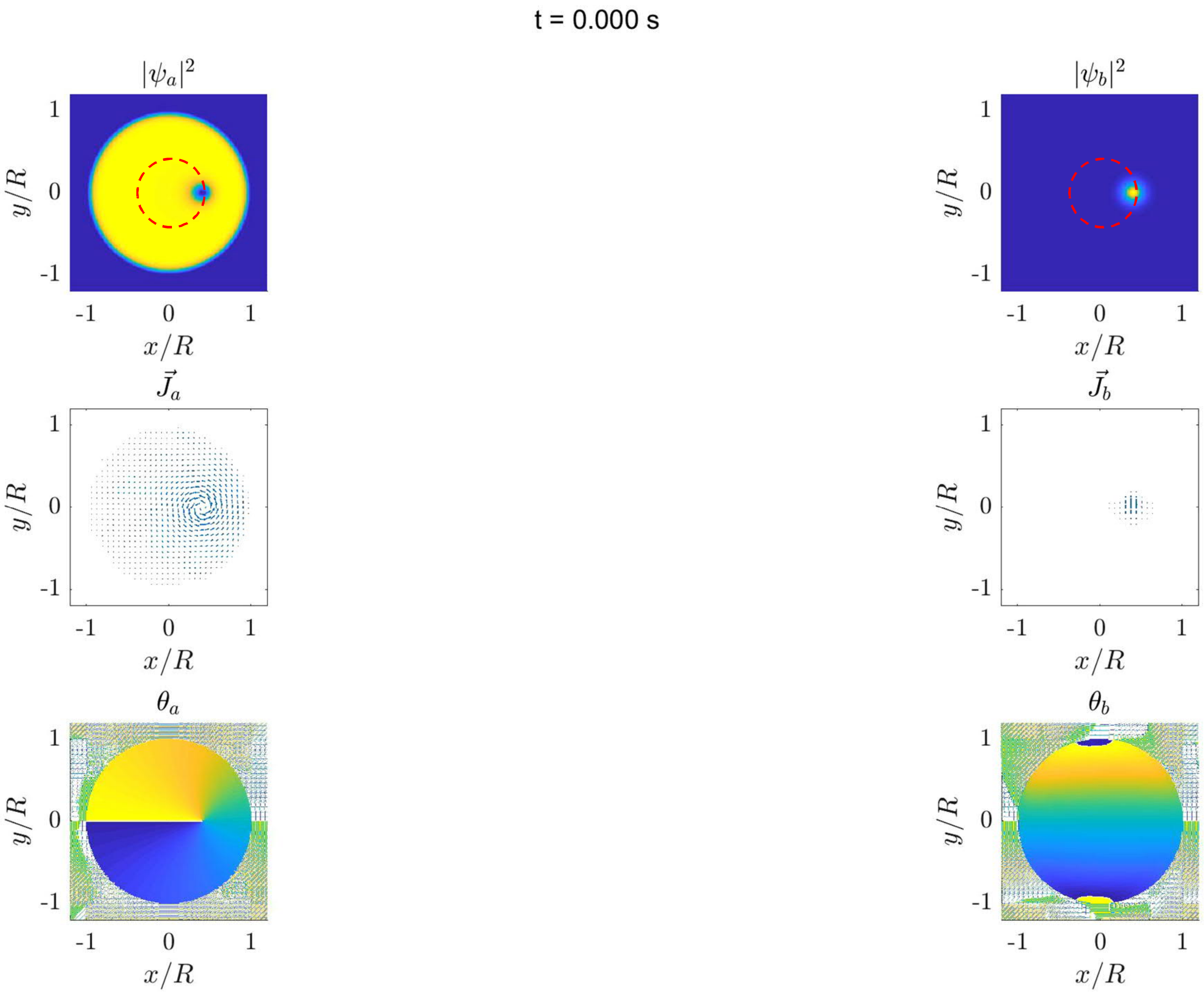
$N_a = 5 \times 10^4$ atoms of ^{23}Na

$R = 50\,\mu m$



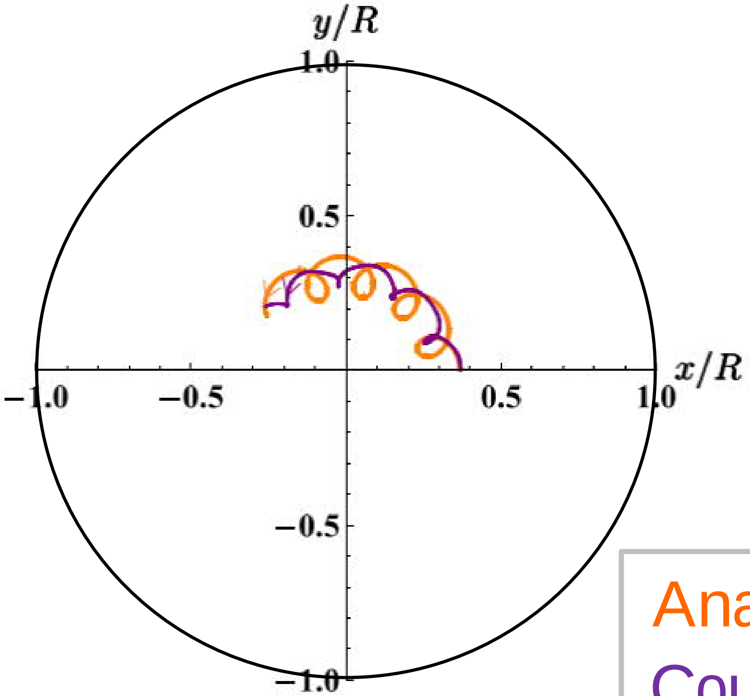
$N_b = 1 \times 10^3$ atoms of ^{39}K

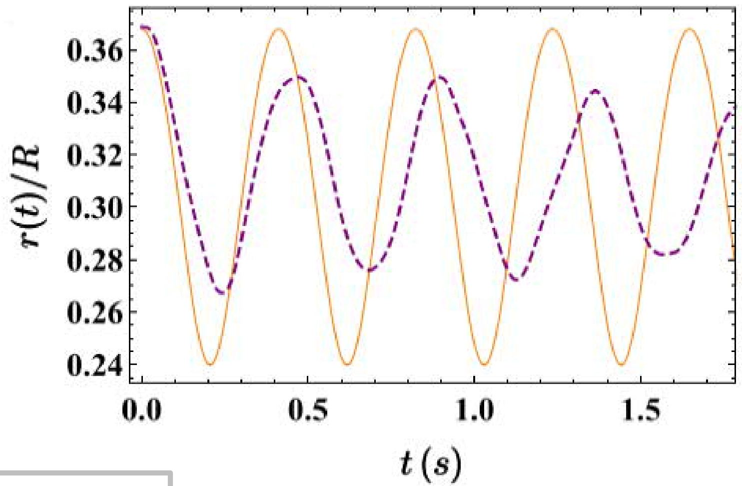
$$\frac{g_{ab}}{\sqrt{g_a g_b}} = 1.25$$



t = 0.000 s
$|\psi_a|^2$
$|\psi_b|^2$
$\vec{J}_a$
$\vec{J}_b$
θ_a
θ_b
x/R
y/R

Comparison GPE vs analytical model





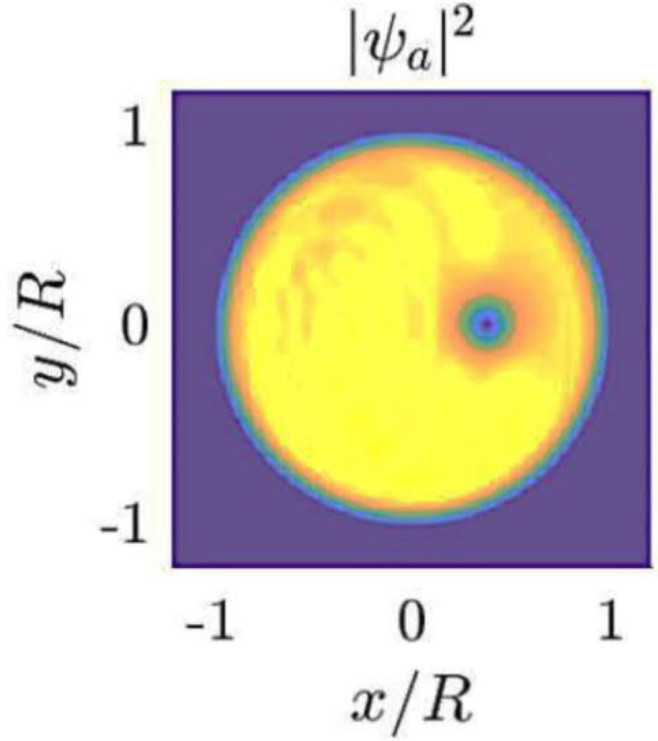
Analytical model
Coupled GPEs

Comparison GPE vs analytical model

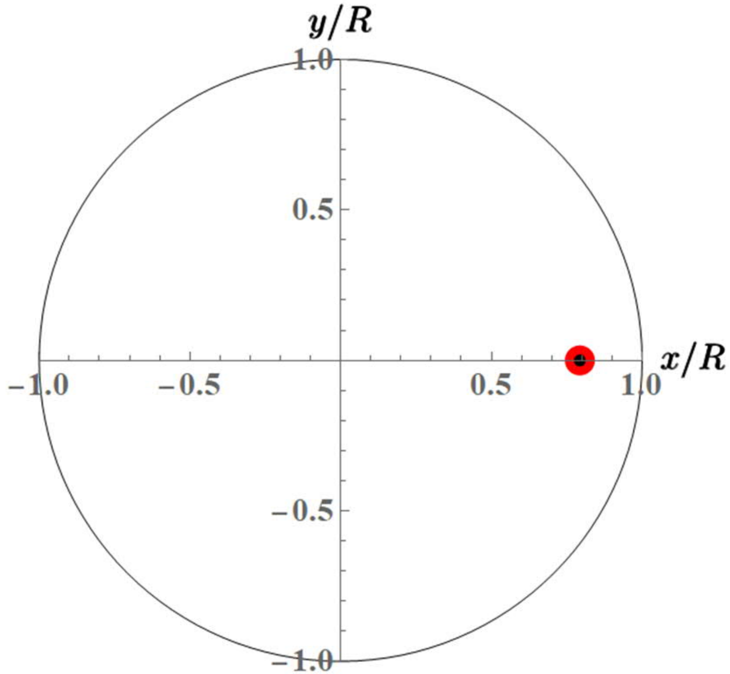
The agreement is good…

… but our analytical approach did not capture a possibly important ingredient:

the **emission of sound waves.**



Signature of the core mass



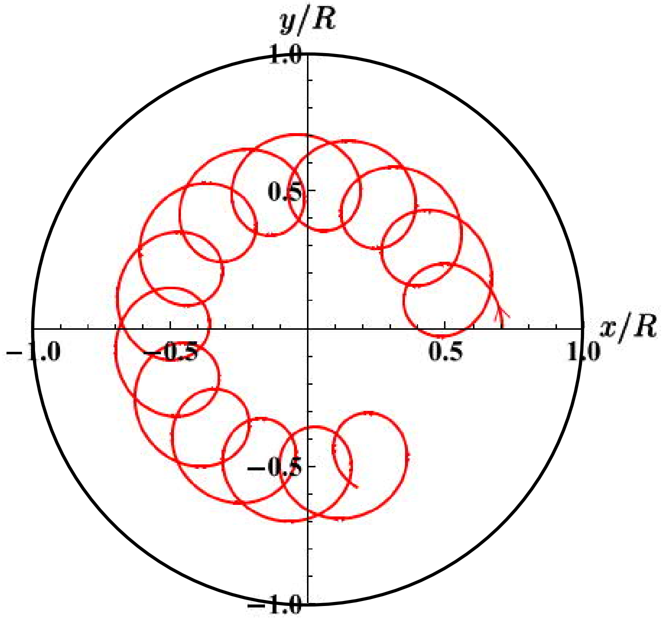
Usual circular motion of precession

+

Small-amplitude radial oscillations

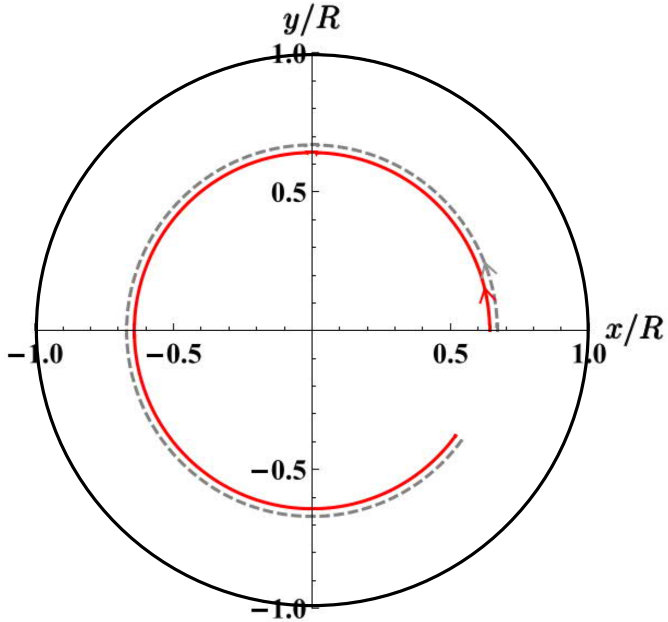
The signature of core mass!

Possible dynamical regimes



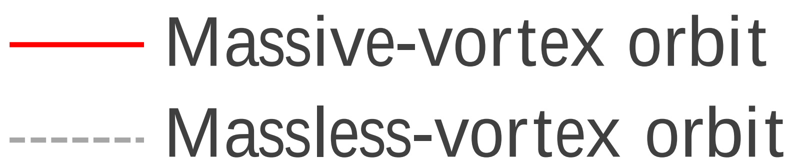
Flower-like orbits (superposition of uniform circular orbits and small radial oscillations)

Possible dynamical regimes

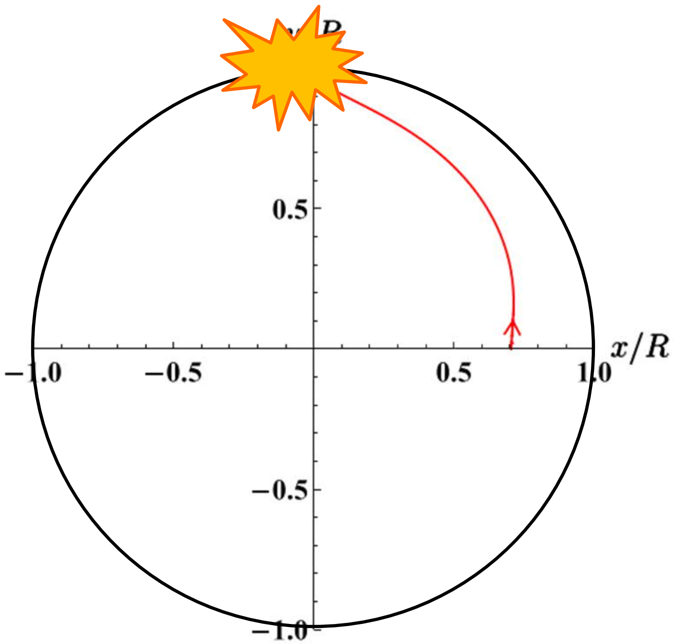


Uniform circular orbits exist provided that position and velocity match a precise condition.

Orbit radius changed by core mass.



Possible dynamical regimes



If big, core mass can push the vortex to collide against the outer wall.

Massive vortices and massive charges in 2D

Effective Lagrangian of the Massive Point-Vortex Model:

$$L = \sum_j \left[\boxed{\frac{m_j}{2}\left(\dot{x}_j^2 + \dot{y}_j^2\right)} + \frac{k_j \rho_*}{2}\left(y_j \dot{x}_j - x_j \dot{y}_j\right) \right] - \boxed{H}$$

Novel kinetic term ensuing from the core inertial mass

Usual inter-vortex interaction, e.g.:

$$= -\frac{\rho_*}{4\pi} \sum_{i=1}^{M} \sum_{j \neq i} k_i k_j \ln\left(\frac{|\mathbf{r}_i - \mathbf{r}_j|}{\lambda}\right)$$

Upon adding the core mass, x_j and y_j are no longer canonically conjugate!

Each coordinate will be associated to an actual momentum p_{x_j} and p_{y_j}.

Massive vortices and massive charges in 2D

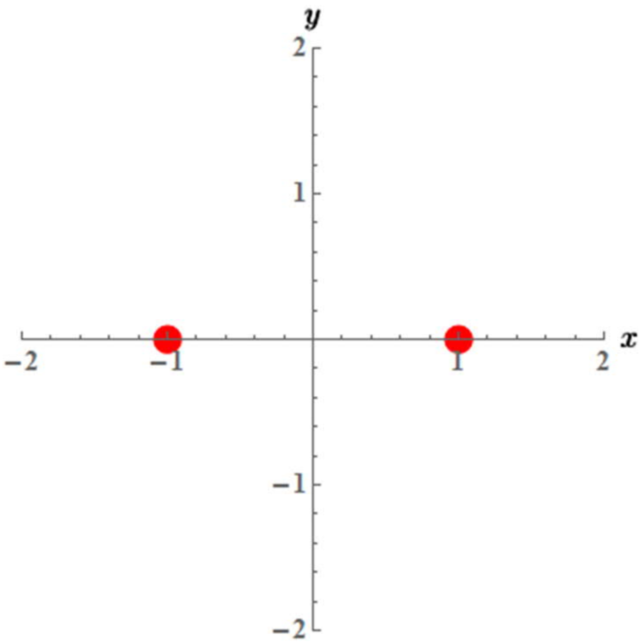
Effective Lagrangian of the Massive Point-Vortex Model:

$$L = \sum_j \left[\frac{m_j}{2} \left(\dot{x}_j^2 + \dot{y}_j^2 \right) + \frac{k_j \rho_*}{2} \left(y_j \dot{x}_j - x_j \dot{y}_j \right) \right] + \frac{\rho_*}{4\pi} \sum_{i=1}^{M} \sum_{j \neq i} k_i k_j \ln \left(\frac{|\mathbf{r}_i - \mathbf{r}_j|}{\lambda} \right)$$

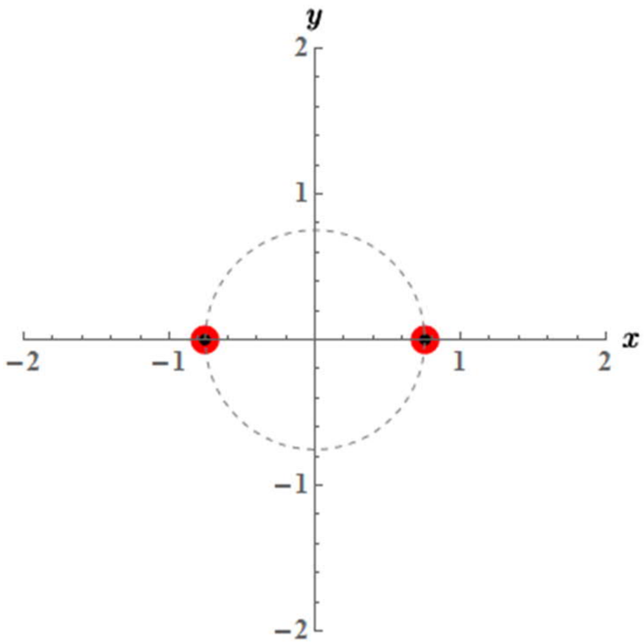
Lagrangian of M massive charges in 2D:

$$L = \sum_{i=1}^{M} \left[\frac{m |\vec{v}_i|^2}{2} - \frac{q_i}{2} B \left(y_i v_{x,i} - x_i v_{y,i} \right) \right] - \frac{1}{2} \sum_{i=1}^{M} \sum_{j \neq i} q_i q_j \ln \frac{|\vec{r}_i - \vec{r}_j|}{\lambda}$$

Massless vs Massive vortex pairs

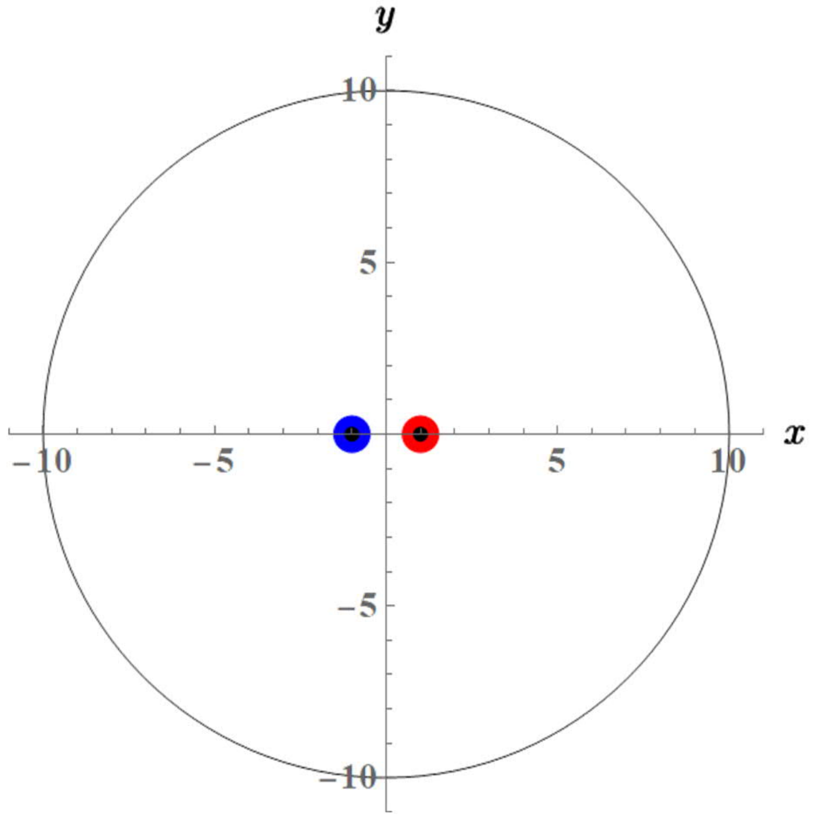


Massless



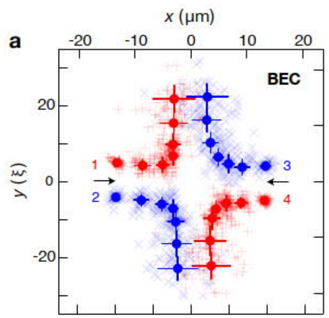
Massive

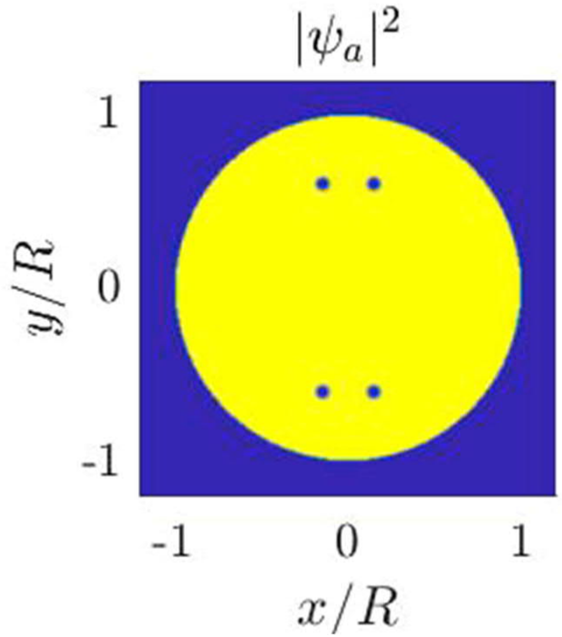
Massive vortex dipole



Experimental observation?

Experimental observation of small oscillations (the signature of inertial mass)?



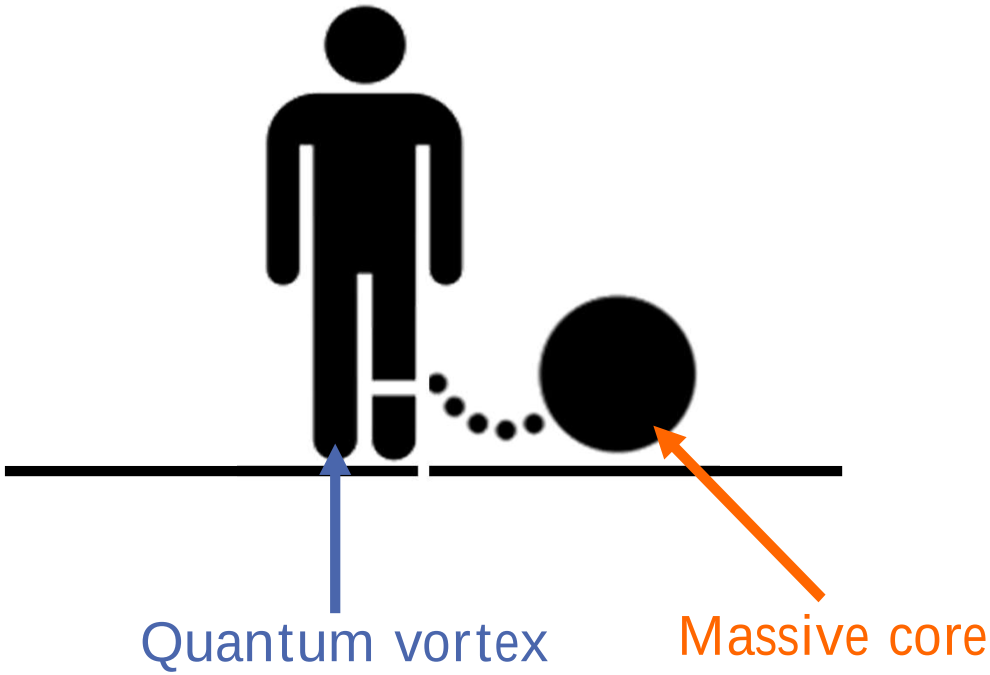


W. J. Kwon et al., Nature **600**, 64 (2021)

Massive-core-driven collisions

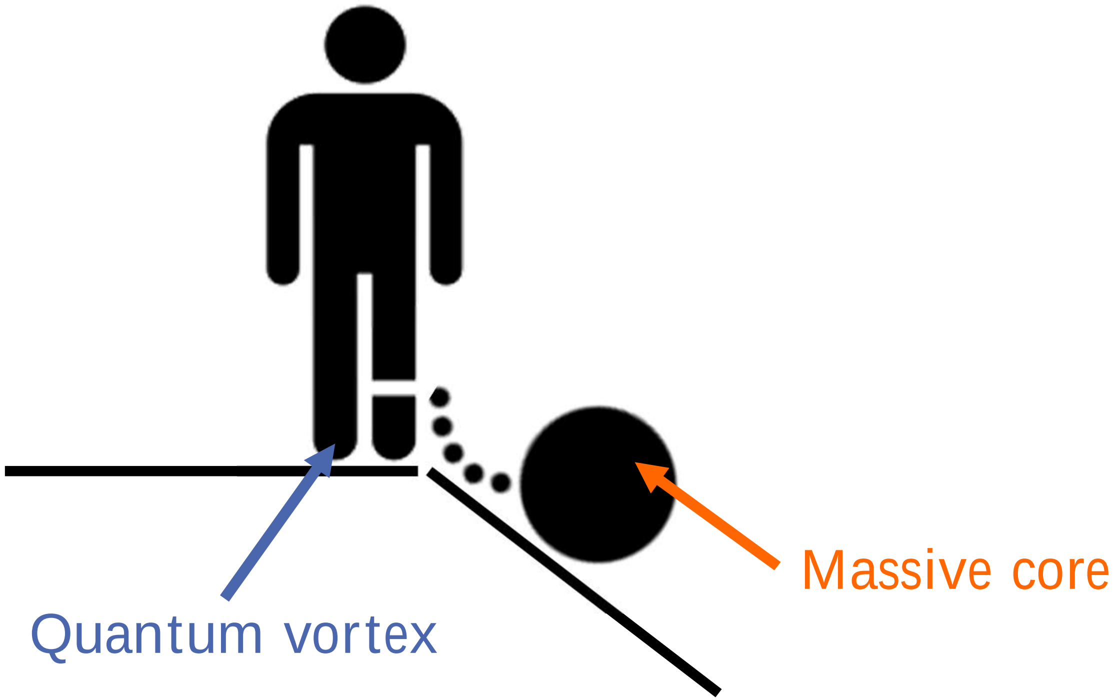
[A. Richaud, G. Lamporesi, M. Capone, A. Recati, *to be published*.]

A change of perspective



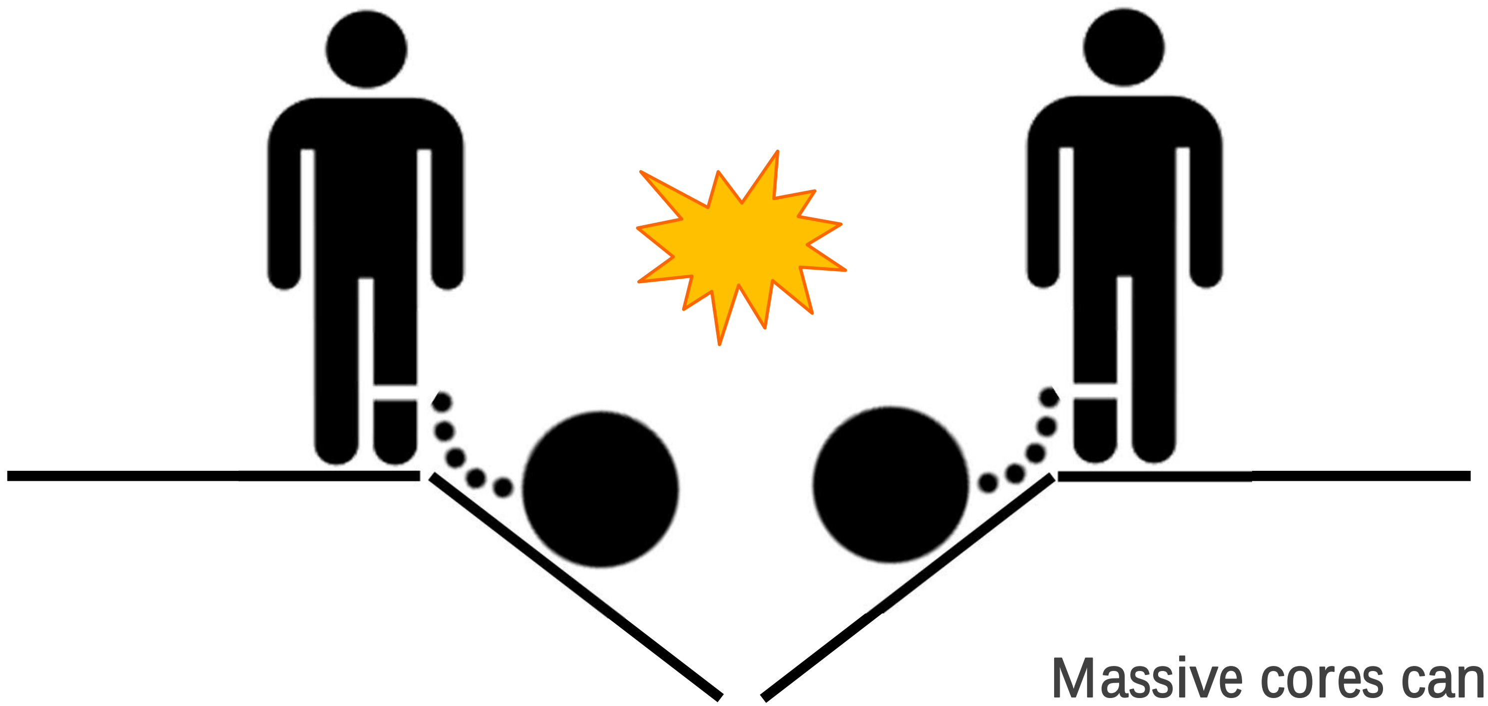
So far, the massive core has played a ‘passive’ role, meaning that it is like a **burden** which quantum vortices, deliberately or accidentally, have to live with.

A change of perspective



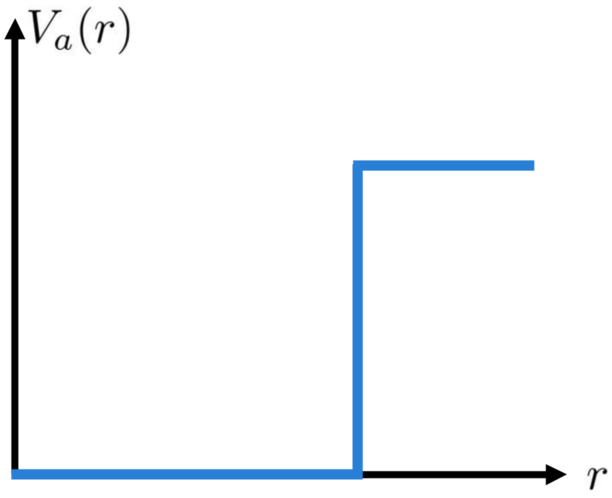
But the massive core can actually **drive** the hosting vortex!

A change of perspective

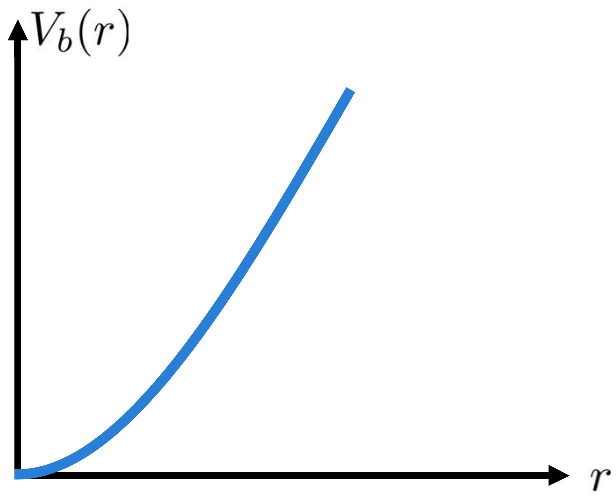


Massive cores can drive vortex collisions!

Species-selective traps

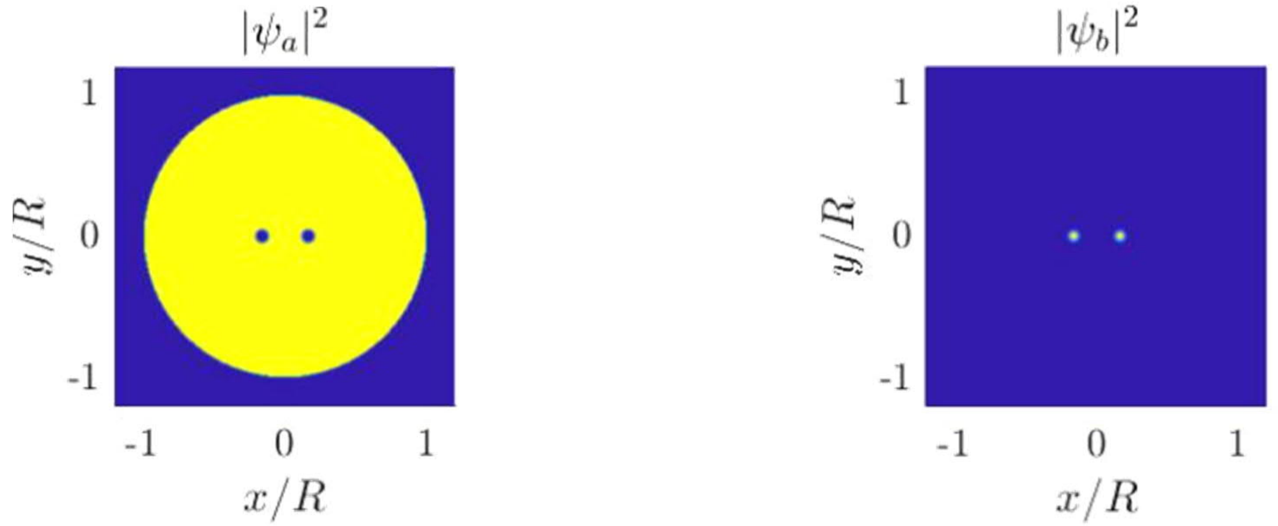


Species-a feels a **hard-wall** circular potential well

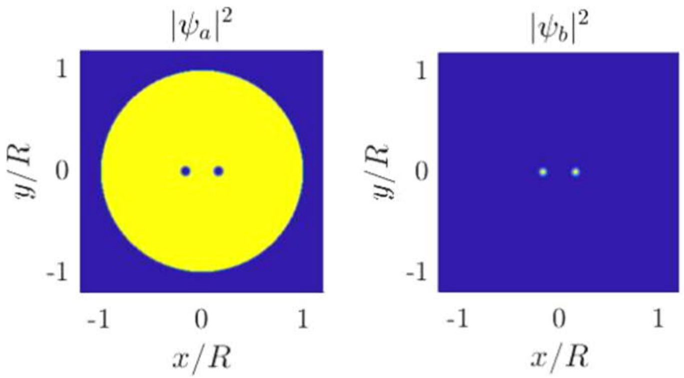


Species-b feels a **harmonic** potential

Massive Point-Vortex Model



Massive Point-Vortex Model



$$L = \sum_{j=1}^{2}\left[\boxed{\frac{m_j}{2}(\dot{x}_j^2 + \dot{y}_j^2)} + \boxed{\frac{k_j\rho_*}{2}(y_j\dot{x}_j - x_j\dot{y}_j)}\right] - \boxed{\frac{\rho_*}{4\pi}\left\{k_1k_2\log\frac{|R^2 - z_1\bar{z}_2|^2}{|R(z_1 - z_2)|^2} + k_1^2\log\left(1 - \frac{|z_1|^2}{R^2}\right) + k_2^2\log\left(1 - \frac{|z_2|^2}{R^2}\right)\right\}} + \boxed{\sum_{j=1}^{2}\frac{1}{2}m_j\omega_b^2(x_j^2 + y_j^2)}$$

Usual Superfluid Vortex Dynamics in a circular box

Inertial term ensuing from the core mass

Harmonic trapping of species-b cores

Massive Point-Vortex Model

$$L = \sum_{j=1}^{2} \left[\frac{m_j}{2} (\dot{x}_j^2 + \dot{y}_j^2) + \frac{k_j \rho_*}{2} (y_j \dot{x}_j - x_j \dot{y}_j) \right] -$$

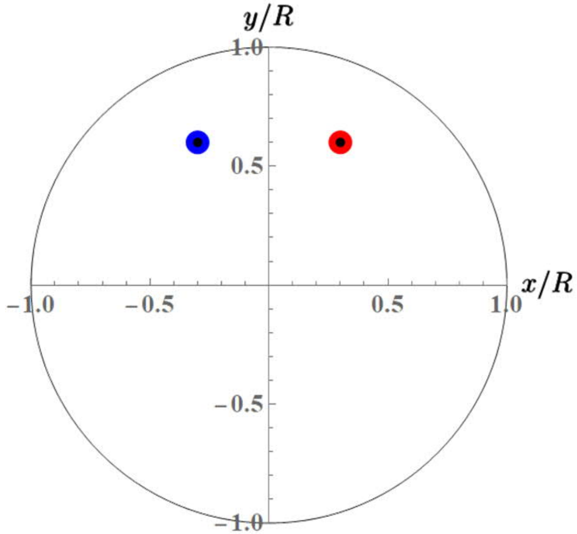
$$\frac{\rho_*}{4\pi} \left\{ k_1 k_2 \log \frac{|R^2 - z_1 \bar{z}_2|^2}{|R(z_1 - z_2)|^2} + k_1^2 \log \left(1 - \frac{|z_1|^2}{R^2} \right) + k_2^2 \log \left(1 - \frac{|z_2|^2}{R^2} \right) \right\} + \sum_{j=1}^{2} \frac{1}{2} m_j \omega_b^2 (x_j^2 + y_j^2)$$

Euler-Lagrange Equations:

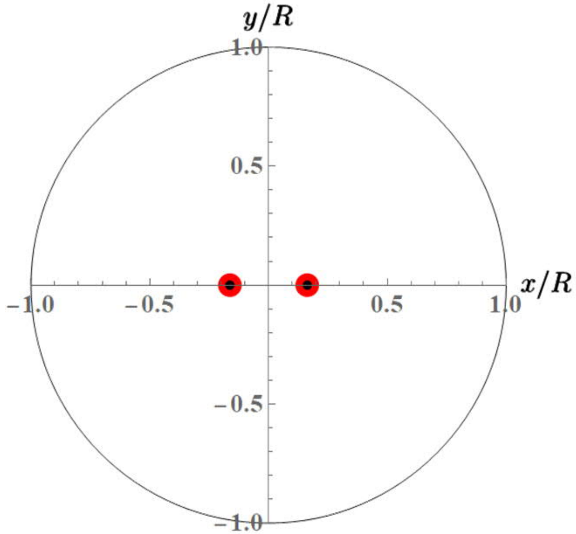
$$m_j \ddot{\vec{r}}_j = k_j \rho_* \vec{u}_3 \wedge \dot{\vec{r}}_j + \rho_* \frac{k_j}{2\pi} \left[k_i \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|^2} + k_j \frac{\vec{r}_j}{R^2 - r_j^2} + k_i \frac{R^2 \vec{r}_i - r_i^2 \vec{r}_j}{R^4 - 2R^2 \vec{r}_i \vec{r}_j + r_i^2 r_j^2} \right] - m_j \omega_b^2 \vec{r}_j ,$$

Predictions of the Massive Point Vortex Model

$$m_j \ddot{\vec{r}}_j = k_j \rho_* \vec{u}_3 \wedge \dot{\vec{r}}_j + \rho_* \frac{k_j}{2\pi} \left[k_i \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|^2} + k_j \frac{\vec{r}_j}{R^2 - r_j^2} + k_i \frac{R^2 \vec{r}_i - r_i^2 \vec{r}_j}{R^4 - 2R^2 \vec{r}_i \vec{r}_j + r_i^2 r_j^2} \right] - m_j \omega_b^2 \vec{r}_j ,$$

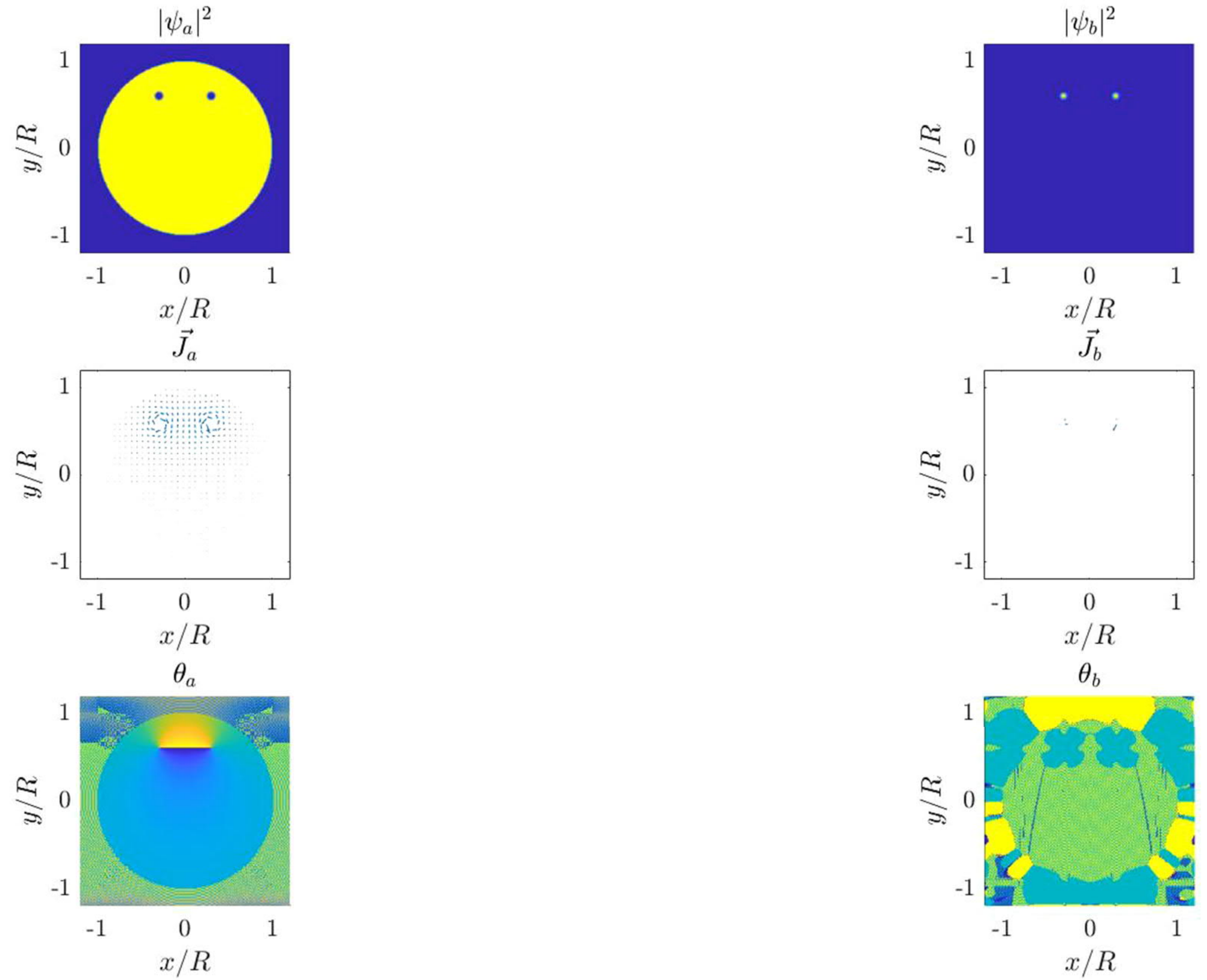


Pair of counter-rotating massive vortices.

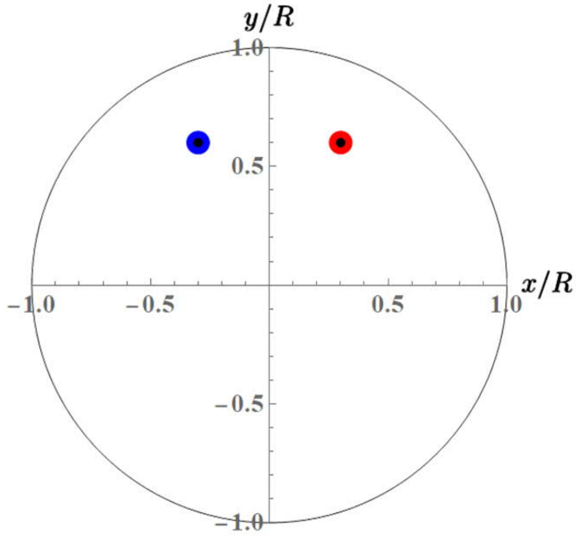


Pair of co-rotating massive vortices.

t = 0.000 s

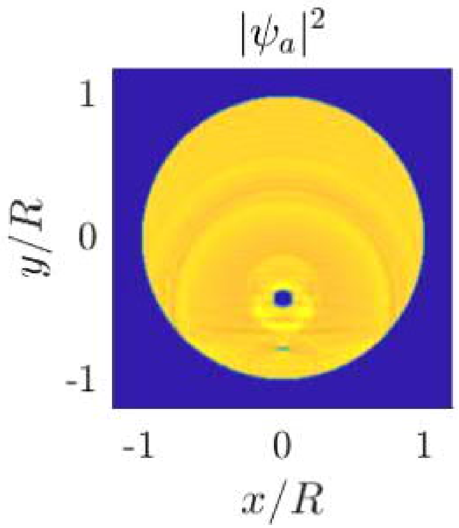


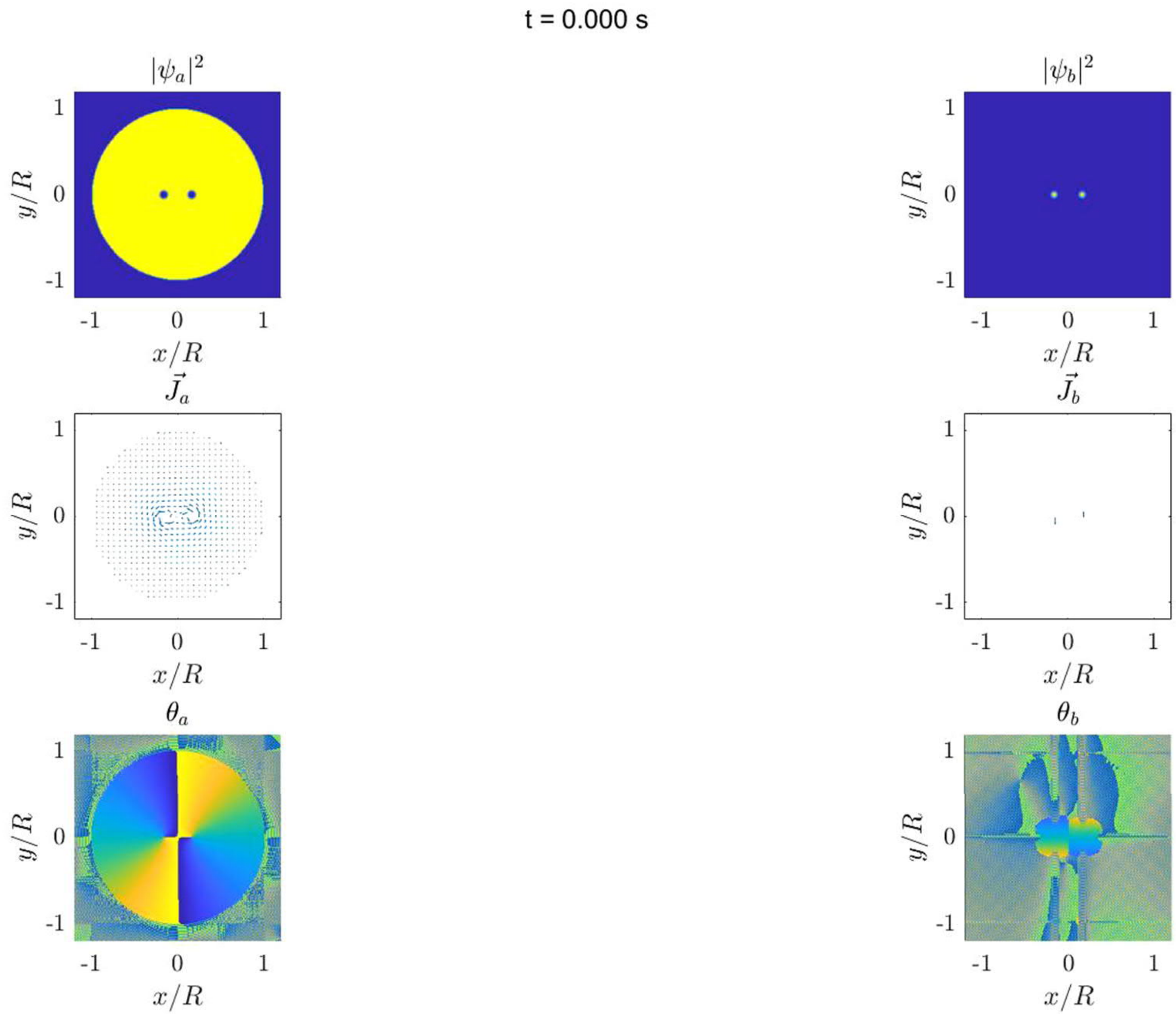
Collision of vortex/antivortex pair



Mutual annihilation of the vortices

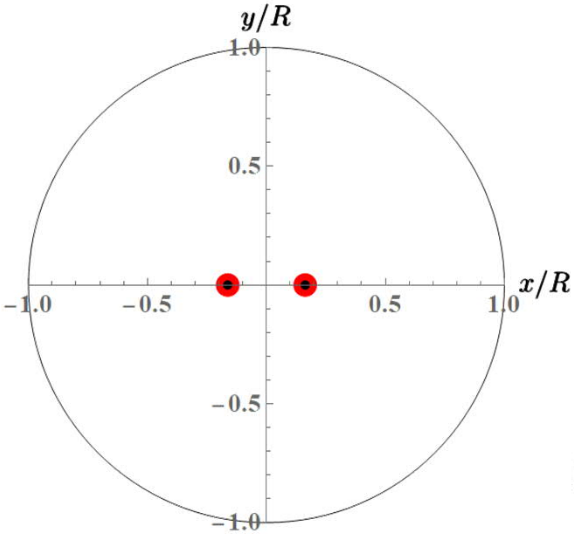
Sound-wave explosion!



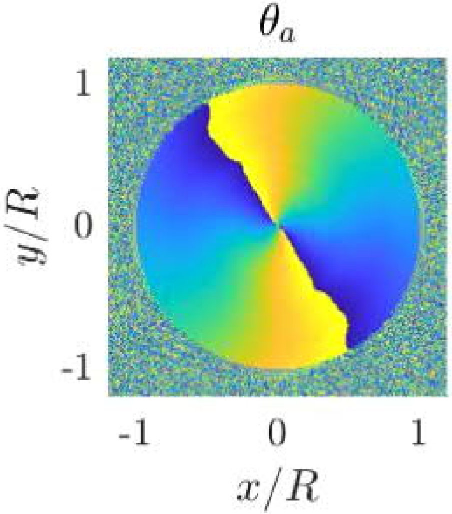


t = 0.000 s
$|\psi_a|^2$
$|\psi_b|^2$
$\vec{J}_a$
$\vec{J}_b$
θ_a
θ_b
x/R
y/R
-1
0
1

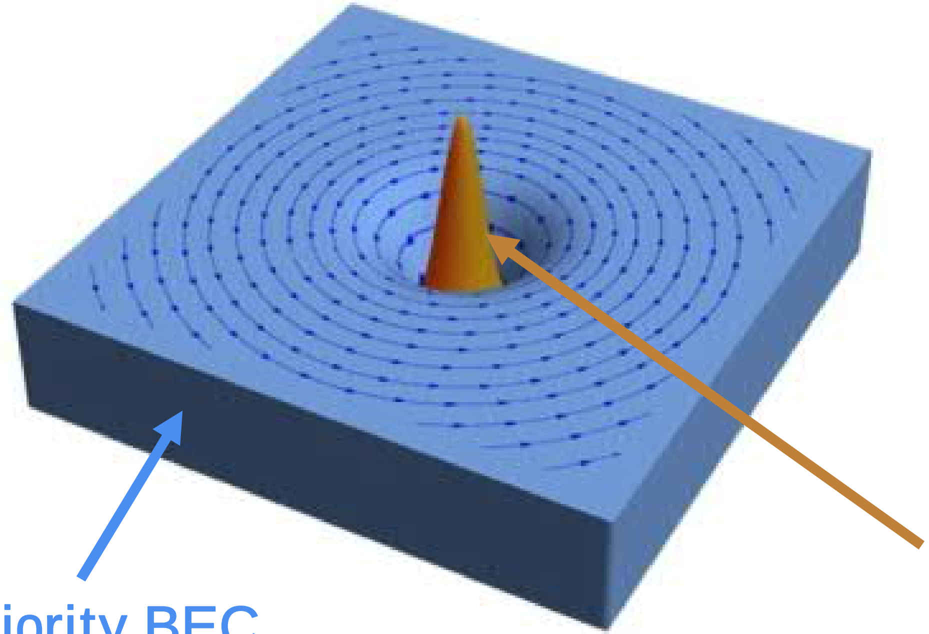
Collision of vortex/vortex pair



Stabilization of a double-charge vortex with filled core!



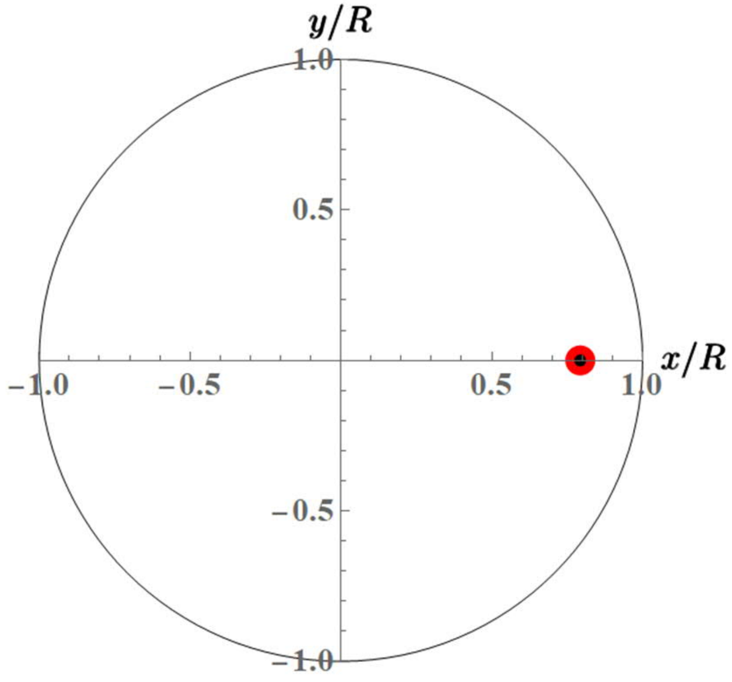
Take-home message 1/3



Superfluid vortices are often filled by massive cores (deliberately or accidentally!)

- Tracers particles
- Thermal atoms
- Quasiparticle bound states
- Another (minority) BEC

Take-home message 2/3



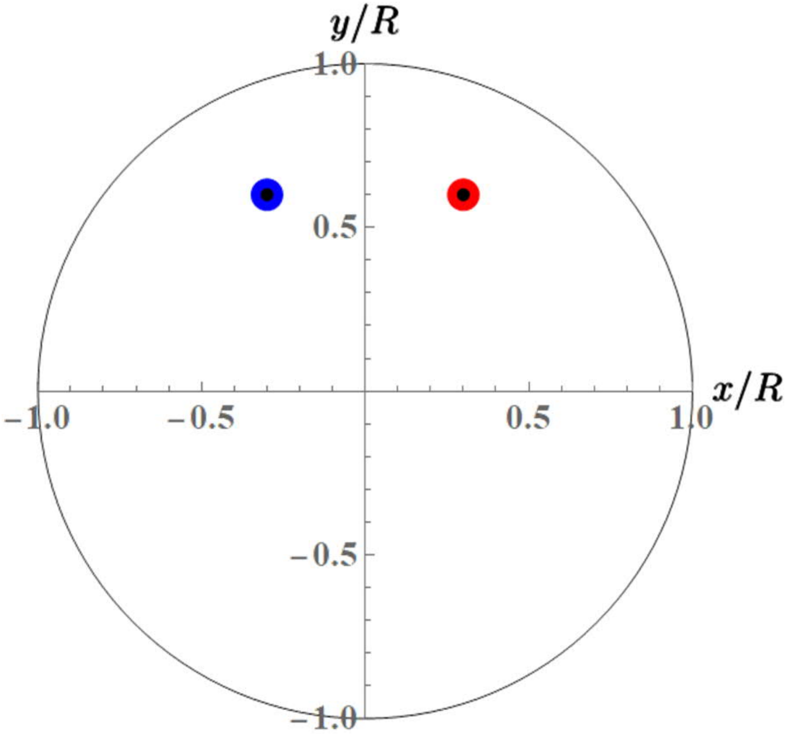
Usual circular motion of precession

+

Small-amplitude radial oscillations

The signature of core mass!

Take-home message 3/3



Massive cores are not only “heavy burdens” but can be used to actively drive the hosting vortices and thus trigger vortex collisions.

Thanks for your attention!

QUESTIONS ?

[A. Richaud, V. Penna, R. Mayol, M. Guilleumas, *Phys. Rev. A* **101**, 013630 (2020)]
[A. Richaud, V. Penna, A. L. Fetter, *Phys. Rev. A* **103**, 023311 (2021)]
[A. Richaud, G. Lamporesi, M. Capone, A. Recati, *in preparation*]

Info: **arichaud@sissa.it**