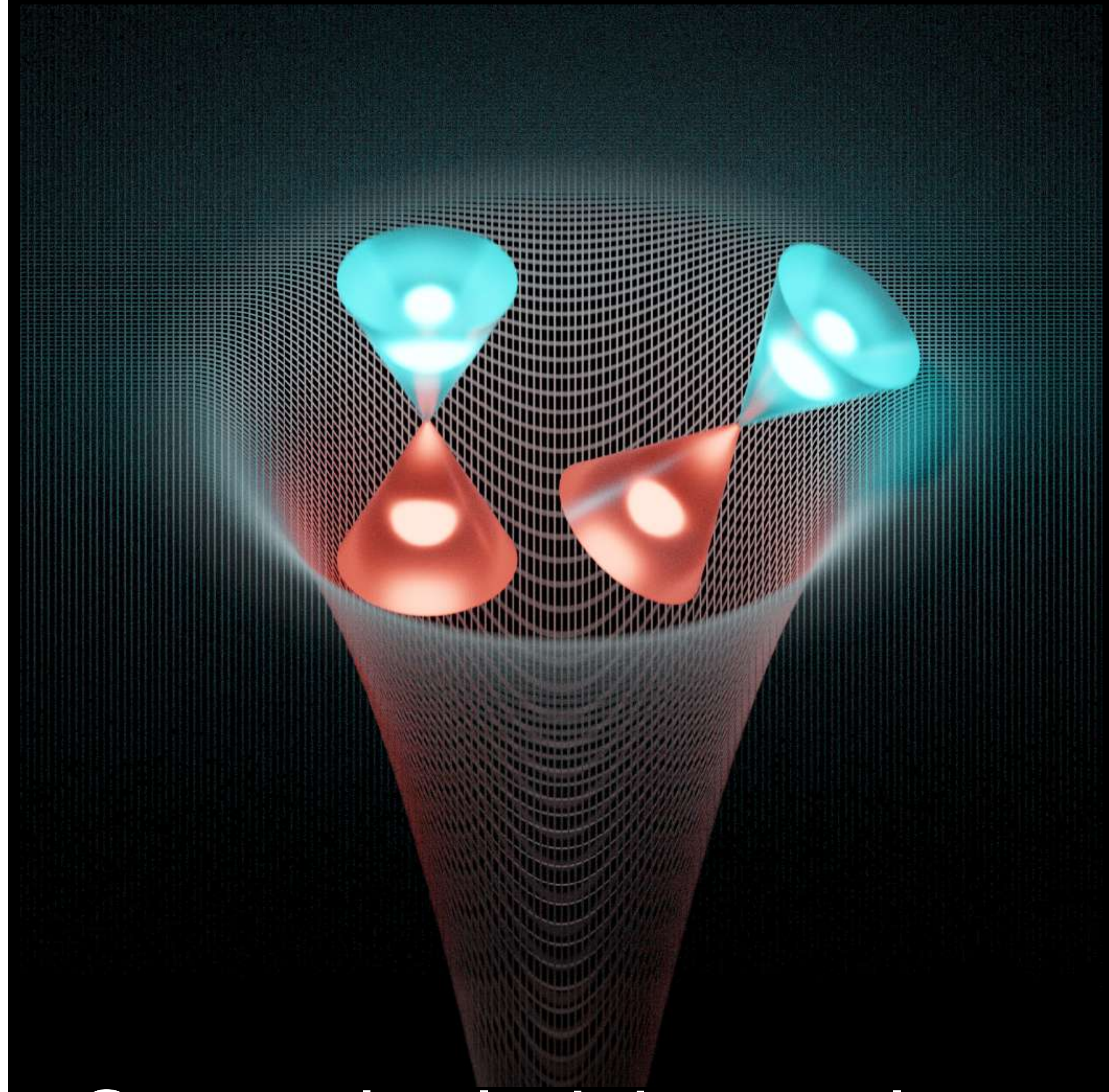
22.02.2022, UPC Barcelona

Quantum skyrmion lattices in Heisenberg ferromagnets

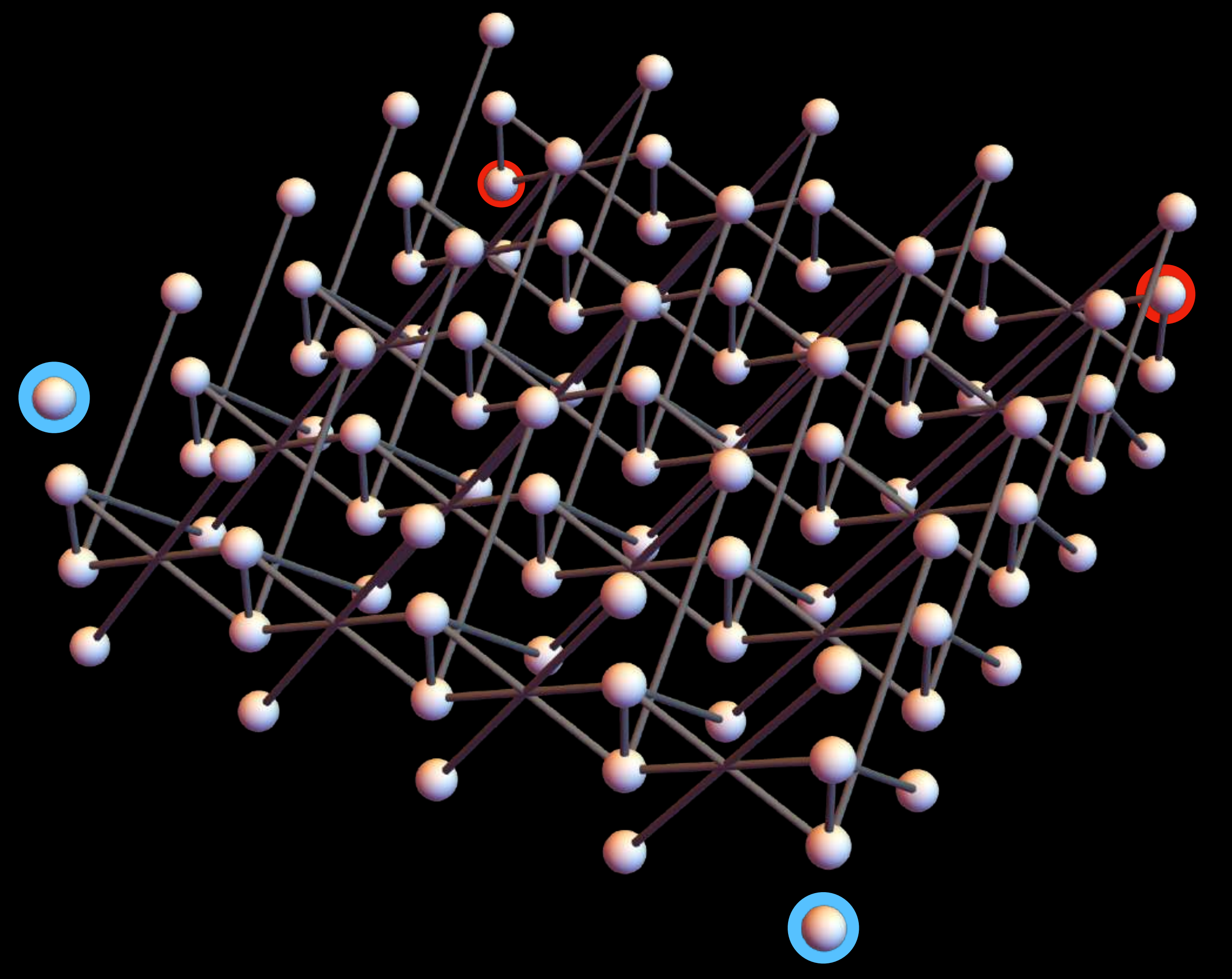
Andreas Haller, Department of Physics and Materials Science, University of Luxembourg

Outline

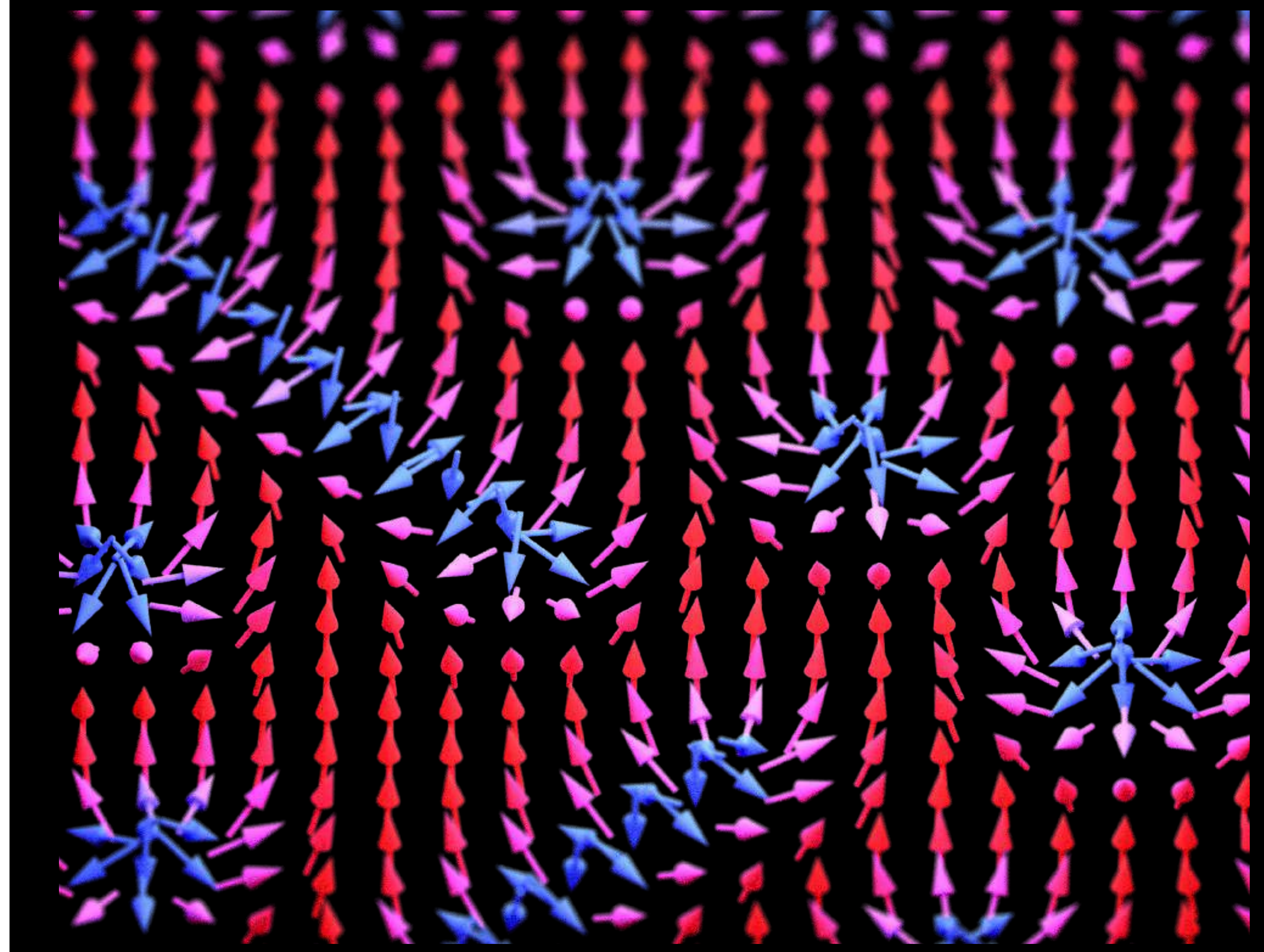
- Skyrmions
- Quantum skyrmion lattices
- Future directions
- Other research projects



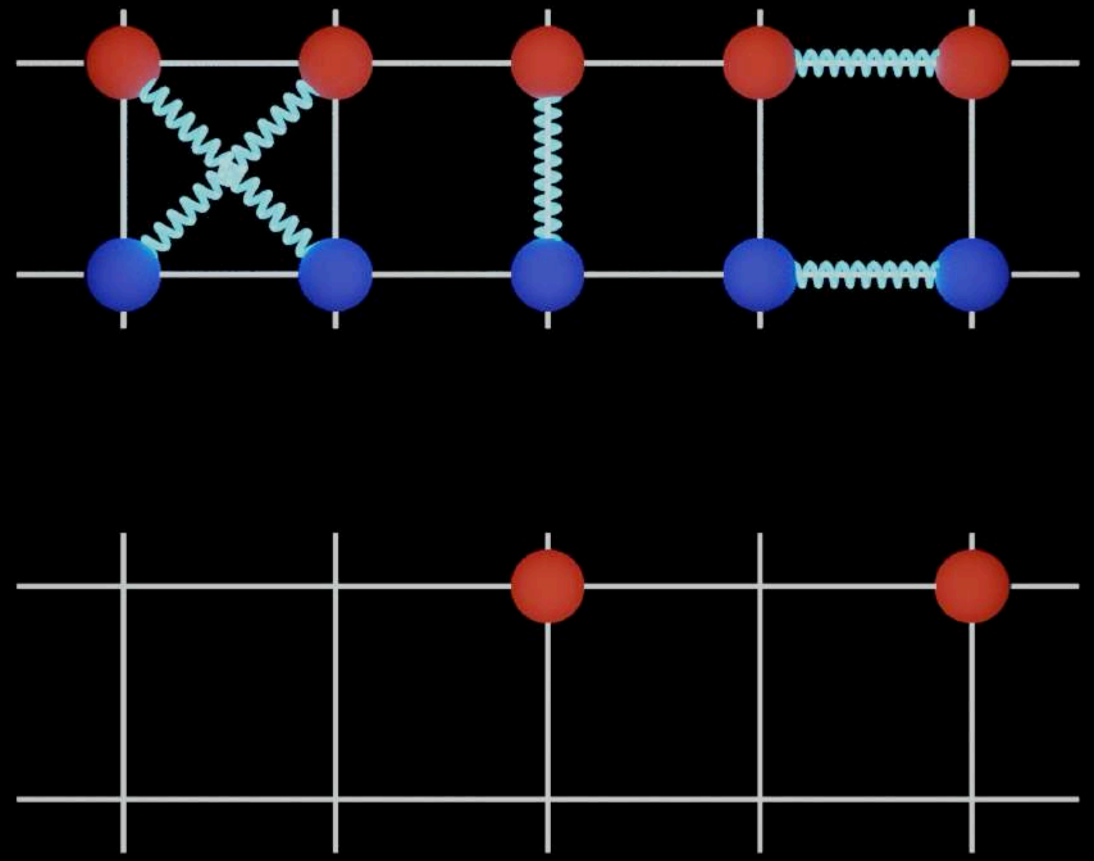
General relativity analogs in tilted Weyl semimetals



Cookbook for parafermion HOTIs



Quantum Spintronics



Interacting ladder systems

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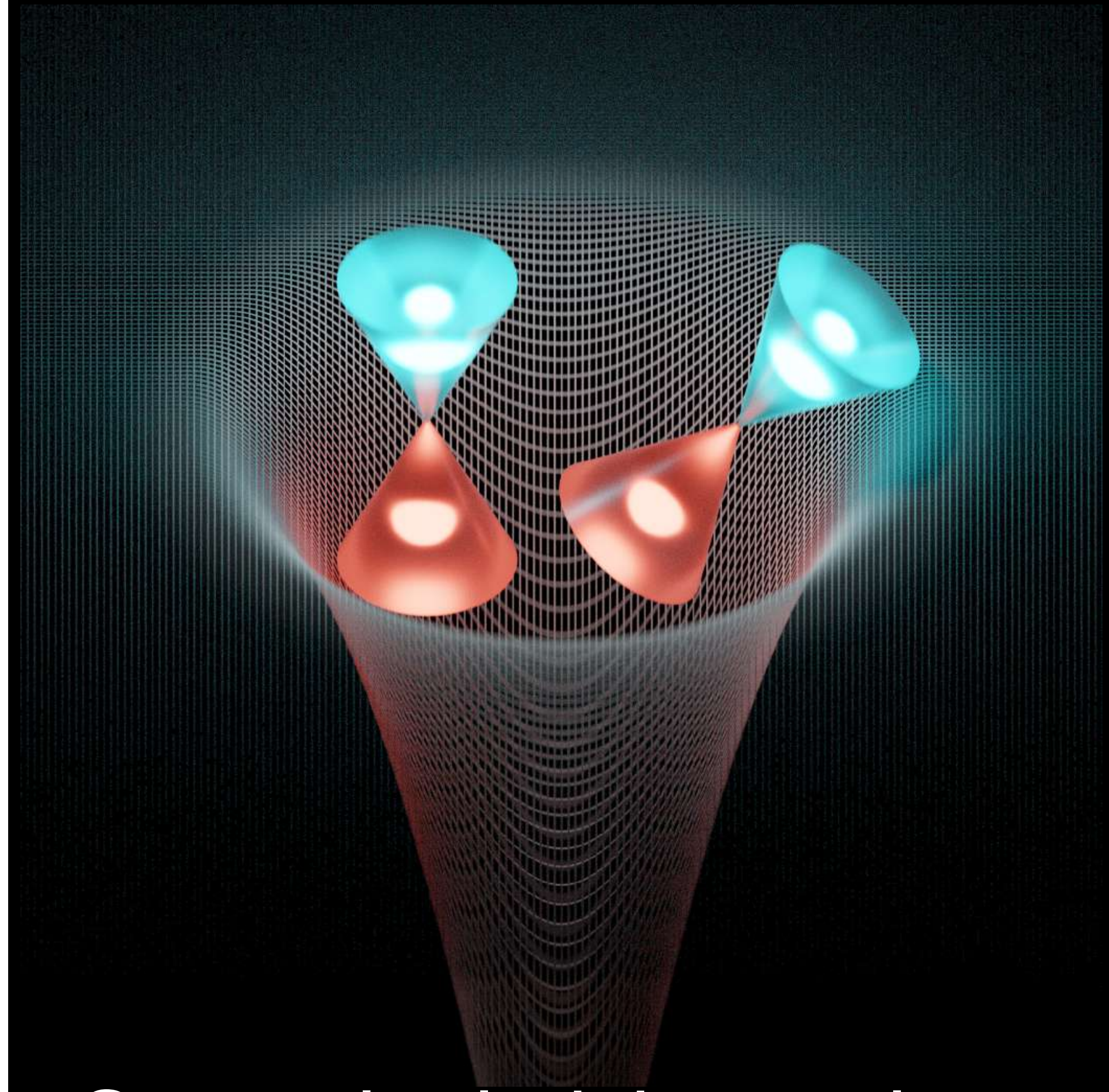

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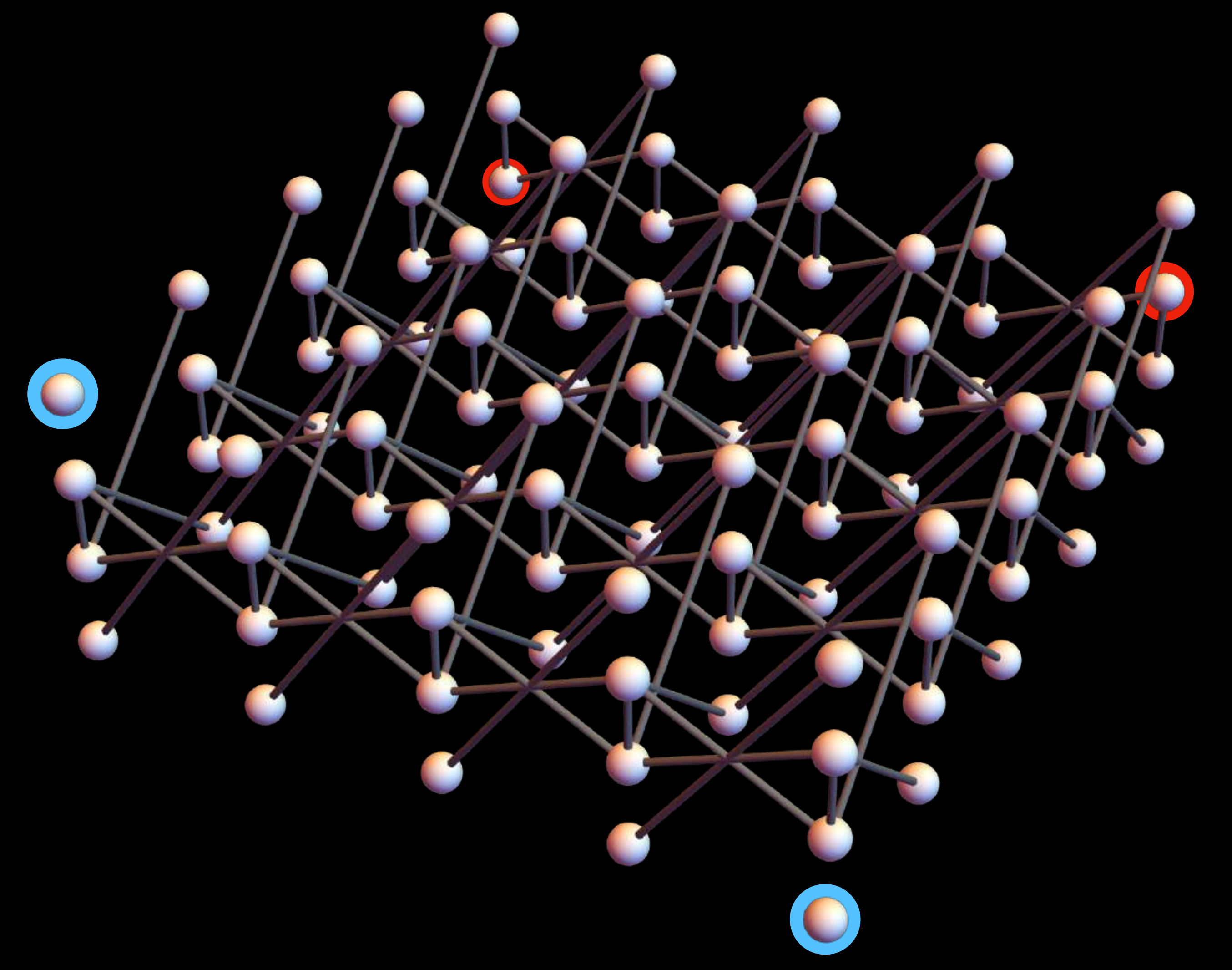

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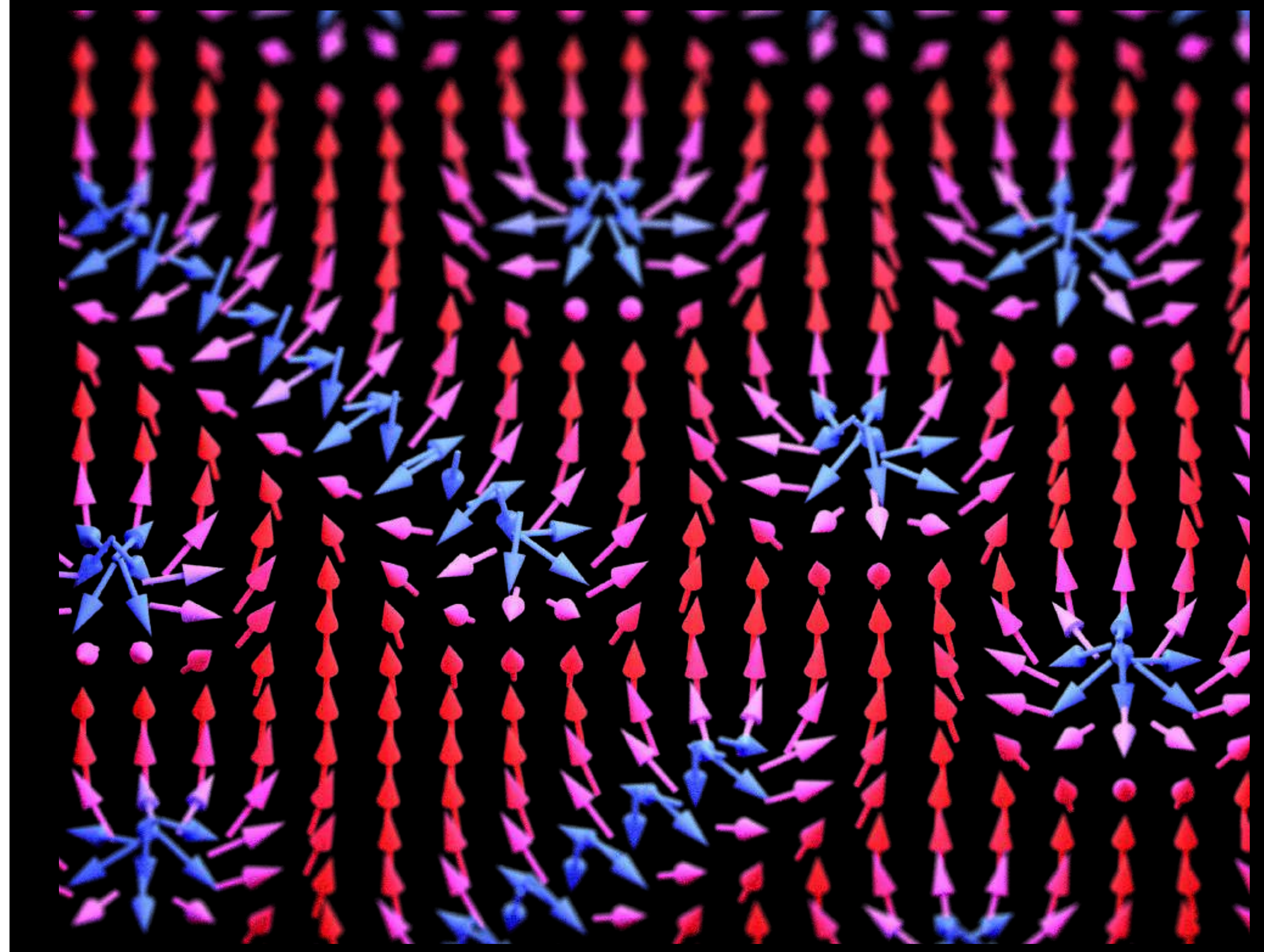

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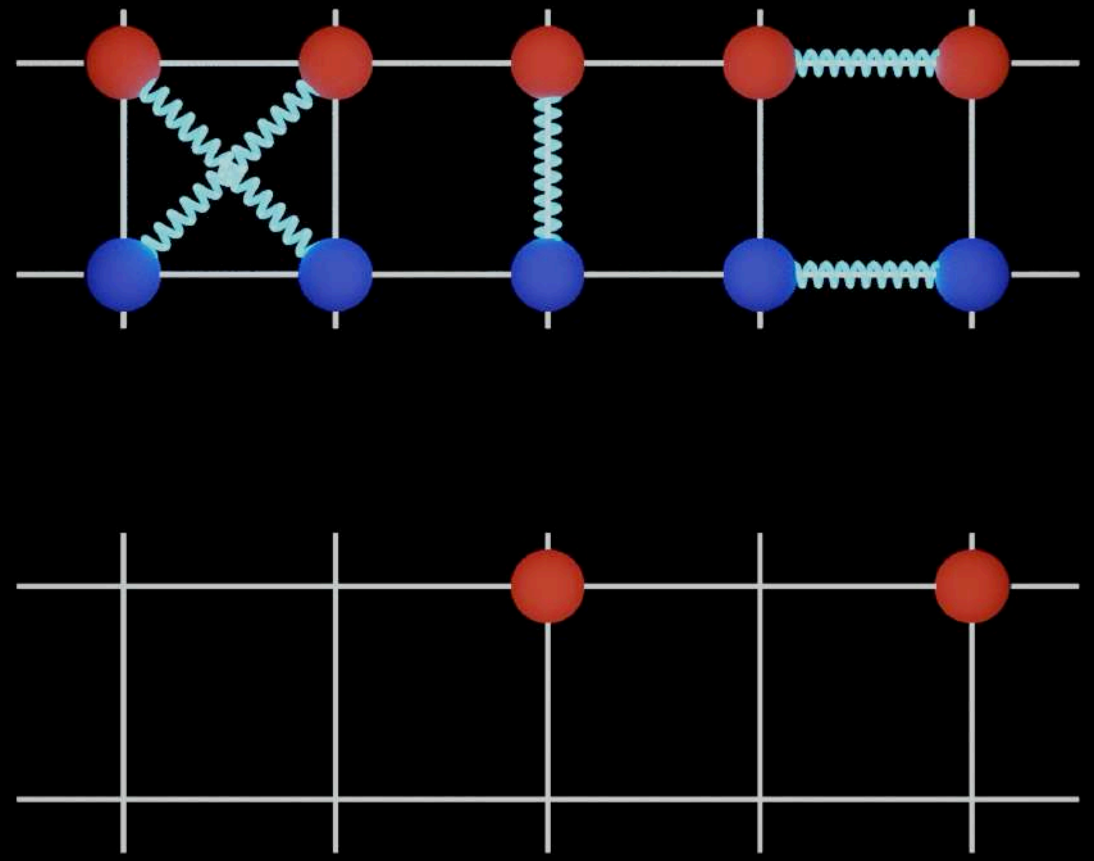

Interacting ladder systems

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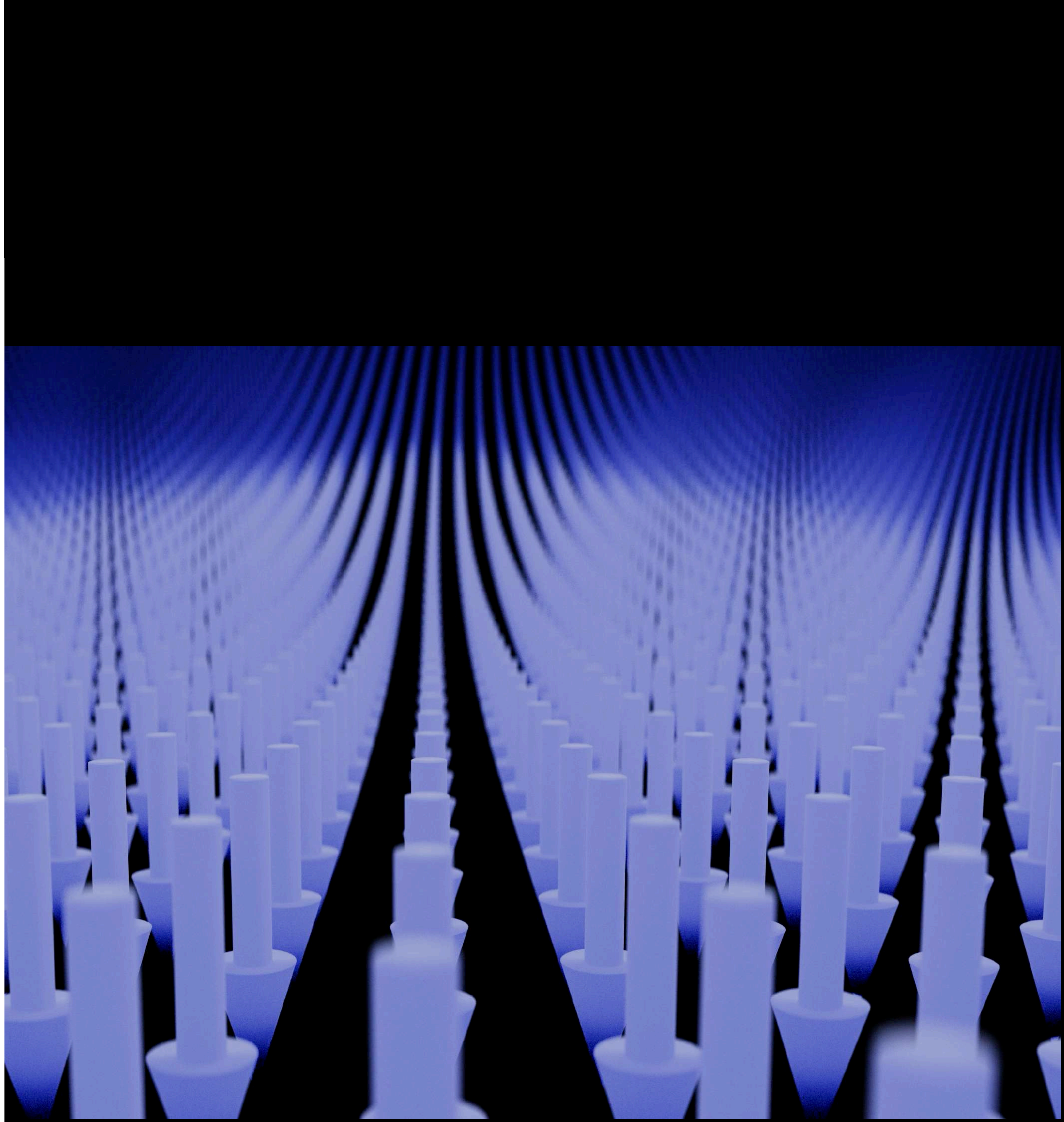
Nature 442, 797 (2006), Science 323, 915 (2009)
Nature Reviews Materials 1, 1 (2016)



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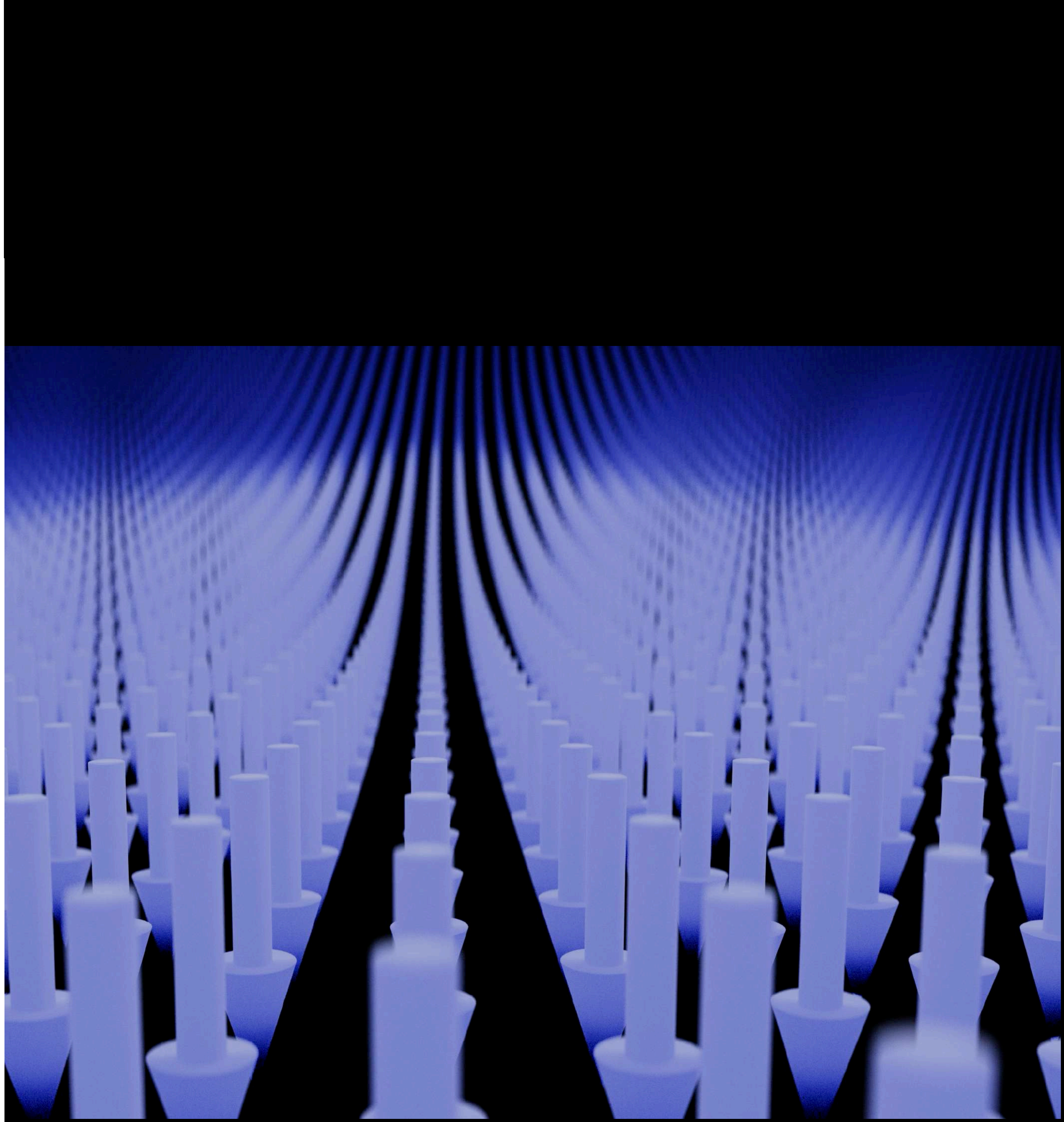

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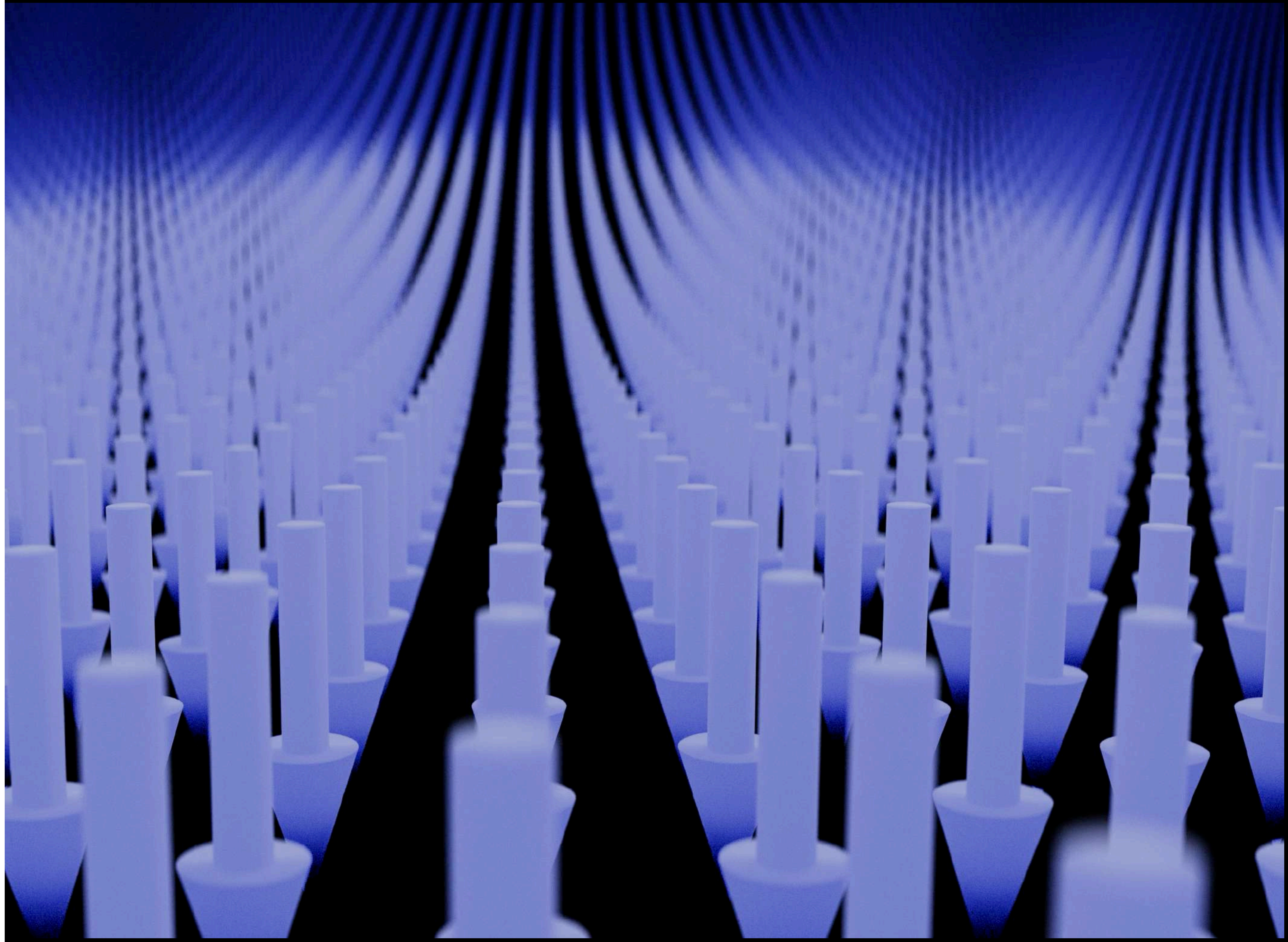

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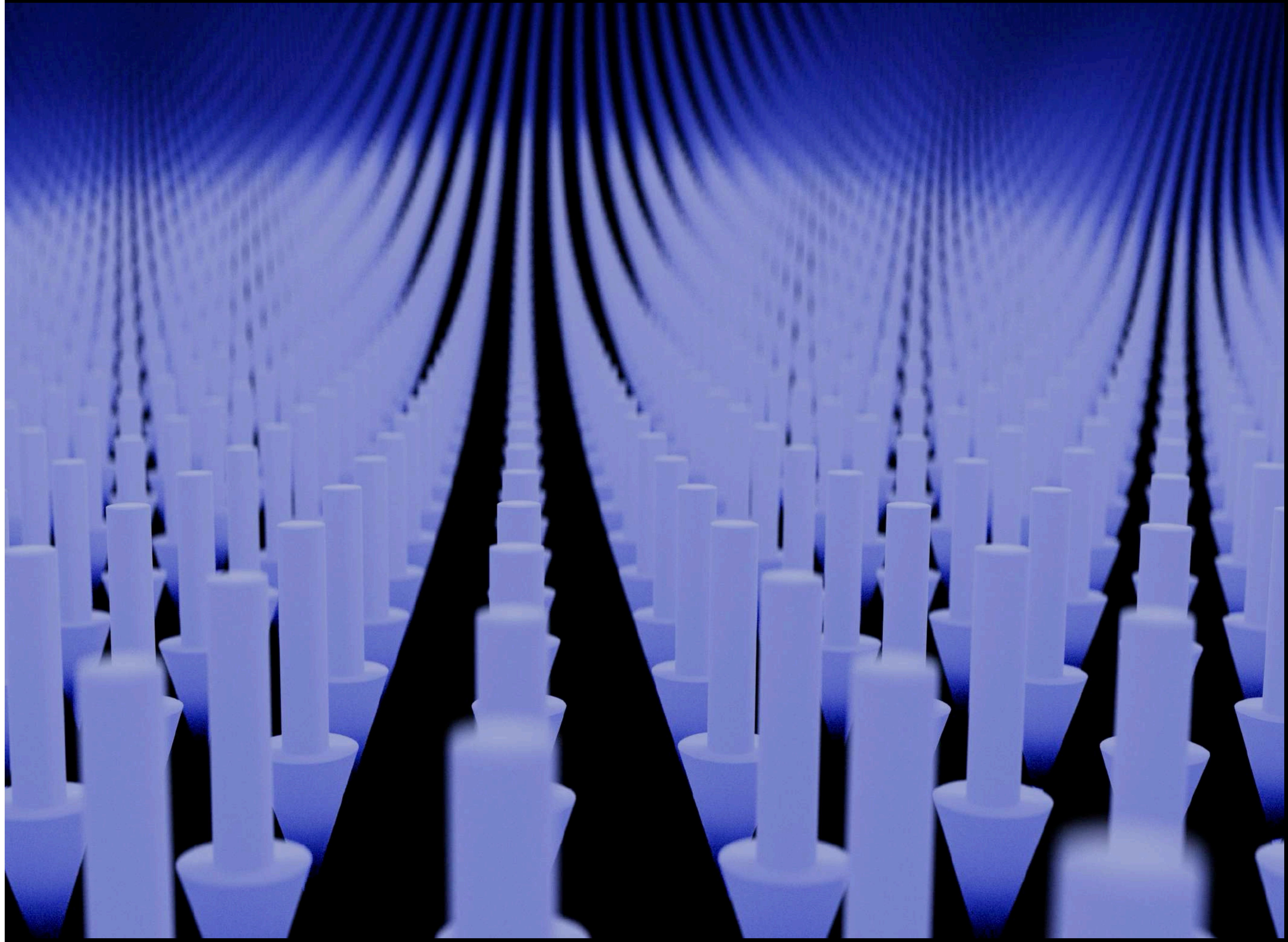

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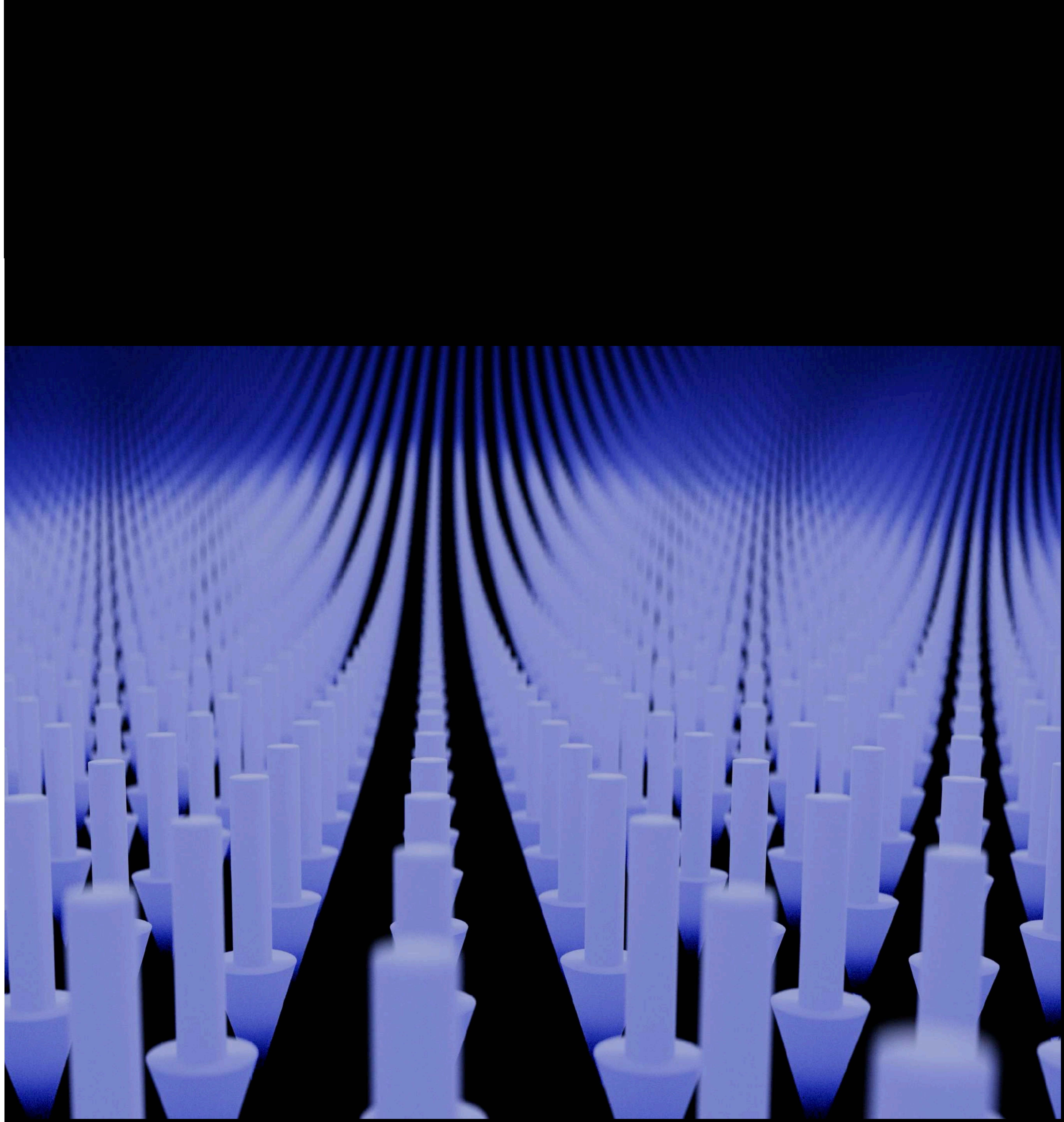

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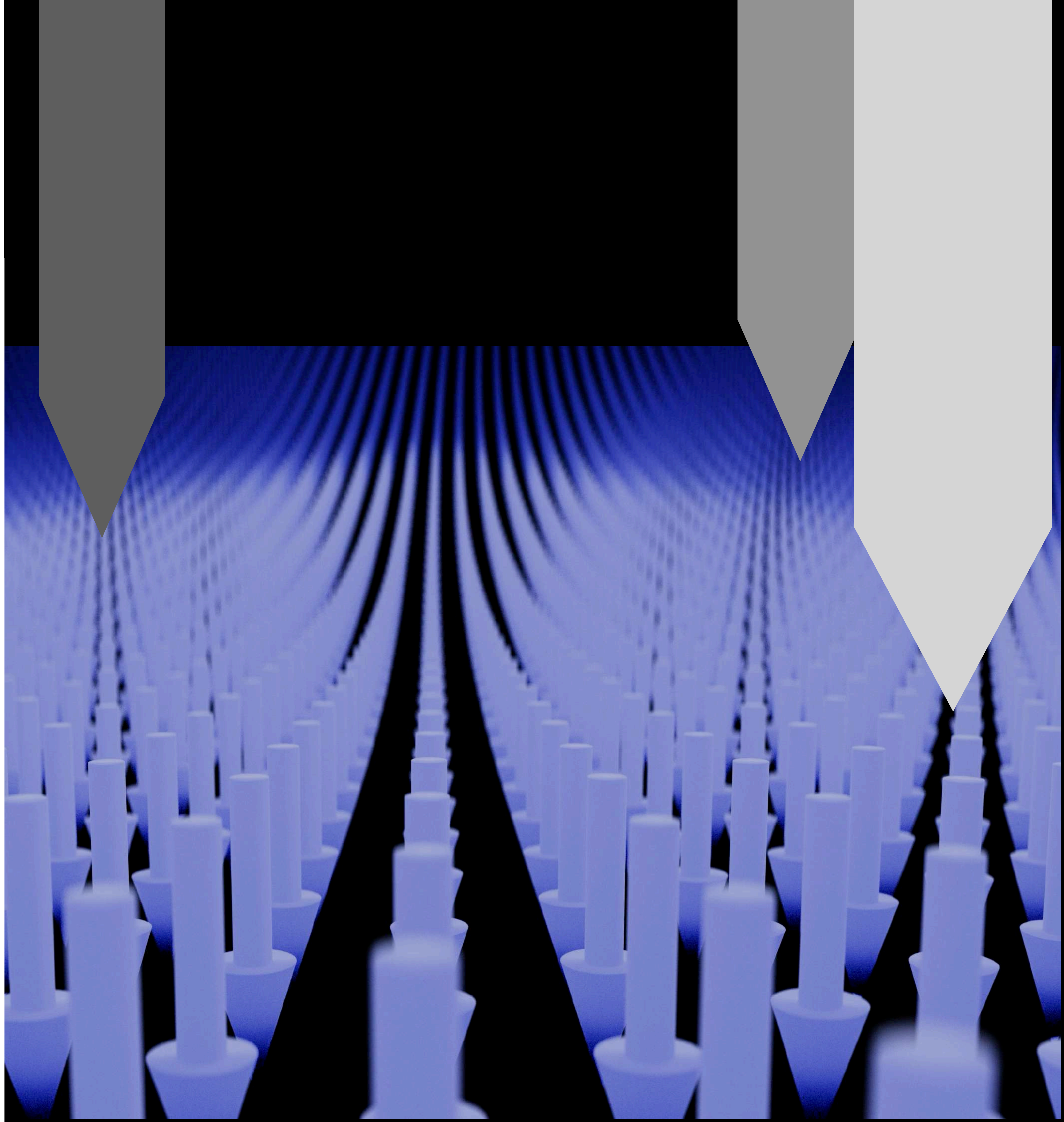

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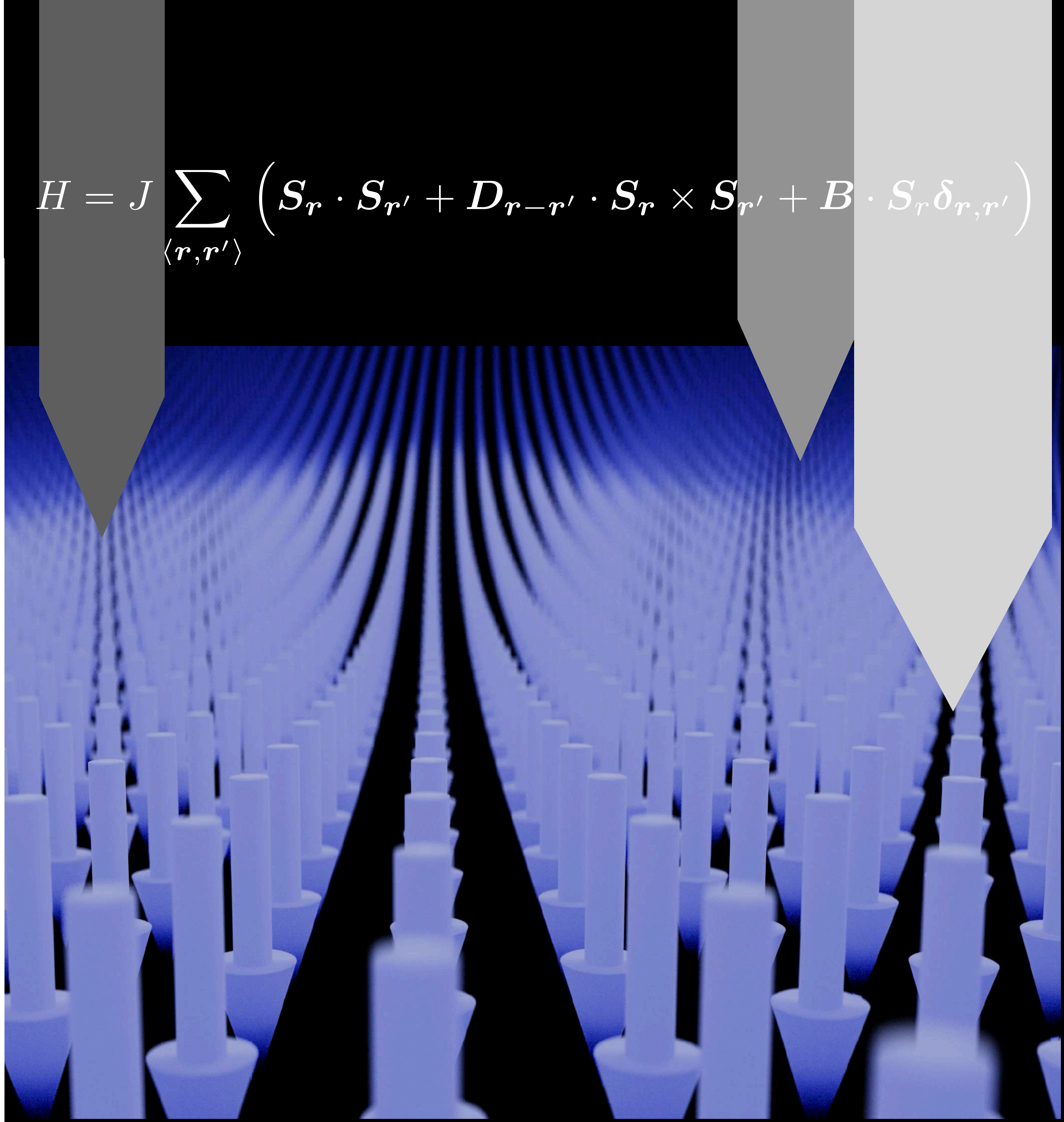

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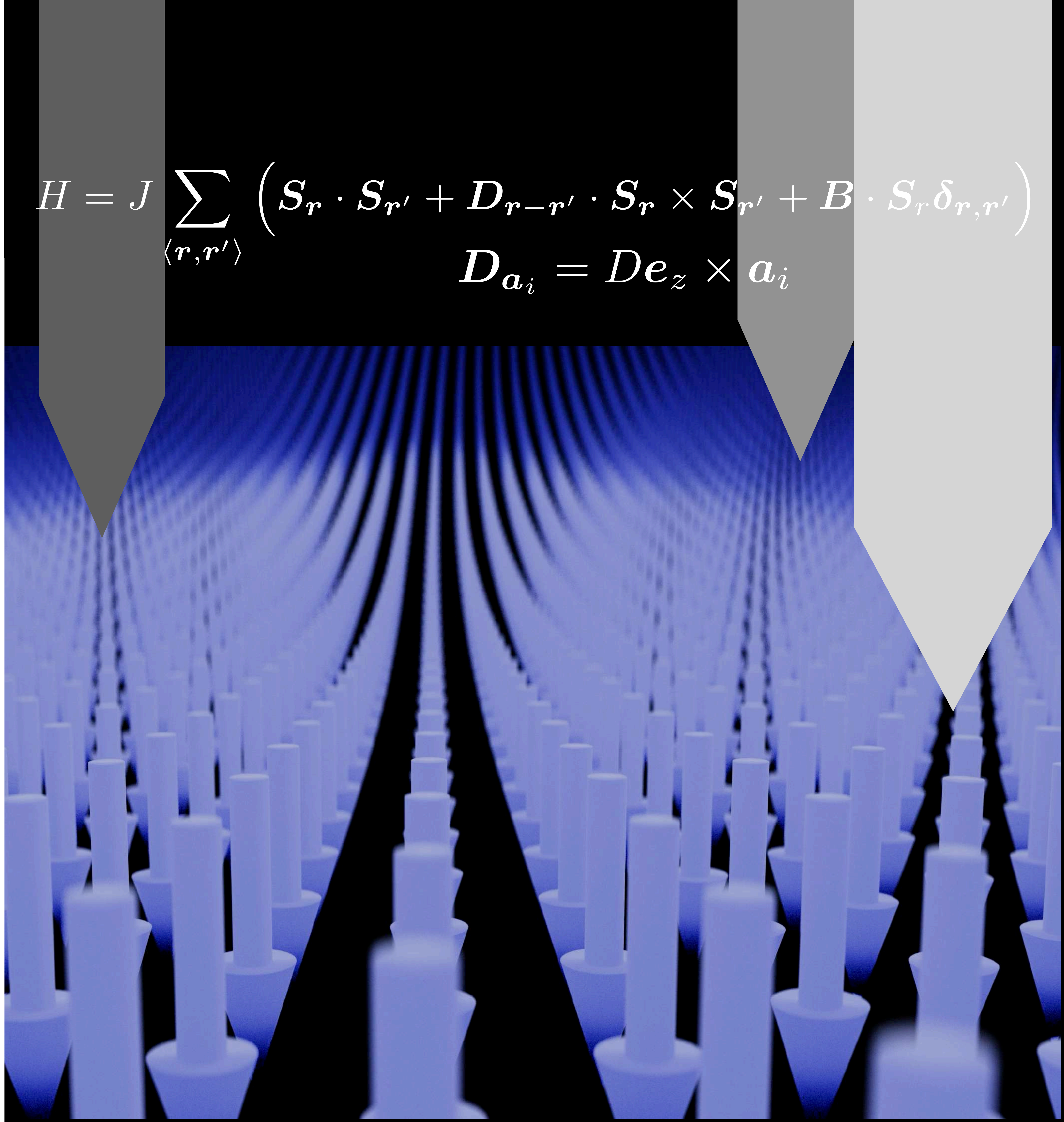

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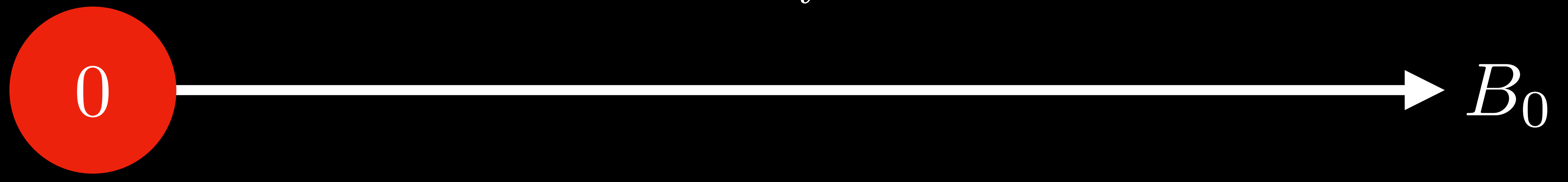

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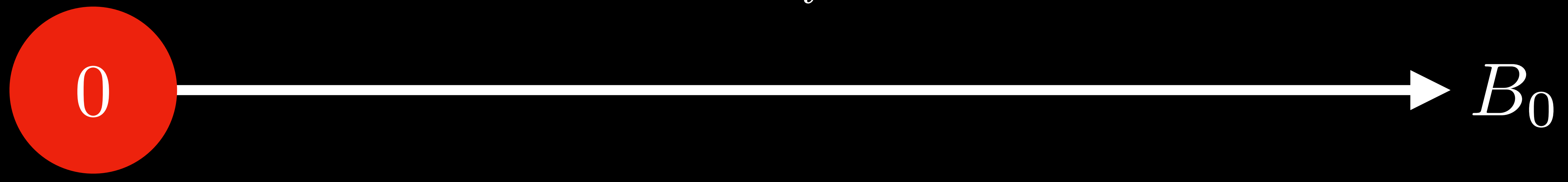
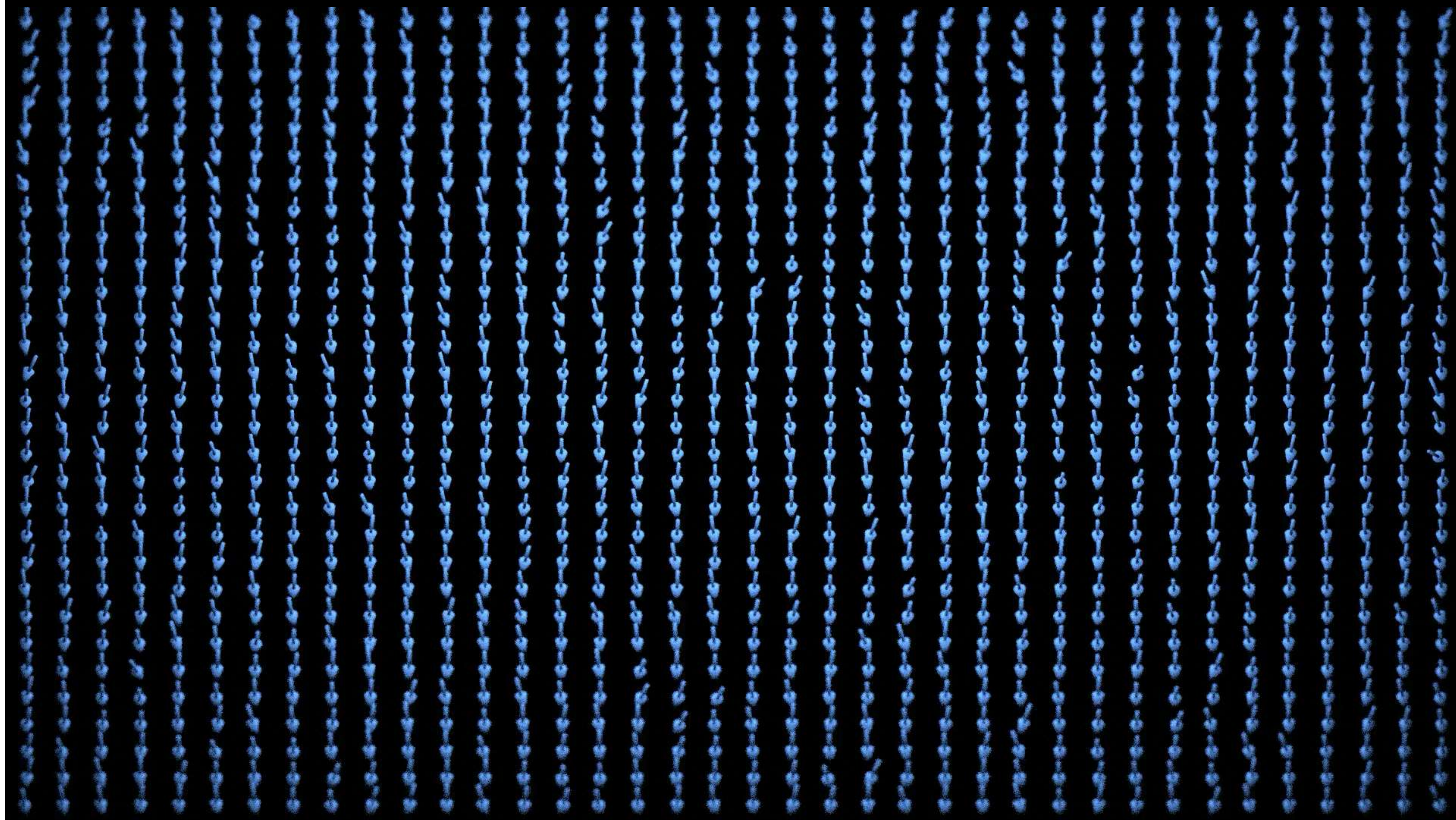


MC sampling with only local rotations



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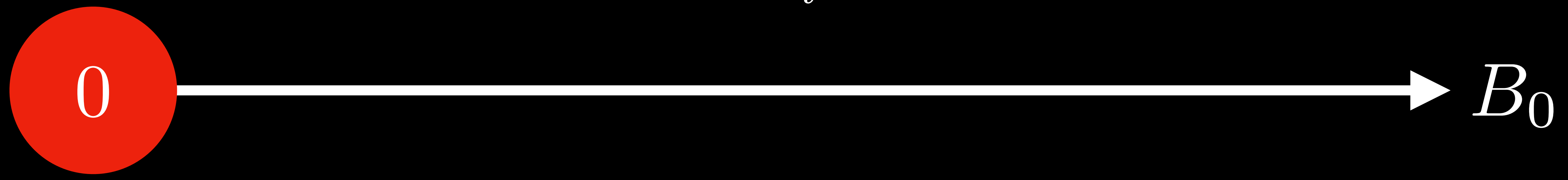

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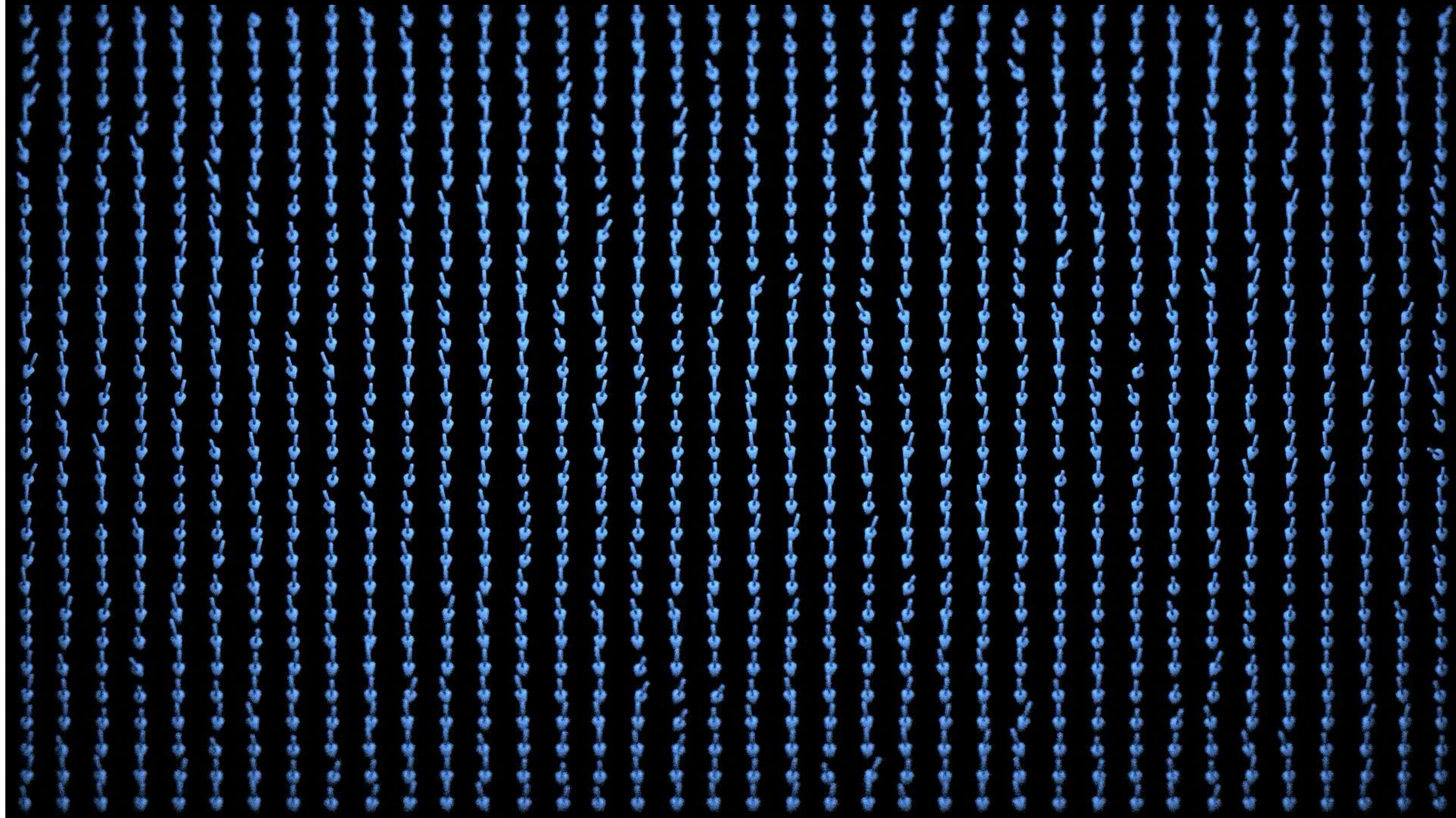

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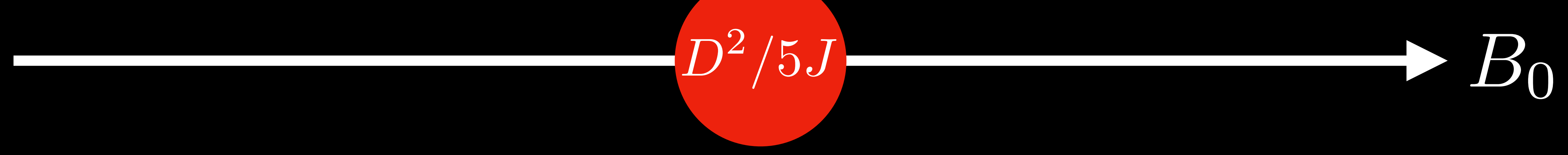

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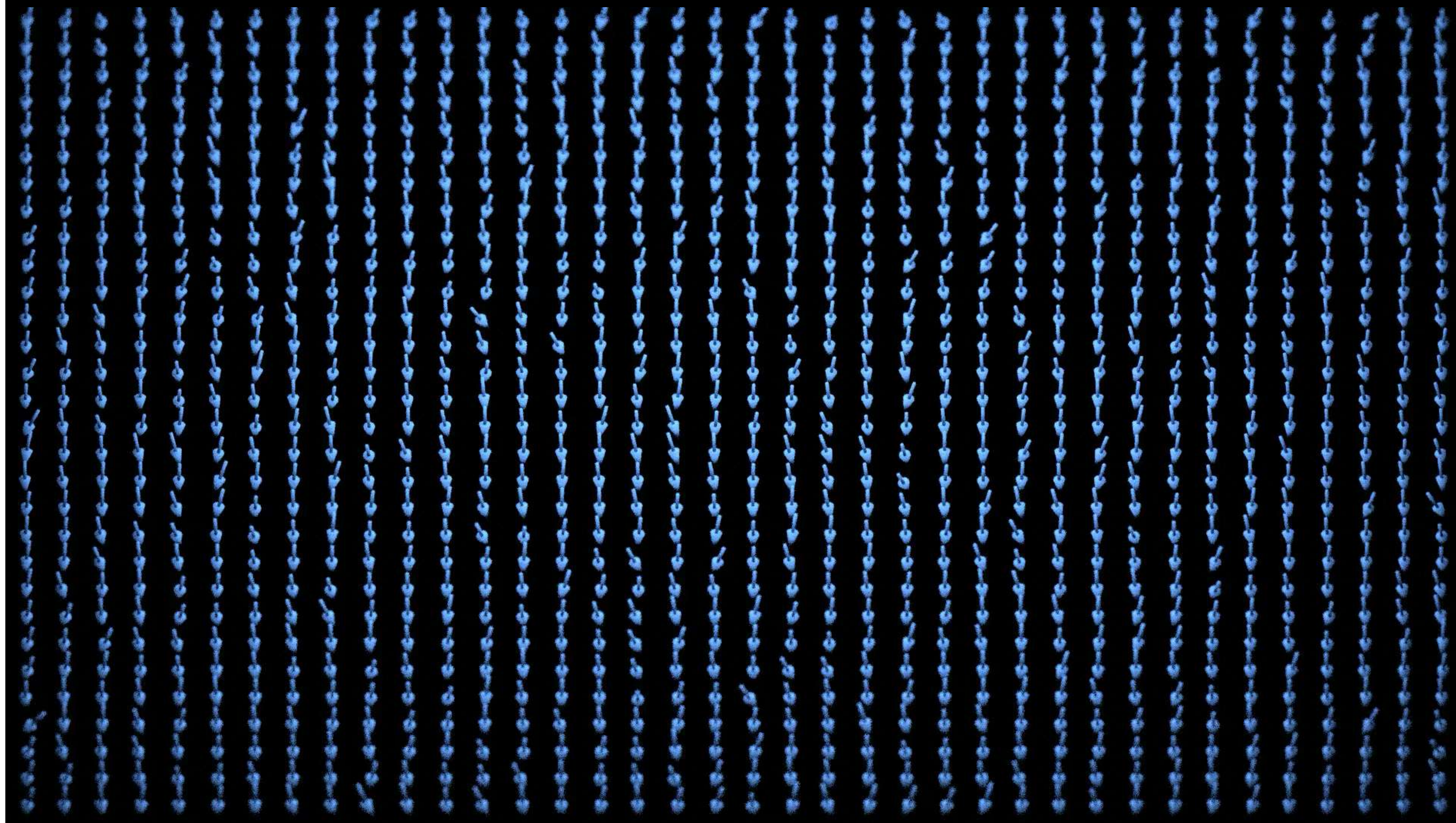

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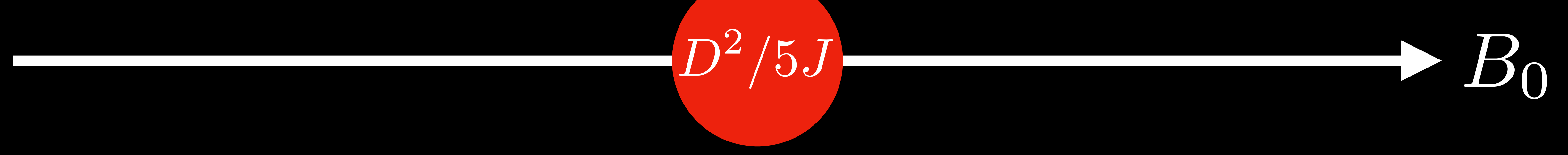

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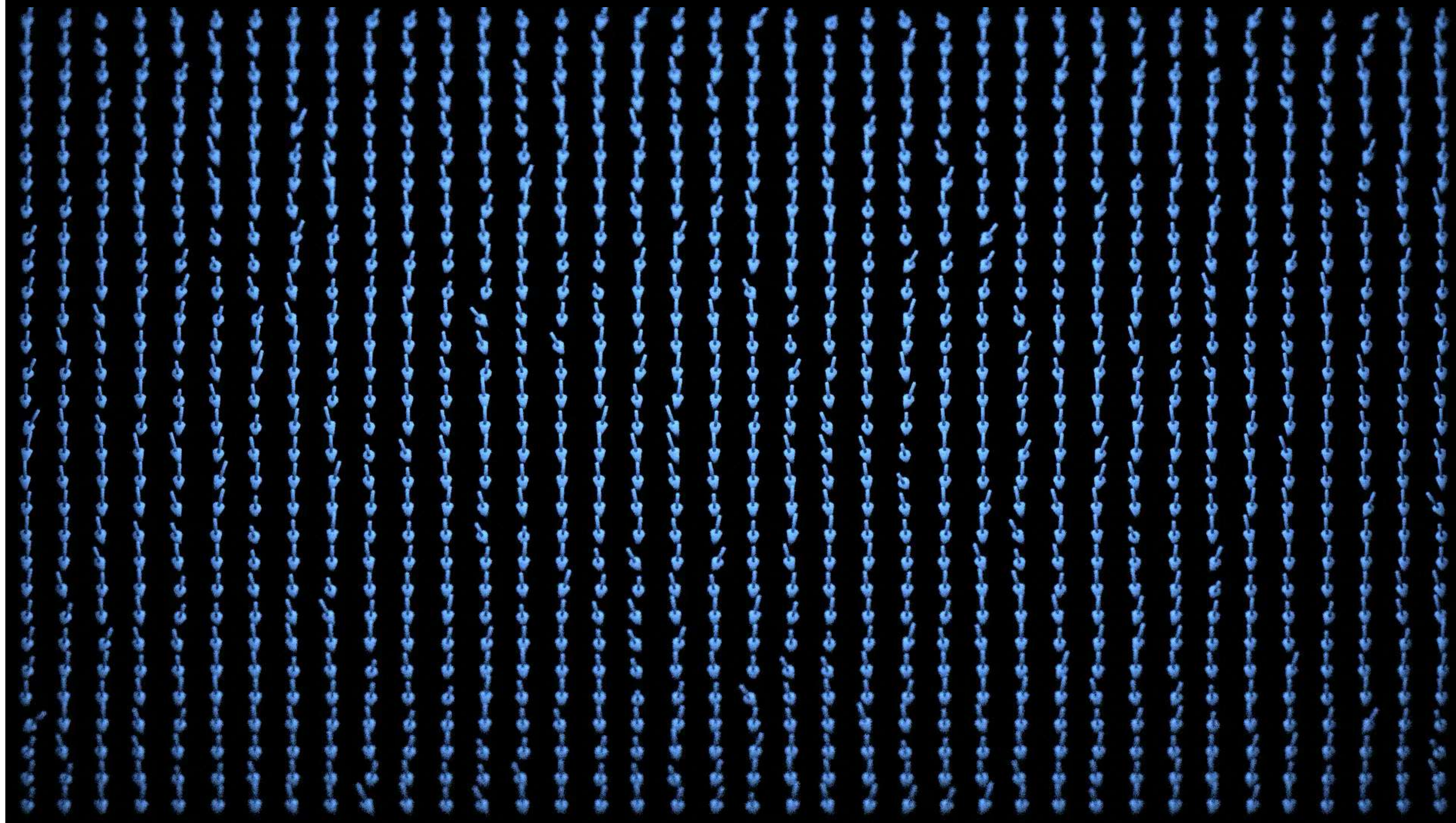

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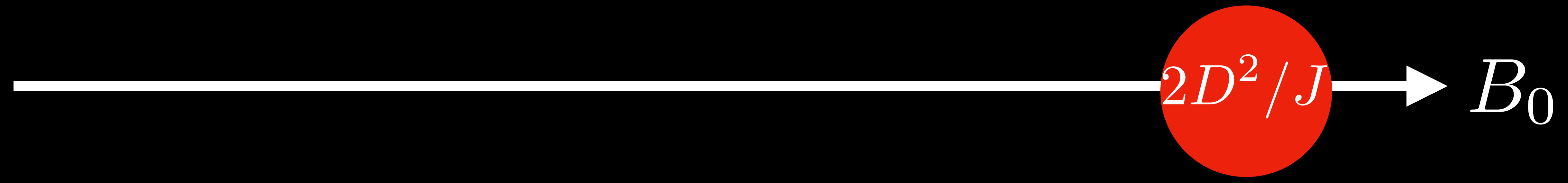

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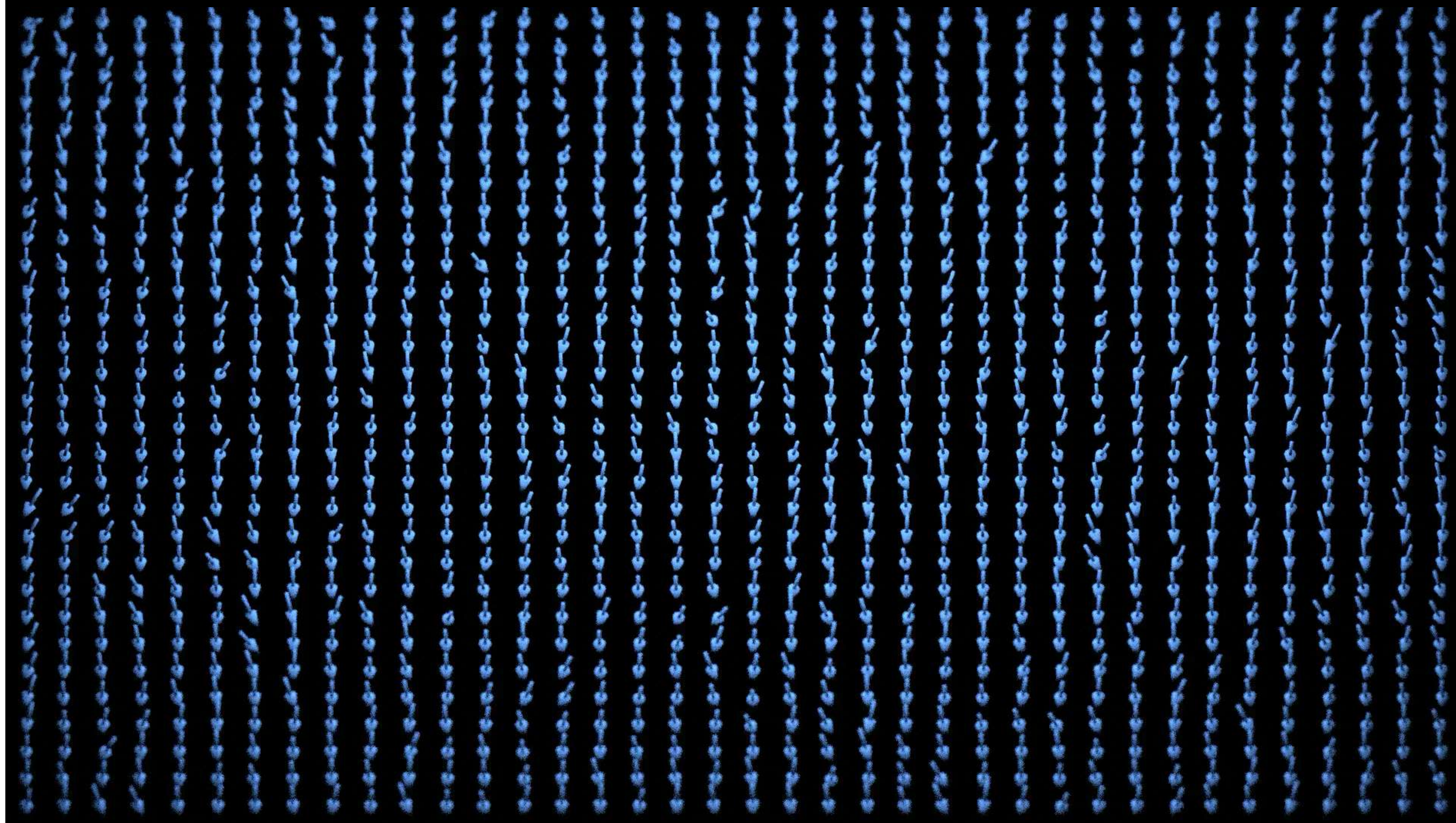

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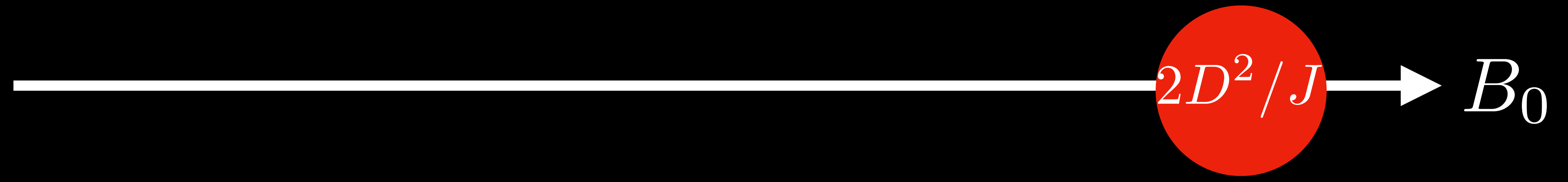

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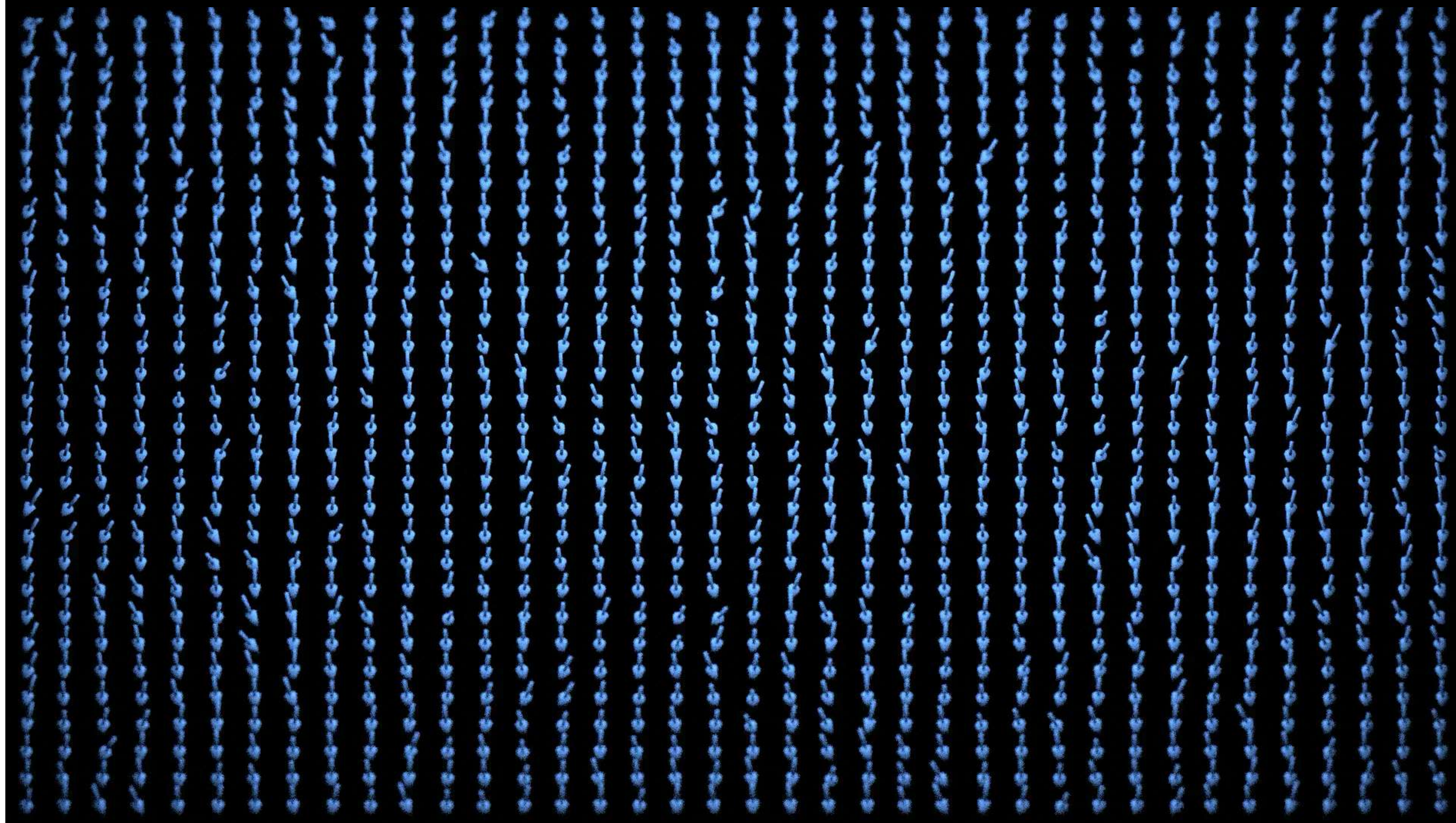

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PRB 92, 245436 (2015), PRX 9, 041063 (2019), PRB 103, L060404 (2021), arXiv:2110.00348(2021)

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- Find the spin configuration $\boldsymbol{\Omega}$ that minimizes H
- Quantize the Hamiltonian $H \to \hat{H}$

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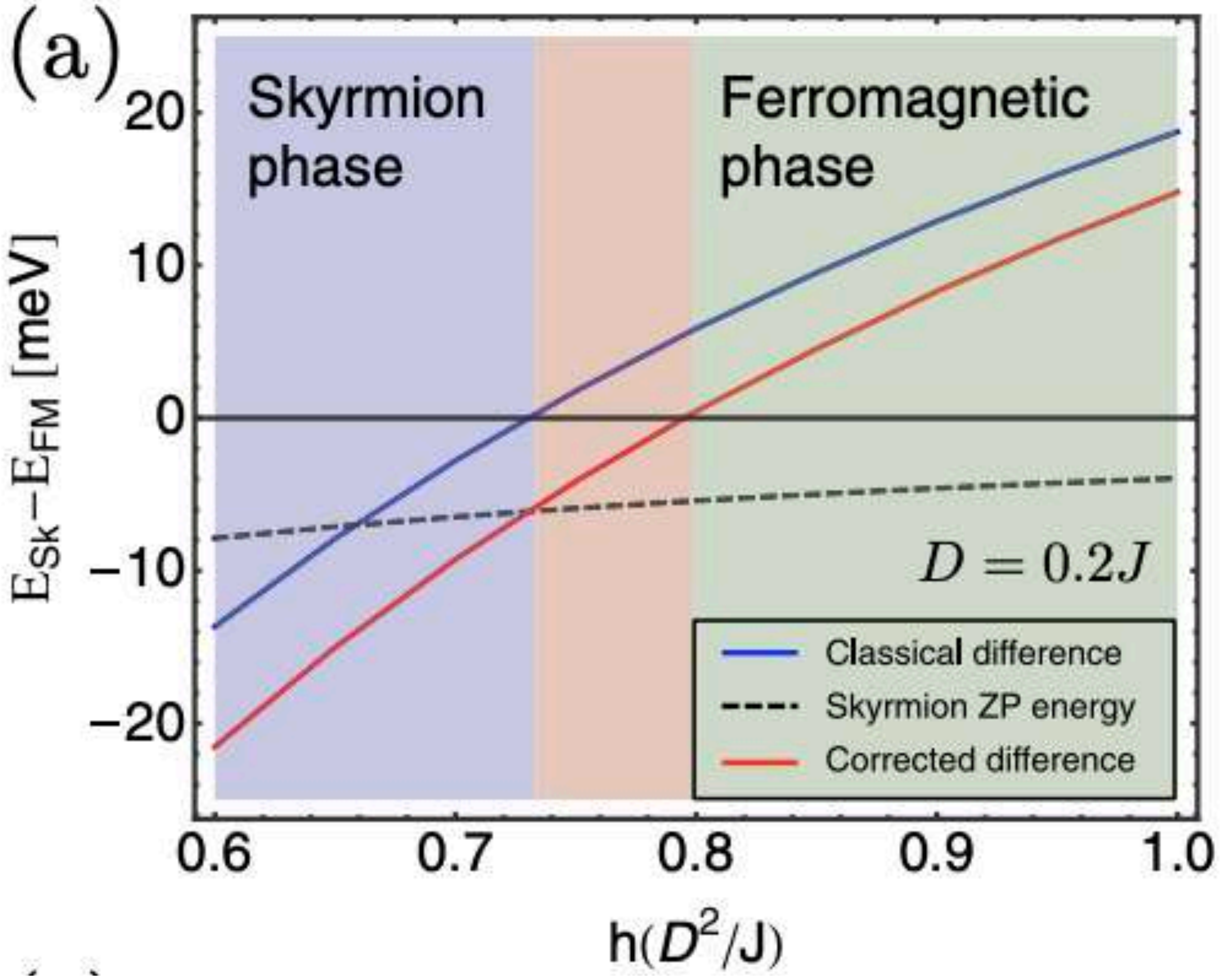

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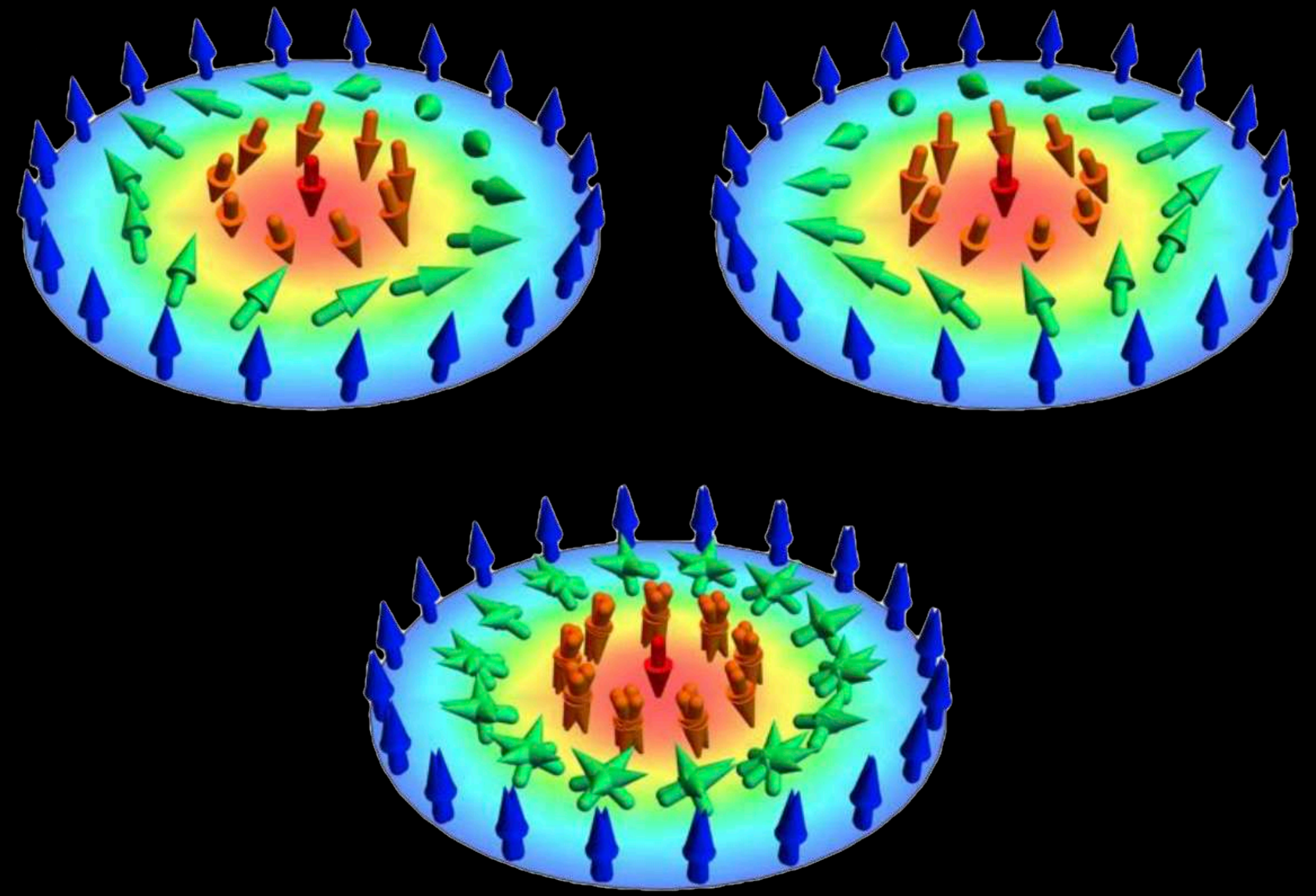

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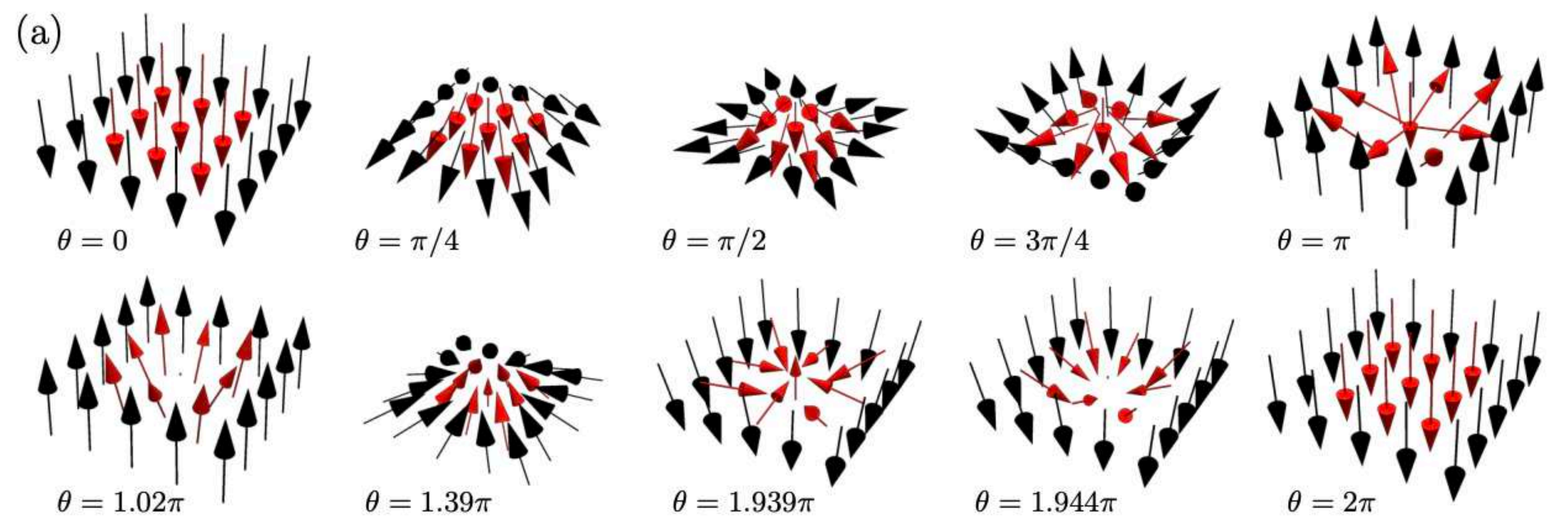

- quantum system (red) coupled to classical magnetic environment (black)

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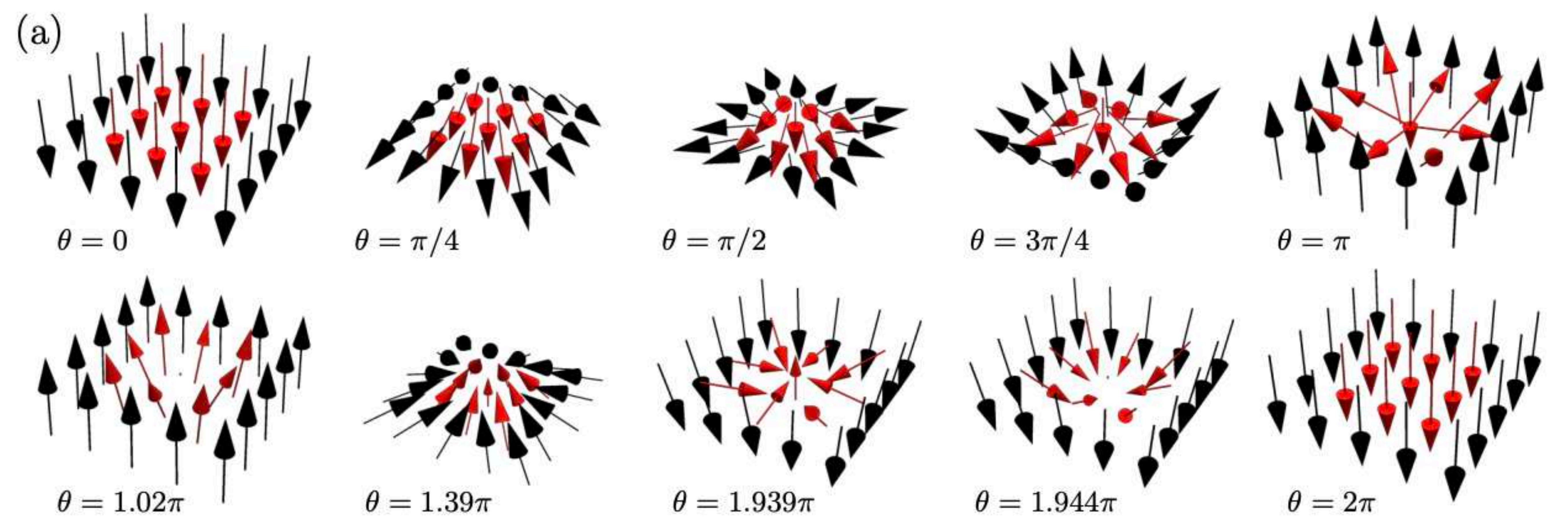

- quantum system (red) coupled to classical magnetic environment (black)
- adiabatic twisting of the environment results in a tiny quantum skyrmion

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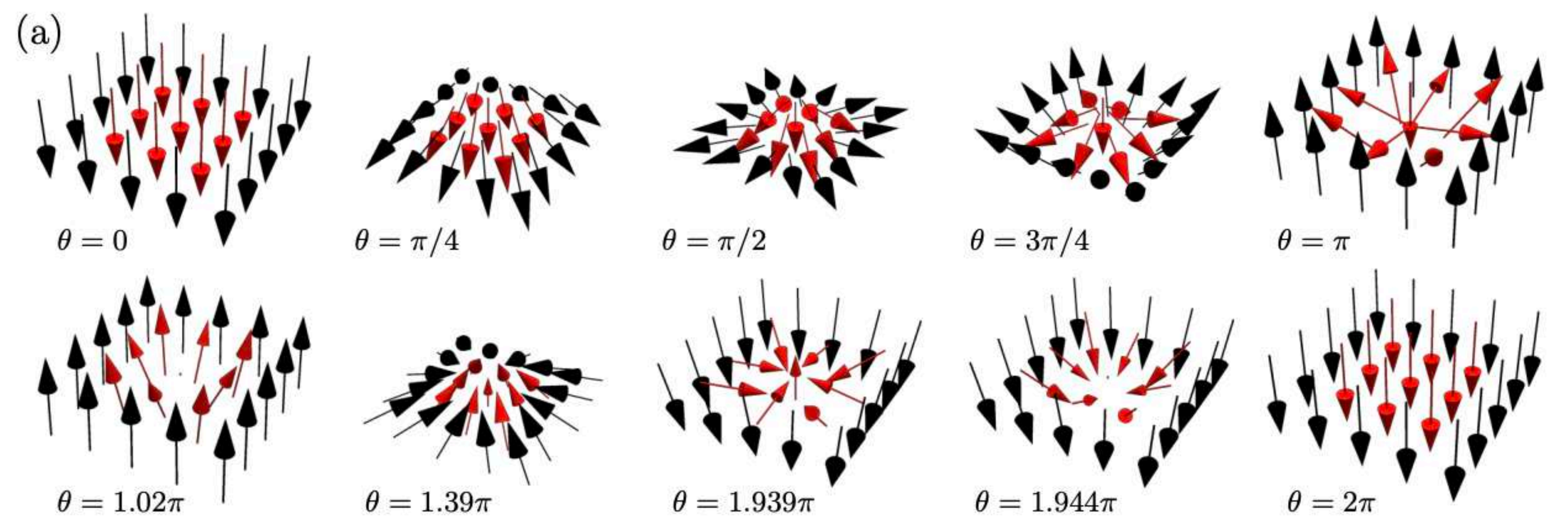

- quantum system (red) coupled to classical magnetic environment (black)
- adiabatic twisting of the environment results in a tiny quantum skyrmion
- controlled creation and annihilation of quantum skyrmions

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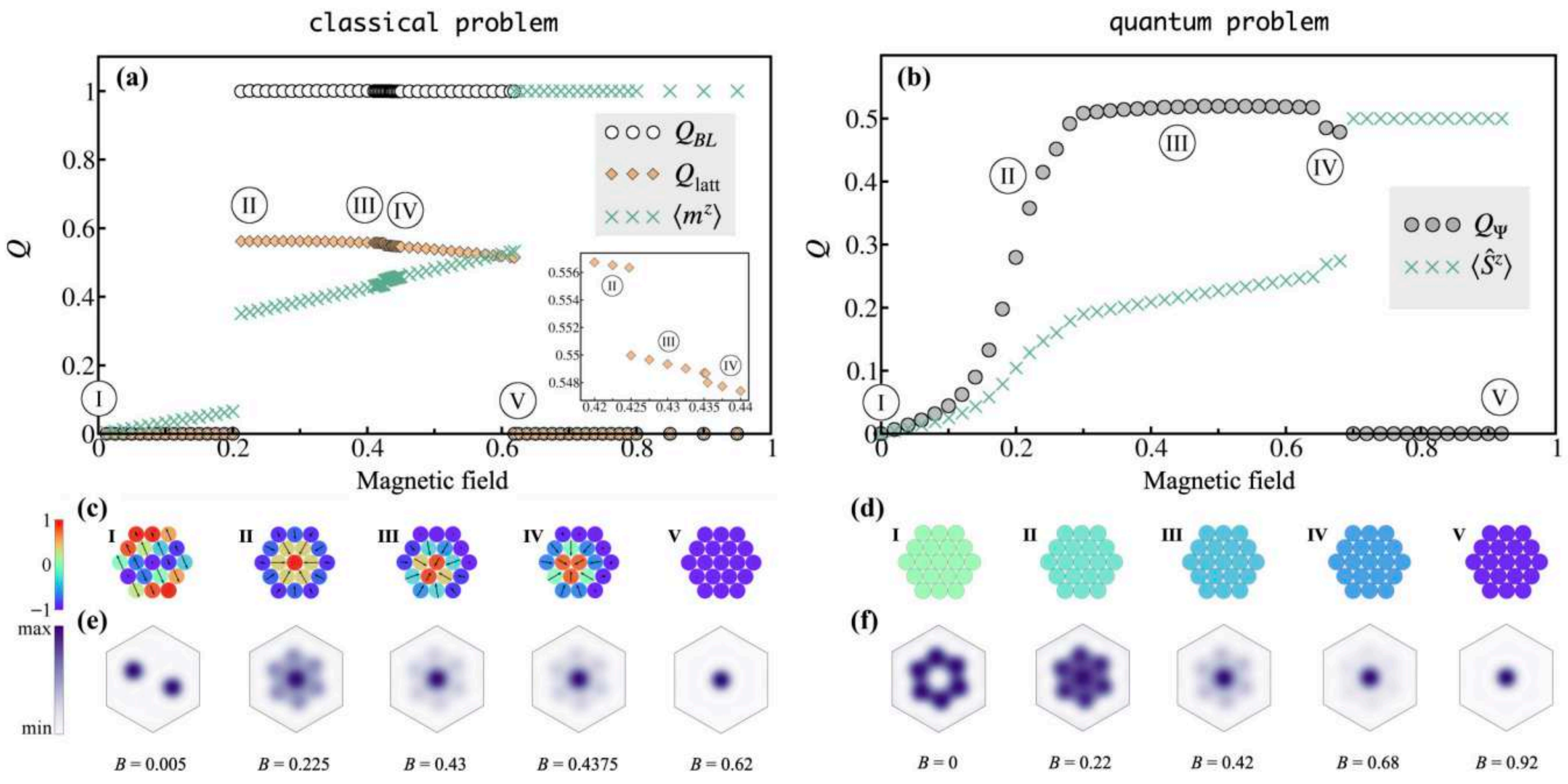

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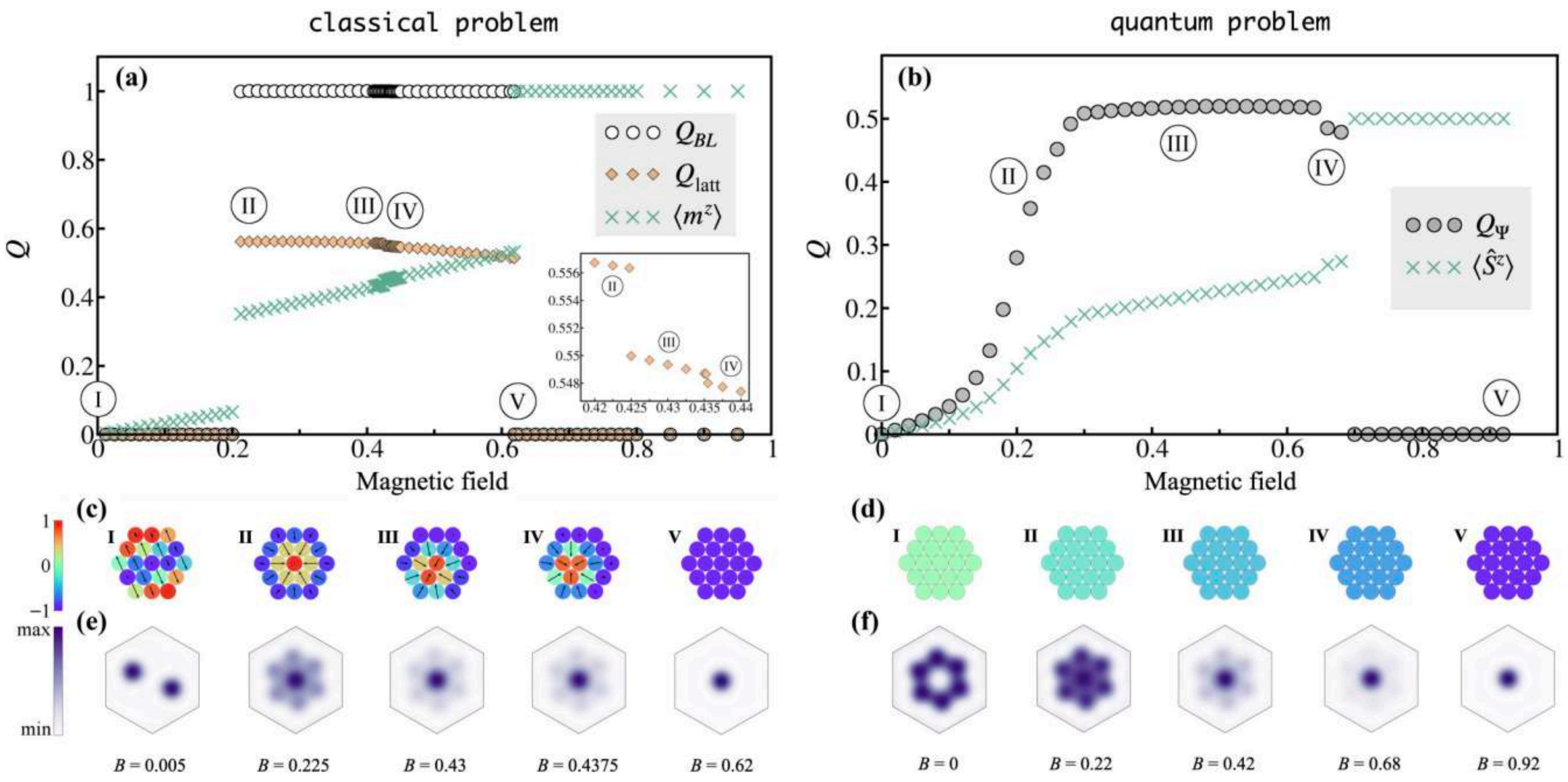

- Skyrmion states are superpositions of classical configurations, skyrmion structure revealed in spin structure factor

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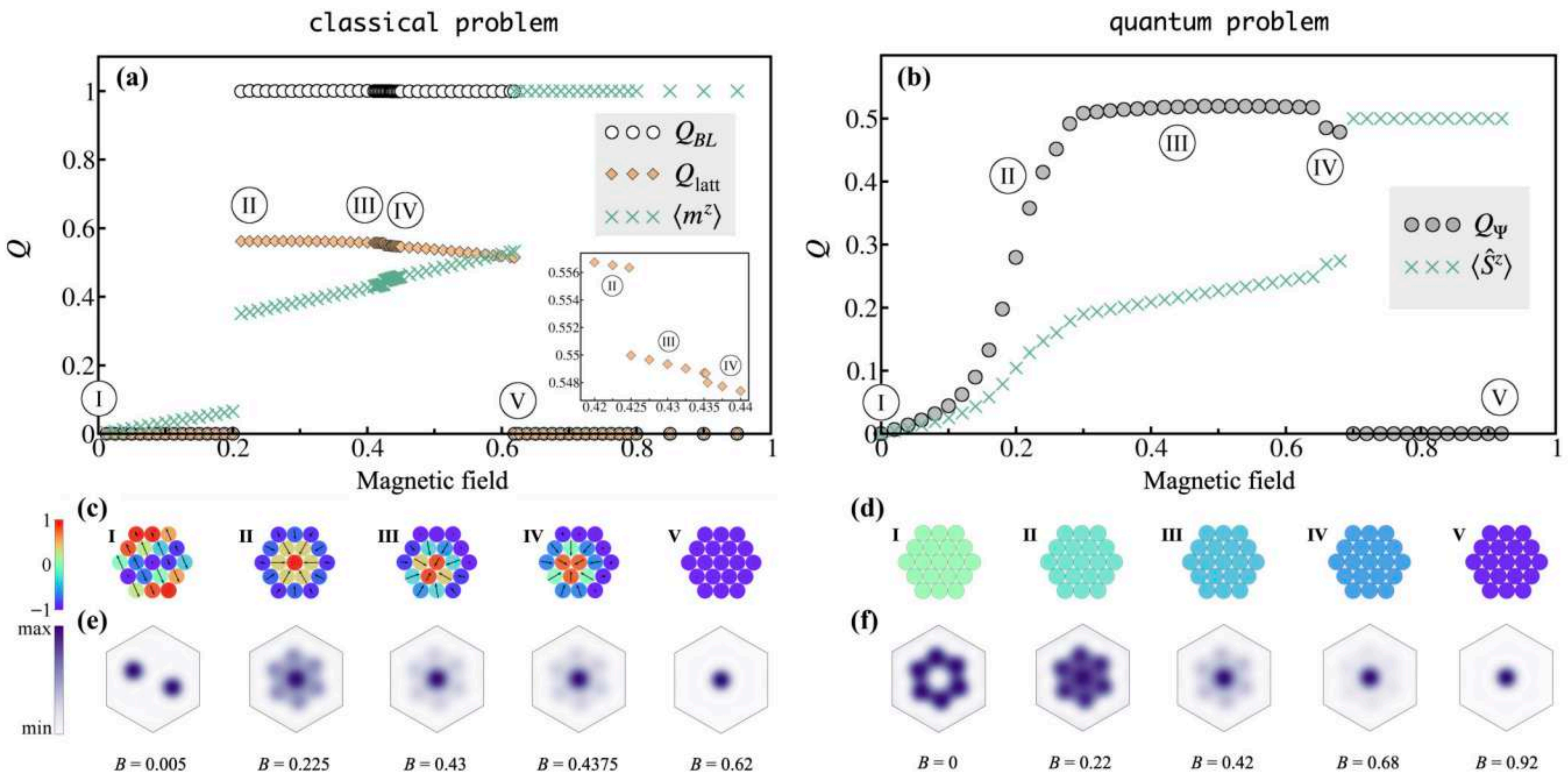

- Skyrmion states are superpositions of classical configurations, skyrmion structure revealed in spin structure factor
- No clean topological classification possible, because invariant quantization breaks down for quantum skyrmions

Quantum Skyrmions

arXiv:2112.12475*

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- Use tensor networks (matrix product states) to overcome size limits of exact diagonalization (ED)

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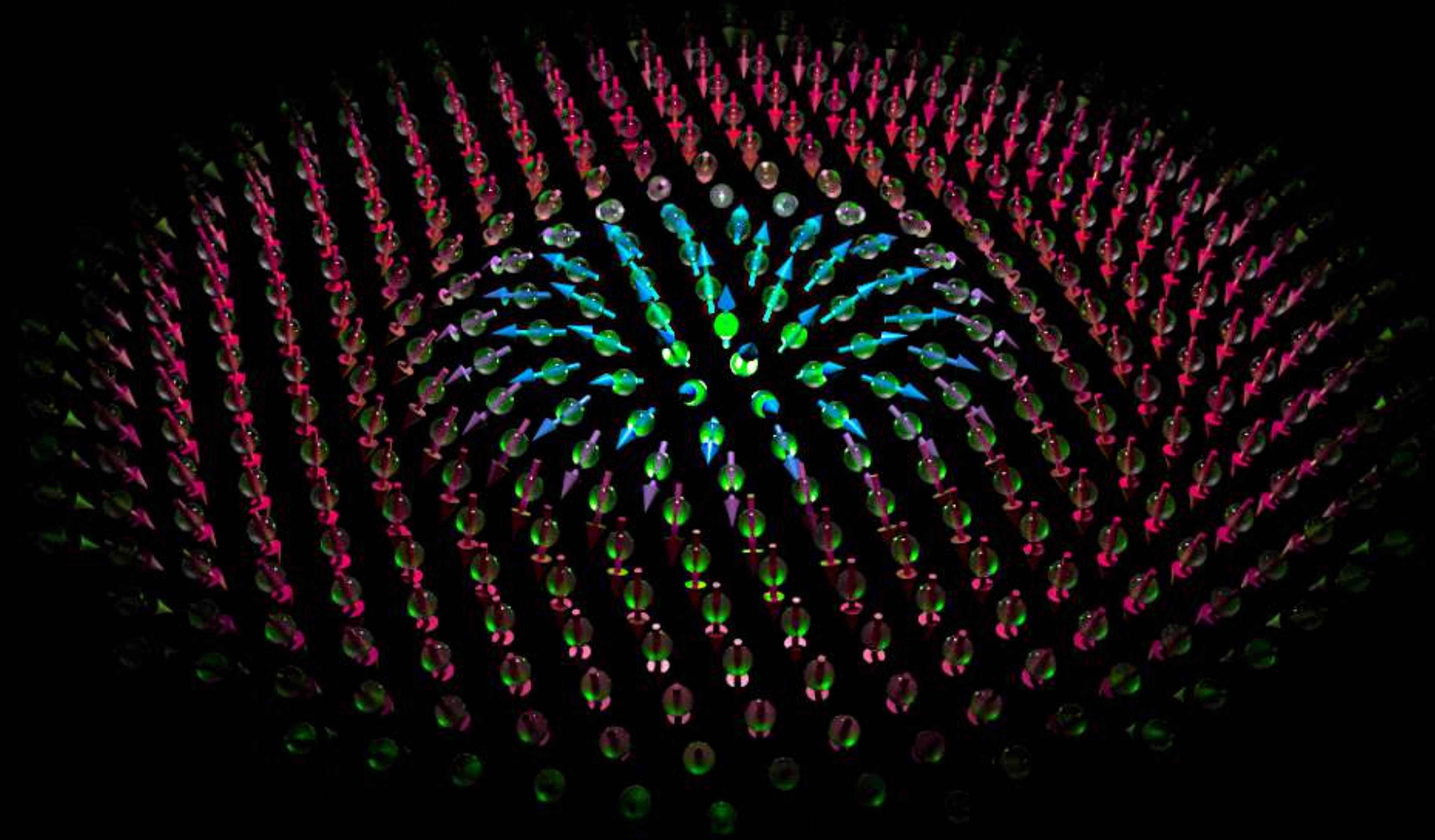

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$$J = -0.5D \qquad \boldsymbol{B} = -B_0 \boldsymbol{e}_z$$

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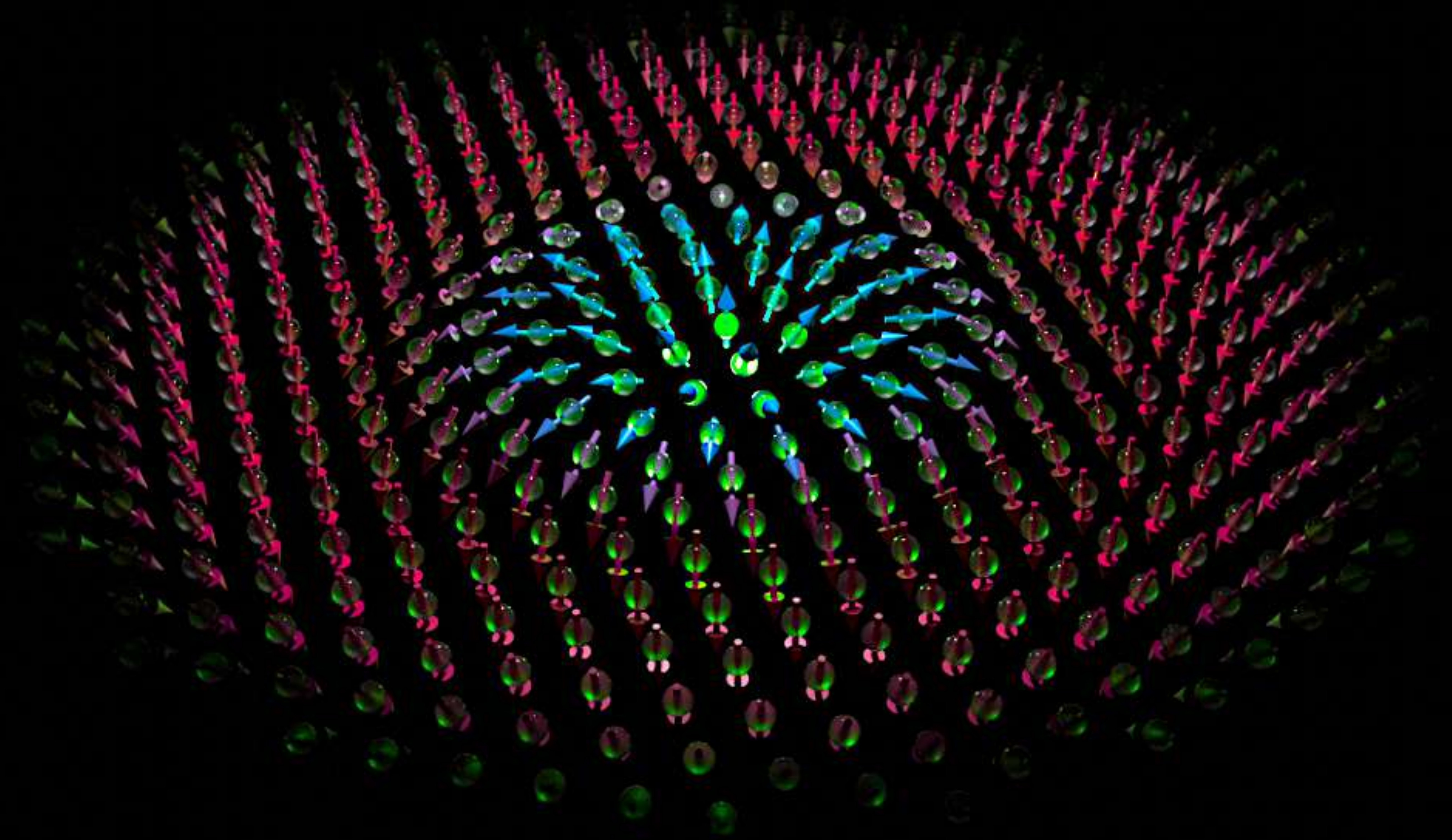

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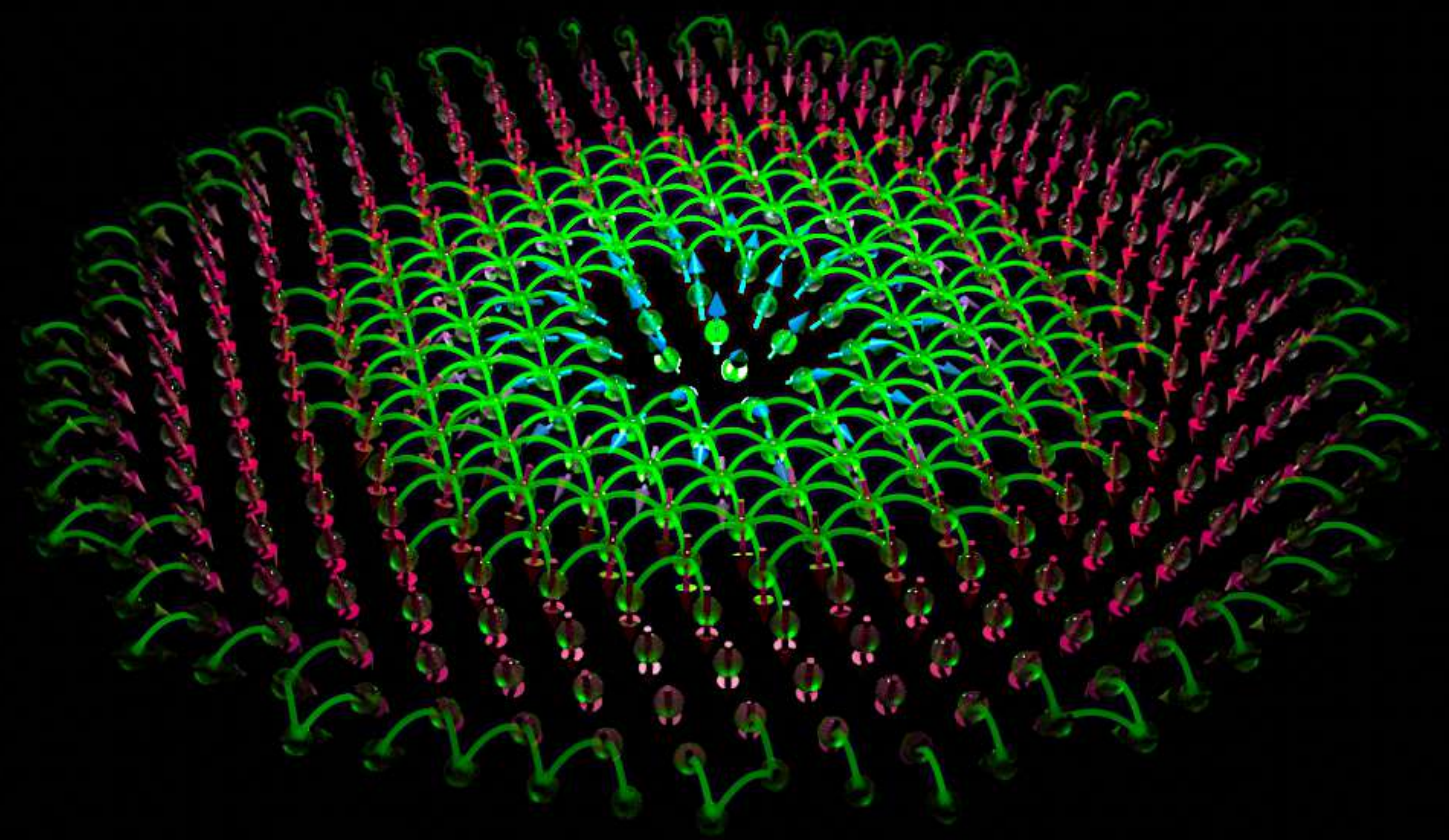

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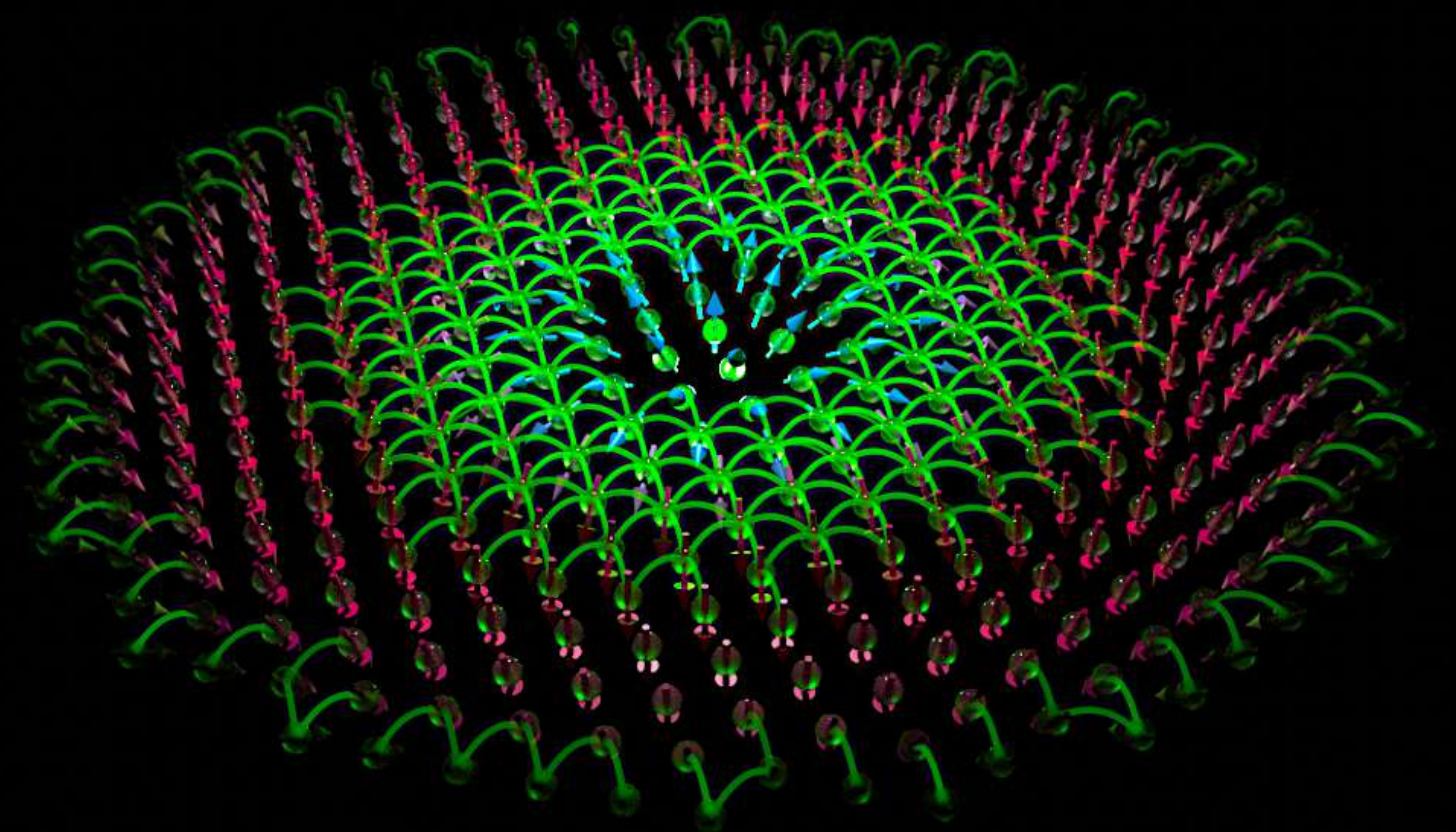

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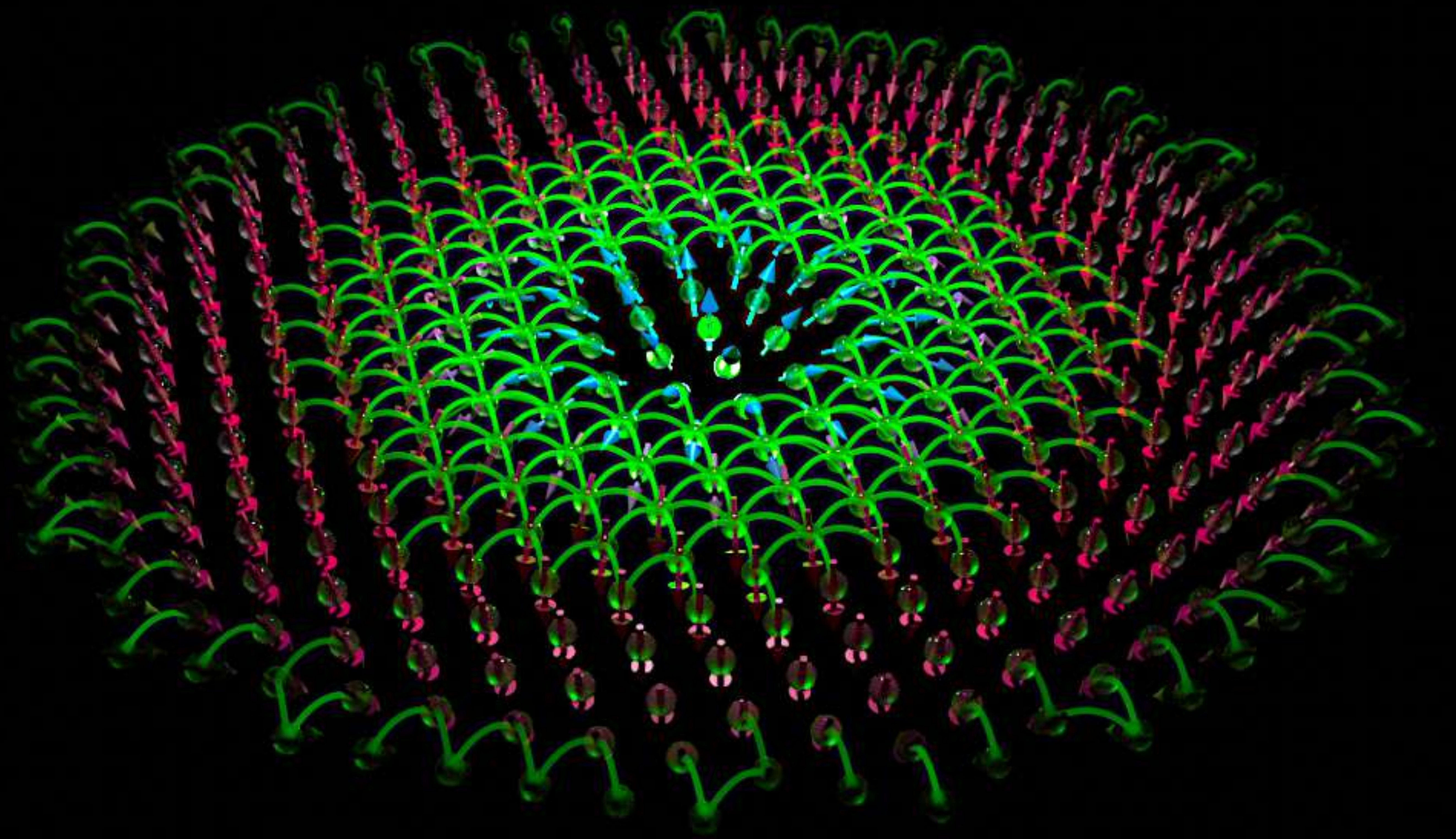

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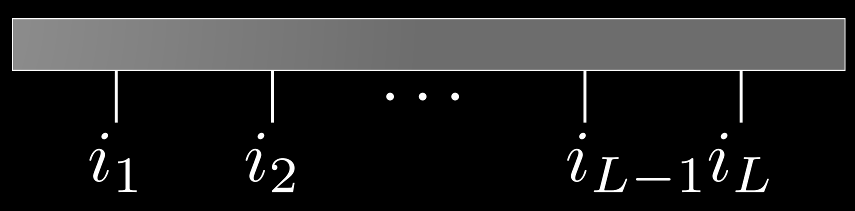

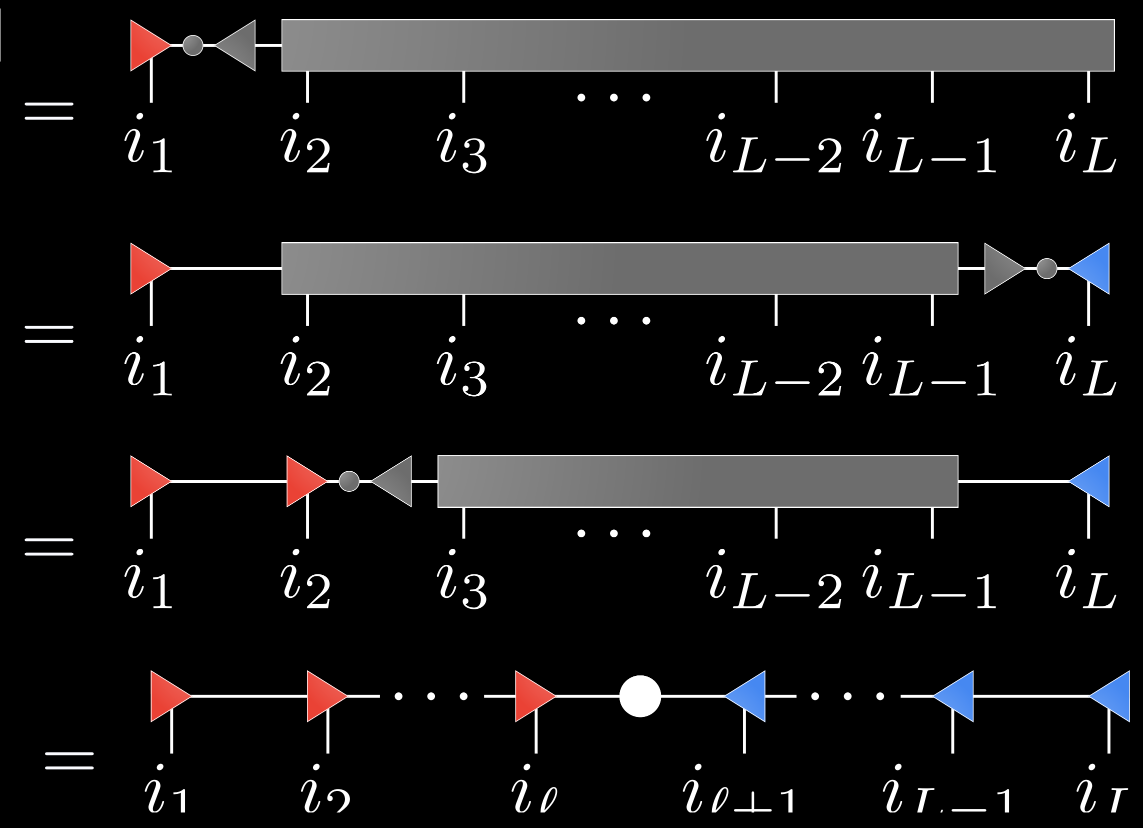

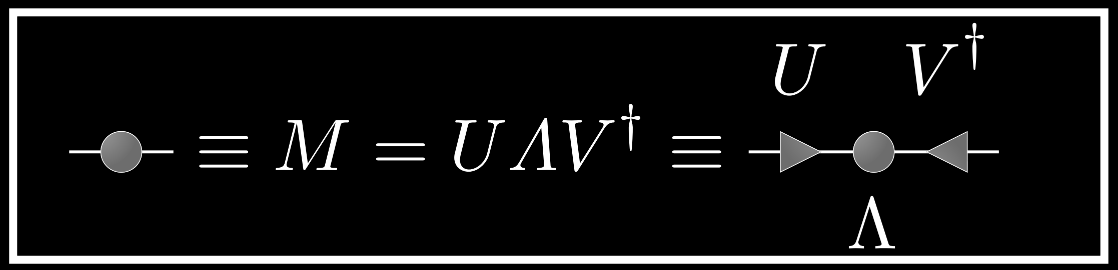

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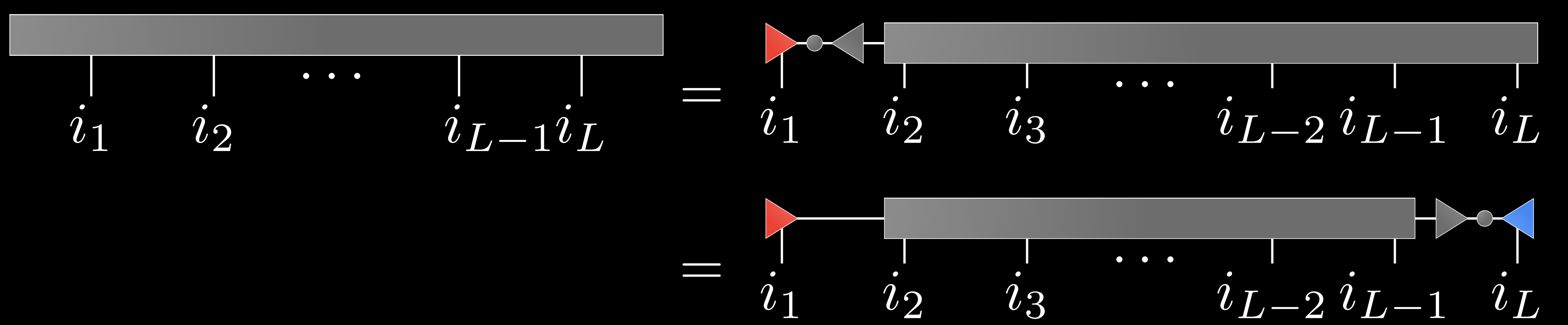

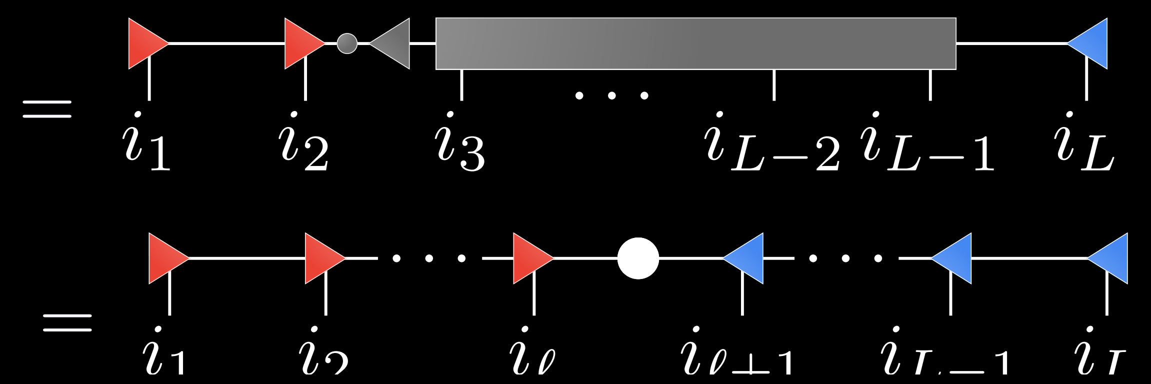

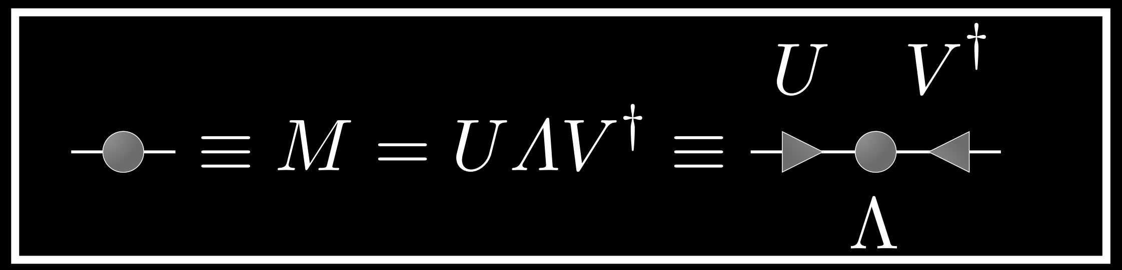

- every quantum state has an MPS form

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- $|\psi_{A(\ell),k}\rangle = U^{[i_1]}_{k_1}U^{[i_2]}_{k_1,k_2}U^{[i_3]}_{k_2,k_3}\ldots U^{[i_\ell]}_{k_{\ell-1},k}\,|i_1,i_2,i_3,\ldots,i_j\rangle = \prod_{k=1}^{\ell} U^{[i_k]}\,|i_1,\ldots,i_\ell\rangle$
  $|\psi_{B(\ell),k}\rangle = V^{\dagger[i_{\ell+1}]}_{k,k_{\ell+1}}V^{\dagger[i_{\ell+2}]}_{k_{\ell+1},k_{\ell+2}}\ldots V^{\dagger[i_L]}_{k_{L-1}}\,|i_{\ell+1},\ldots,i_L\rangle = \prod_{k=\ell+1}^{L} V^{\dagger[i_k]}\,|i_{\ell+1},\ldots,i_L\rangle$

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- MPS form gives access to the eigenvalues of the reduced density matrix
  - measure bipartite entanglement for free

$$S_\alpha = \frac{1}{1-\alpha} \log_2 \mathrm{tr} \hat{\rho}(\ell)^\alpha = \frac{1}{1-\alpha} \log_2 \sum_k s_k(\ell)^\alpha$$

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- canonical truncation: keep the largest contributions to reduced density matrix

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- effective variational energy minimization (density matrix renormalization group)

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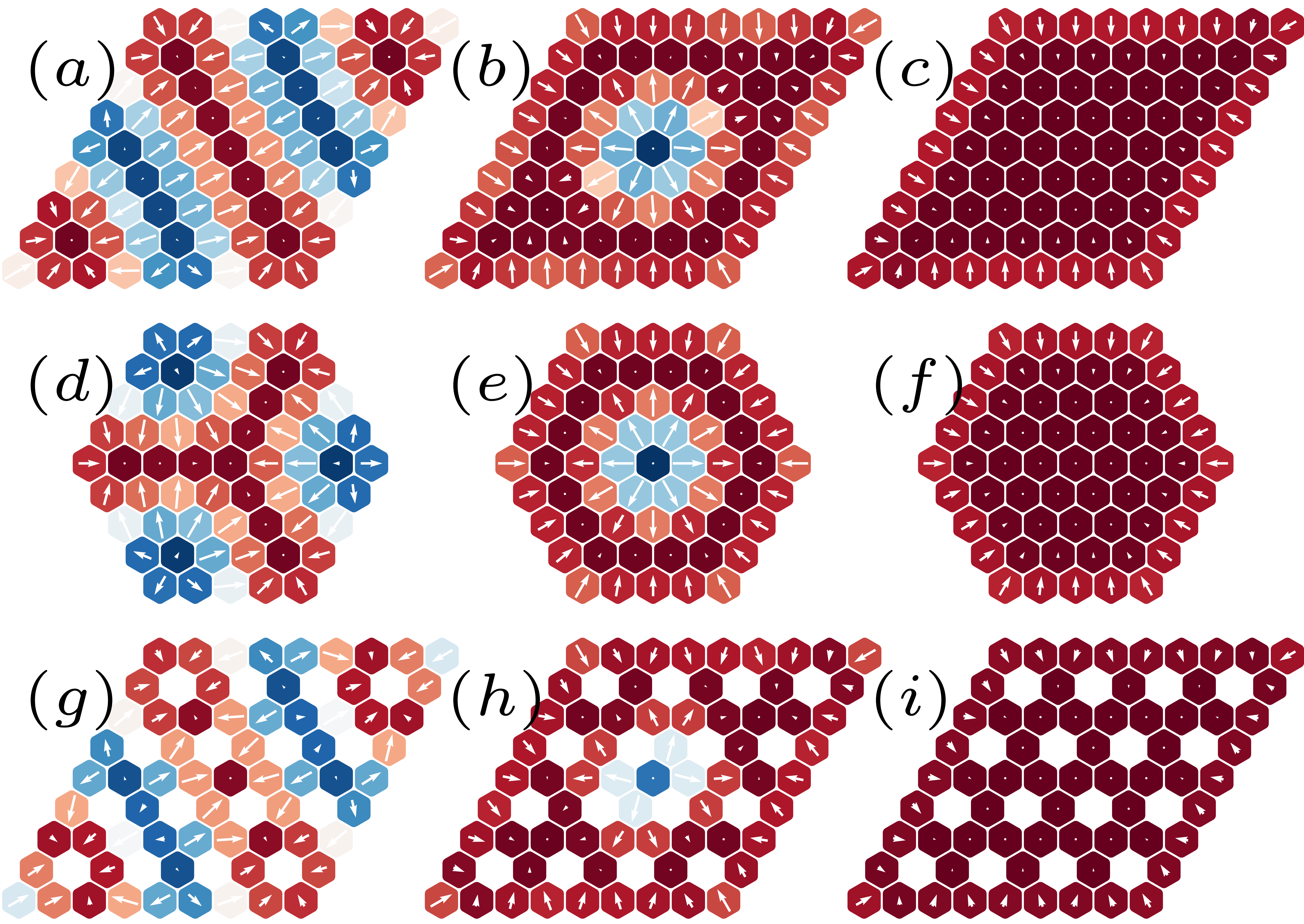

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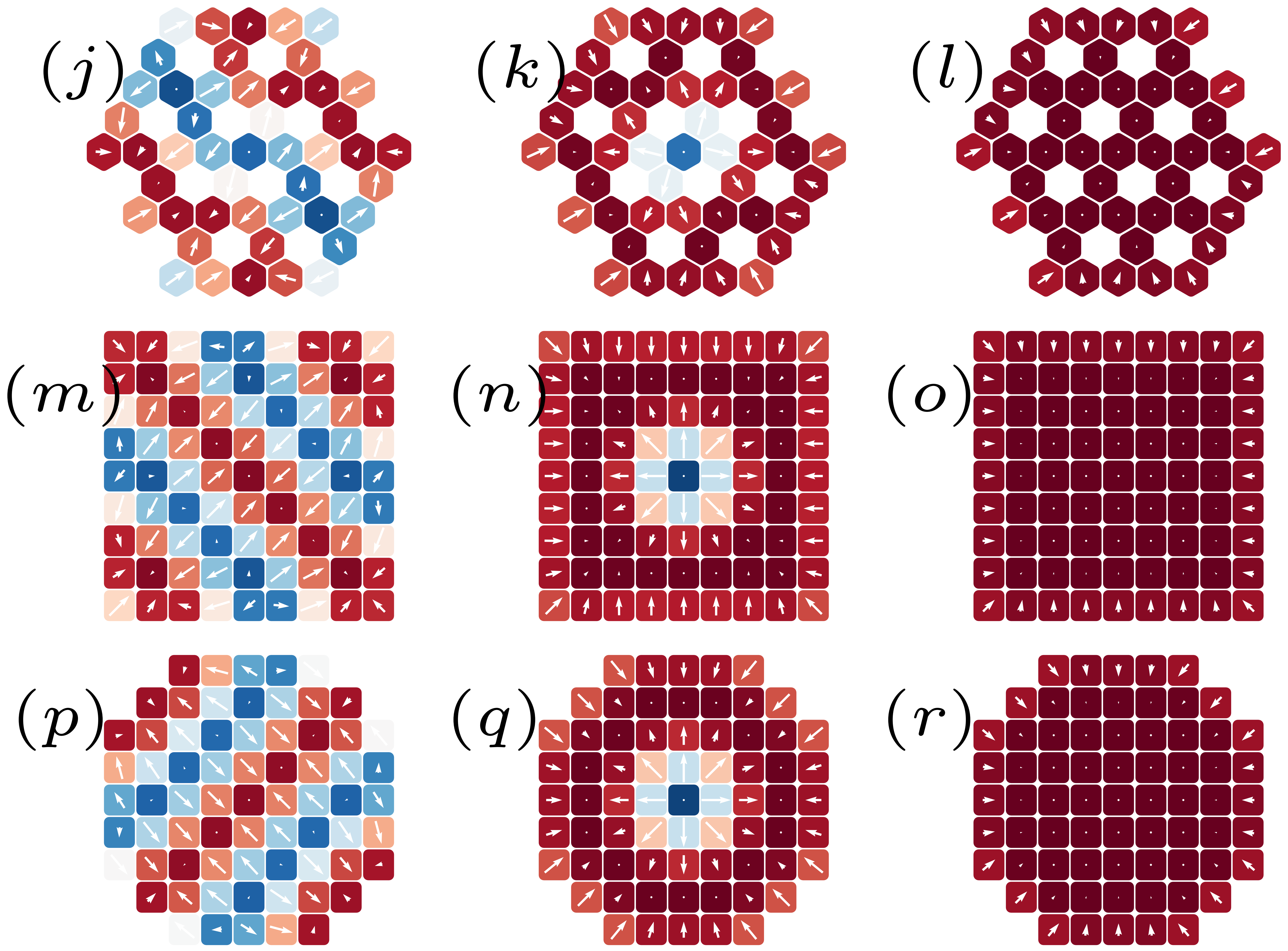

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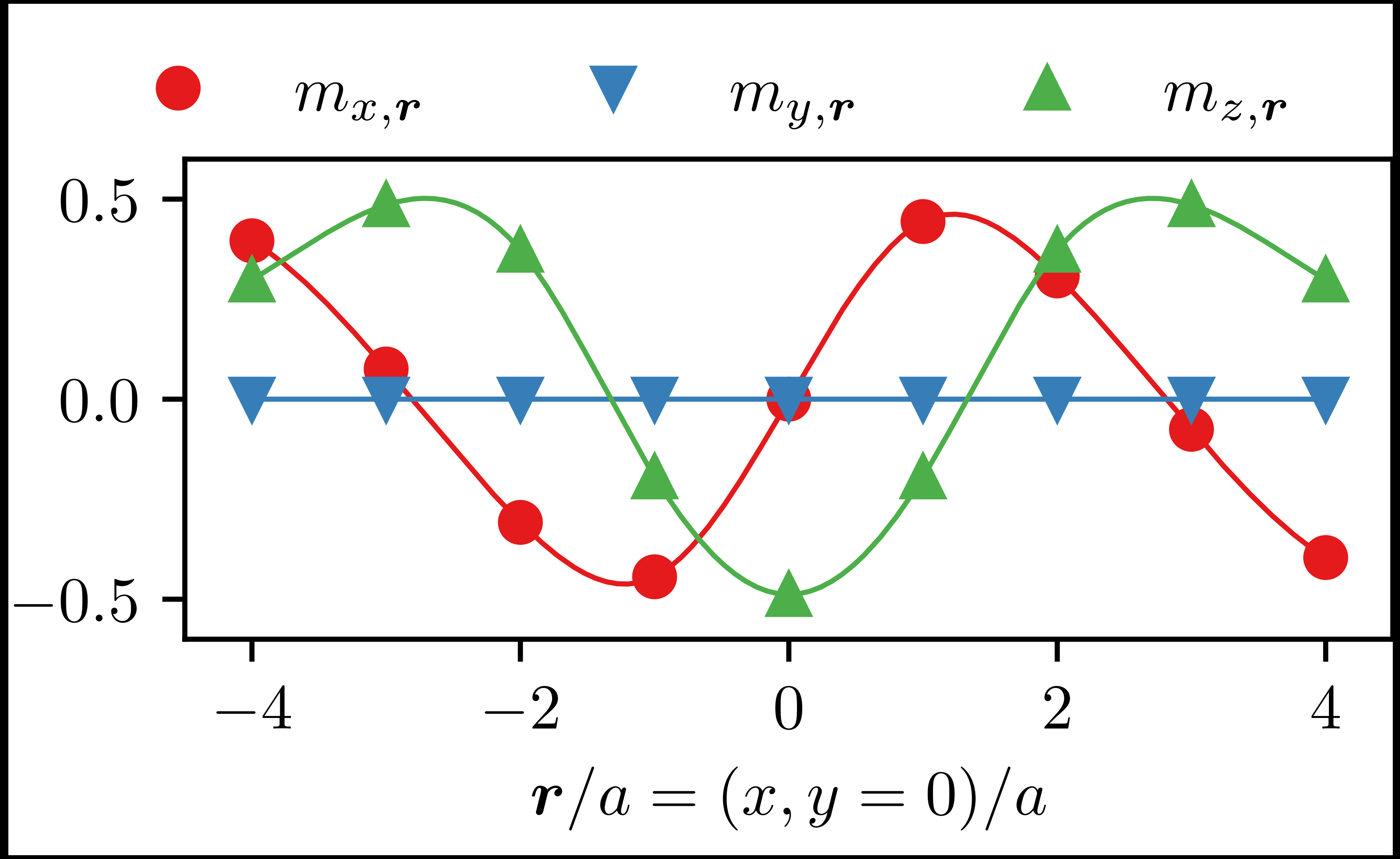

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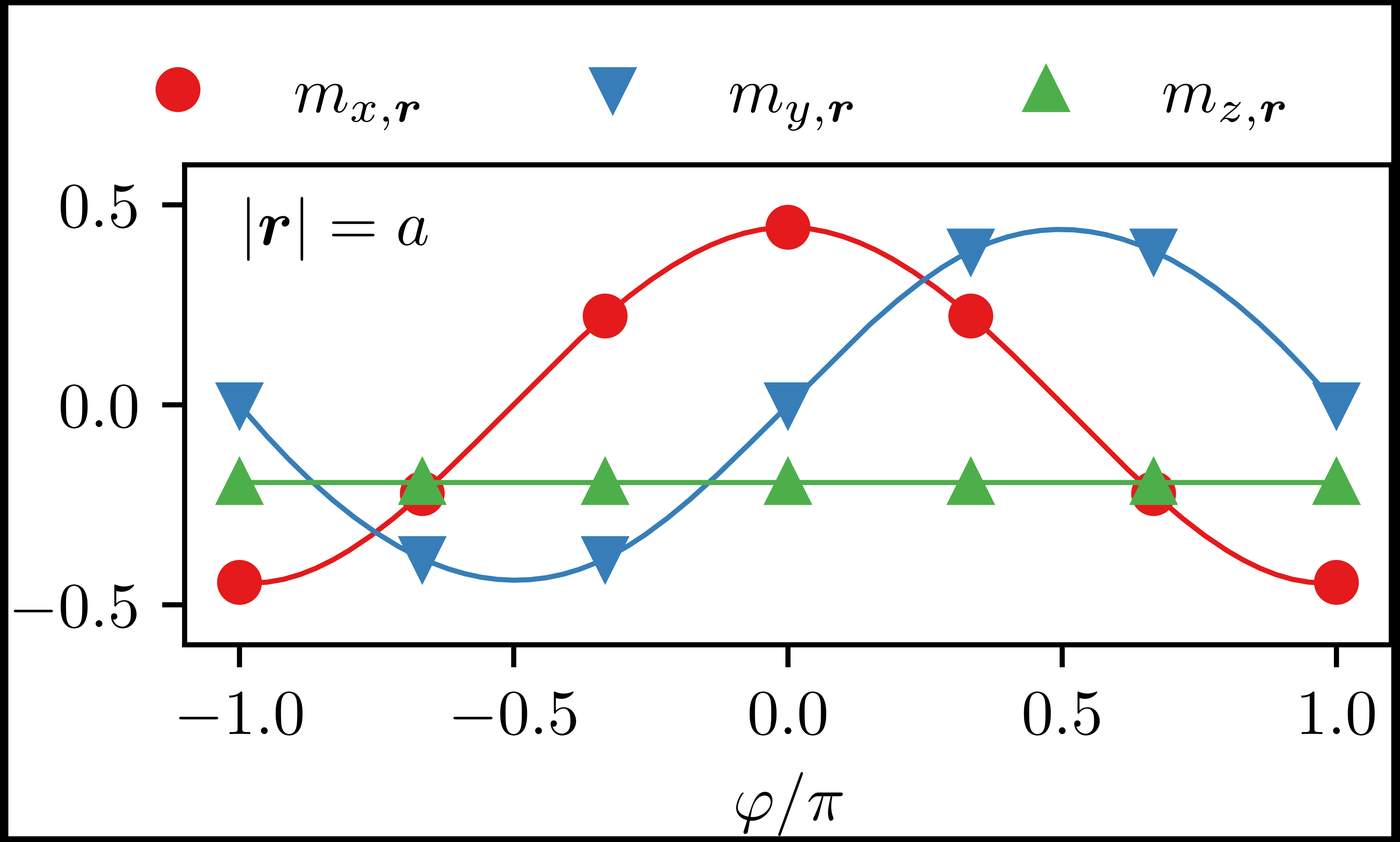

- radial spin winding of 3 lattice sites
- angular spin rotation according to lattice symmetries (here, rotation by π/3)
- Skyrmion type: Hedgehog family

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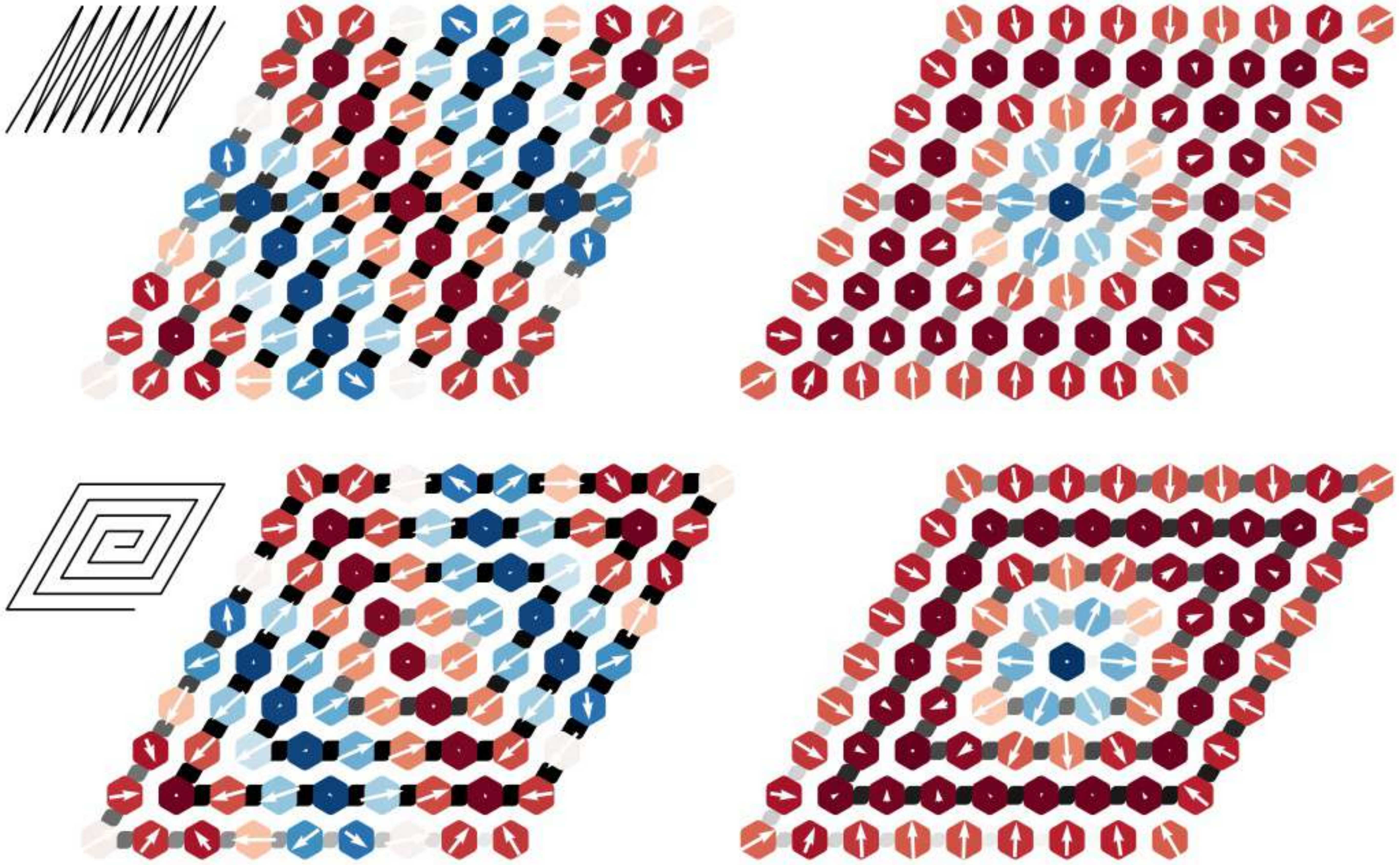

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- use MPS form to confirm bipartite entanglement (von Neumann entropy)
- confirm entanglement shared between two spins (Osterloh et al., Nature 416)
  - $C_{\boldsymbol{r},\boldsymbol{r}'} = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$
    - $R_{\boldsymbol{r},\boldsymbol{r}'} = \rho_{\boldsymbol{r},\boldsymbol{r}'}\tilde{\rho}_{\boldsymbol{r},\boldsymbol{r}'} \qquad \rho_{\boldsymbol{r},\boldsymbol{r}'} = \mathrm{tr}_{\{\boldsymbol{r},\boldsymbol{r}'\}^C}\hat{\rho}$
    - eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$

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- use MPS form to confirm bipartite entanglement (von Neumann entropy)
- confirm entanglement shared between two spins (Osterloh et al., Nature 416)
  - $C_{\boldsymbol{r},\boldsymbol{r}'} = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$
    - $R_{\boldsymbol{r},\boldsymbol{r}'} = \rho_{\boldsymbol{r},\boldsymbol{r}'} \tilde{\rho}_{\boldsymbol{r},\boldsymbol{r}'} \quad \rho_{\boldsymbol{r},\boldsymbol{r}'} = \mathrm{tr}_{\{\boldsymbol{r},\boldsymbol{r}'\}^C} \hat{\rho}$
    - eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$
- side remark:

$$\rho_{\boldsymbol{r},\boldsymbol{r}'} = \sum_{\alpha,\beta} \langle S^\alpha_{\boldsymbol{r}} S^\beta_{\boldsymbol{r}'} \rangle \, \sigma_\alpha \otimes \sigma_\beta$$

Quantum Skyrmions

arXiv:2112.12475*

- Use tensor networks (matrix product states) to overcome size limits of exact diagonalization (ED)
- ***Quantum skyrmions are* entangled** superpositions of classical states
- Detect quantum skyrmions by measuring average polarization or neutron scattering cross section
- ***Quantum skyrmion lattices*** in the thermodynamic limit

$$B_0 = 1$$

$$J = -0.5D \qquad \boldsymbol{B} = -B_0 \boldsymbol{e}_z$$

$$\hat{H} = J \sum_{\langle \boldsymbol{r},\boldsymbol{r}' \rangle} \left(\hat{\boldsymbol{S}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}'} + \boldsymbol{D}_{\boldsymbol{r}-\boldsymbol{r}'} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}} \times \hat{\boldsymbol{S}}_{\boldsymbol{r}'} + \boldsymbol{B} \cdot \hat{\boldsymbol{S}}_{r} \boldsymbol{\delta}_{\boldsymbol{r},\boldsymbol{r}'} \right)$$

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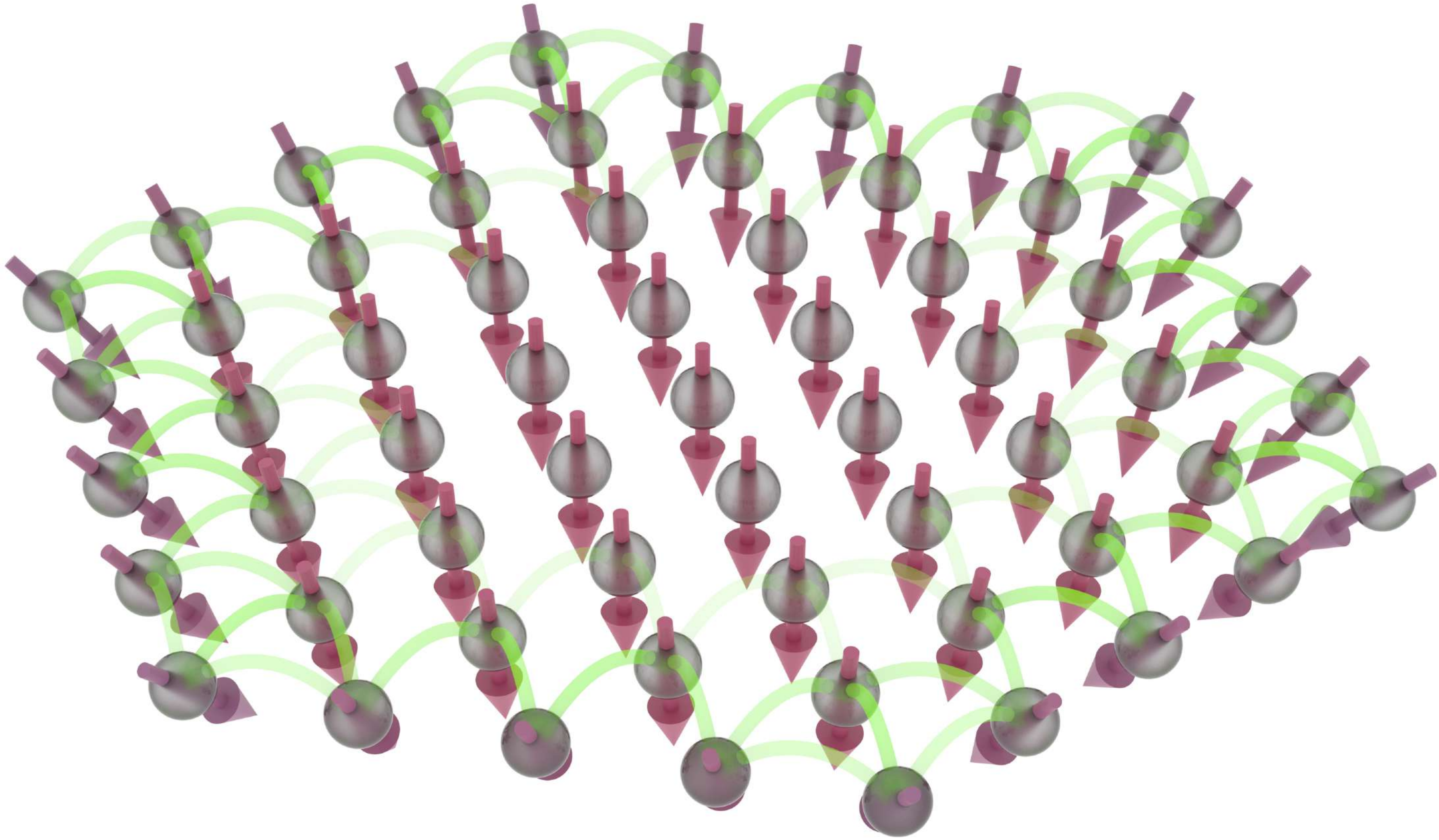

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$$B_0 = 1/8$$

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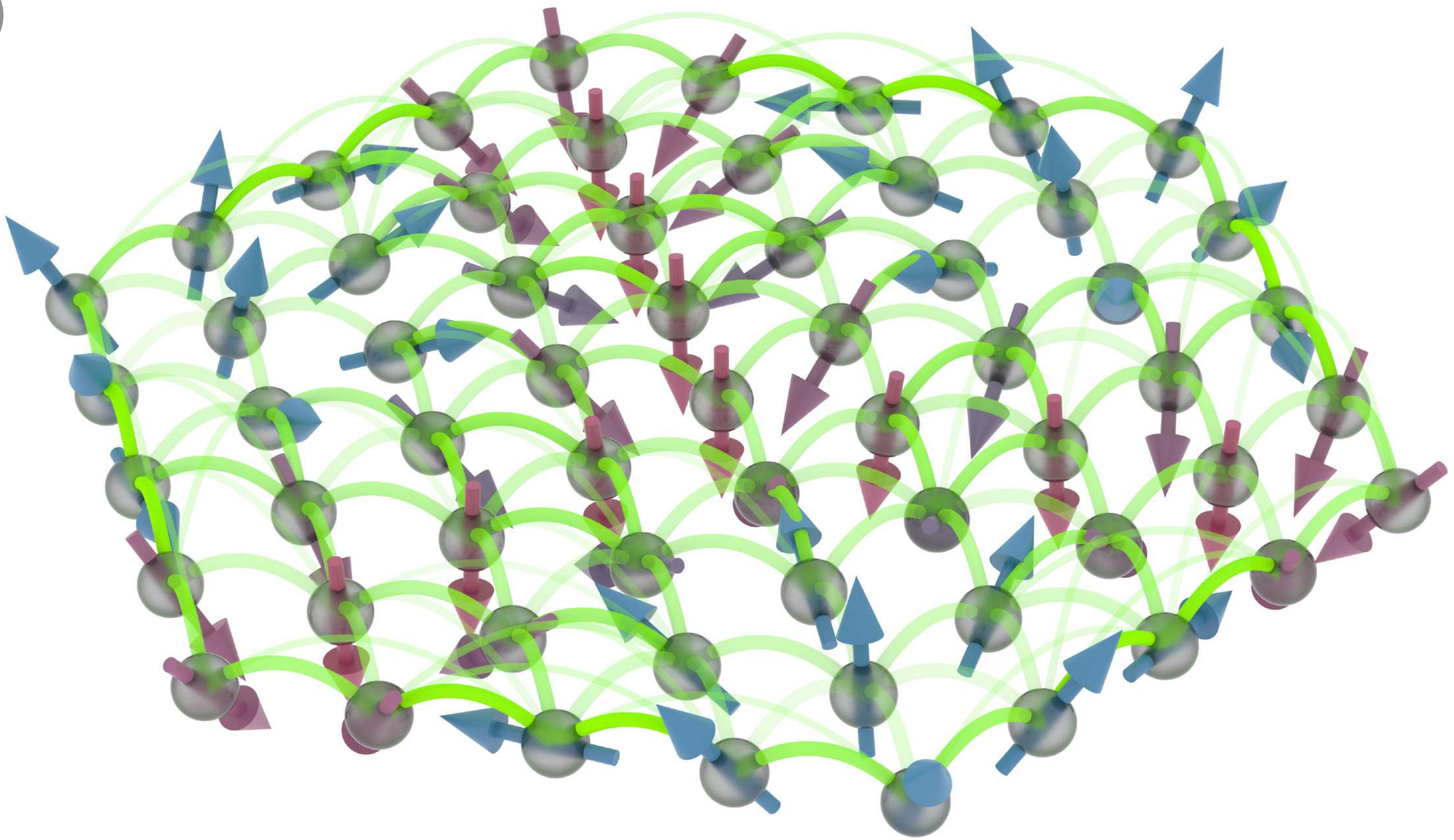

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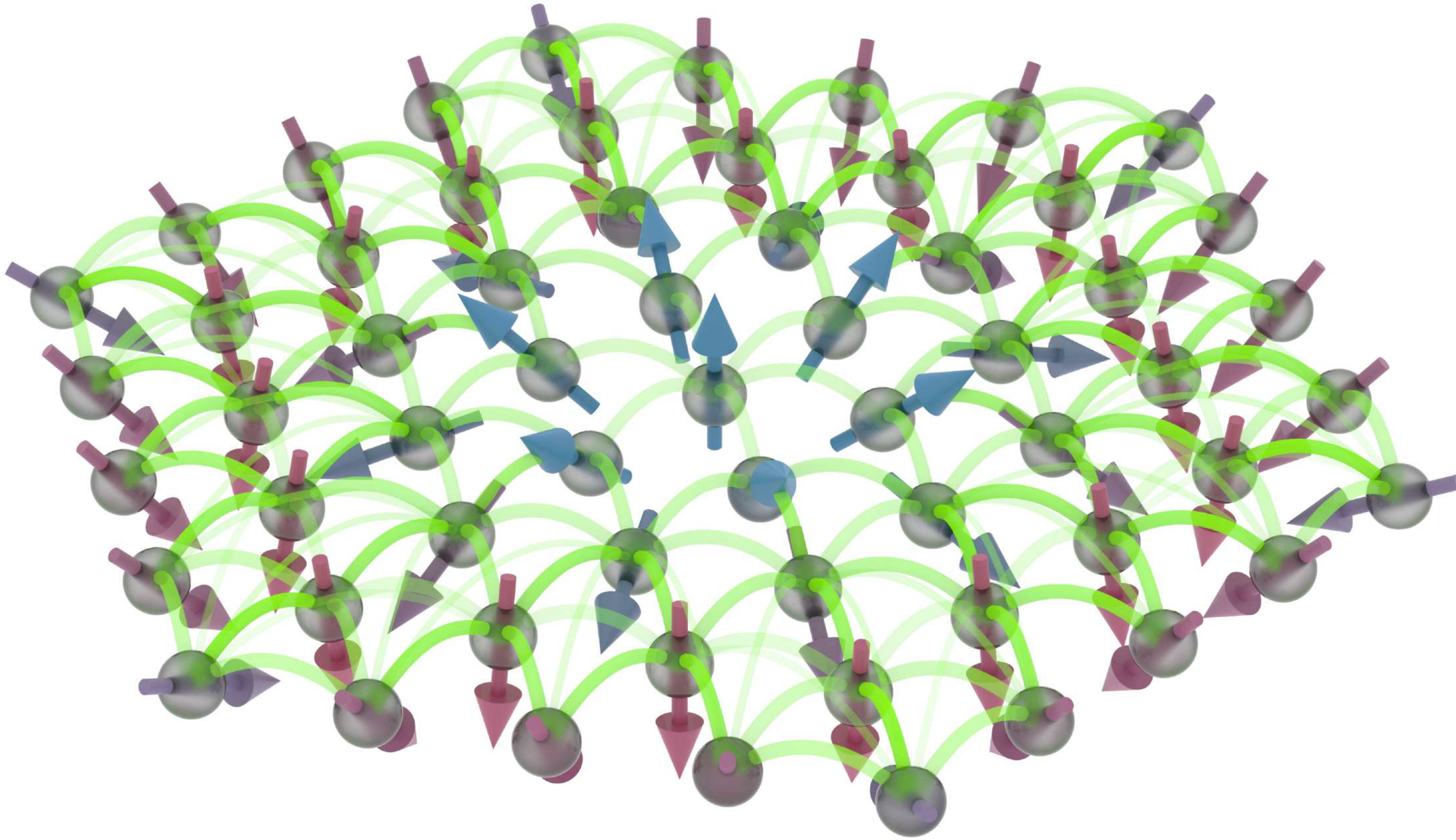

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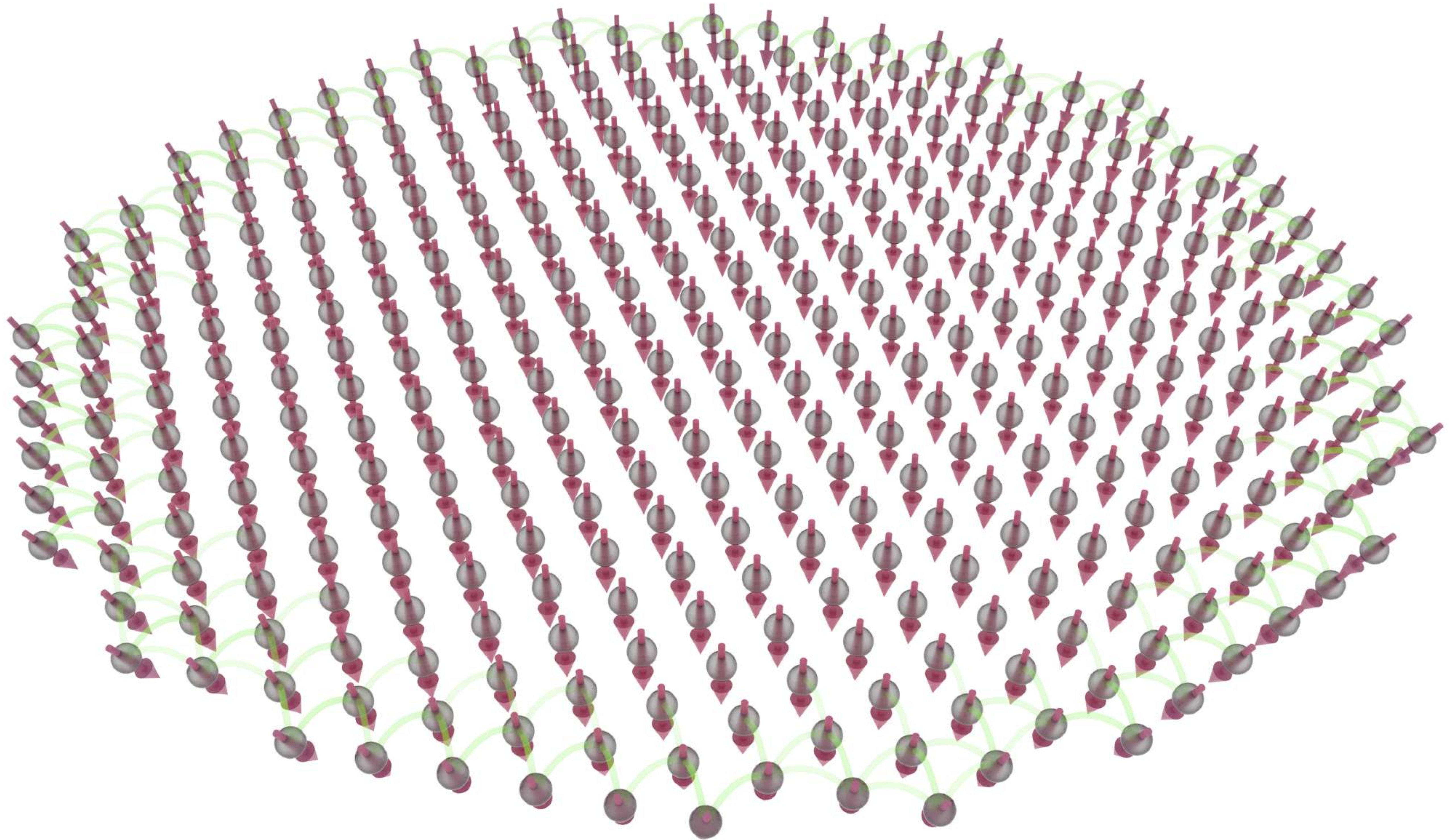
- confirm entanglement shared between two spins (Osterloh et al., Nature 416)
  - field-polarized ground state



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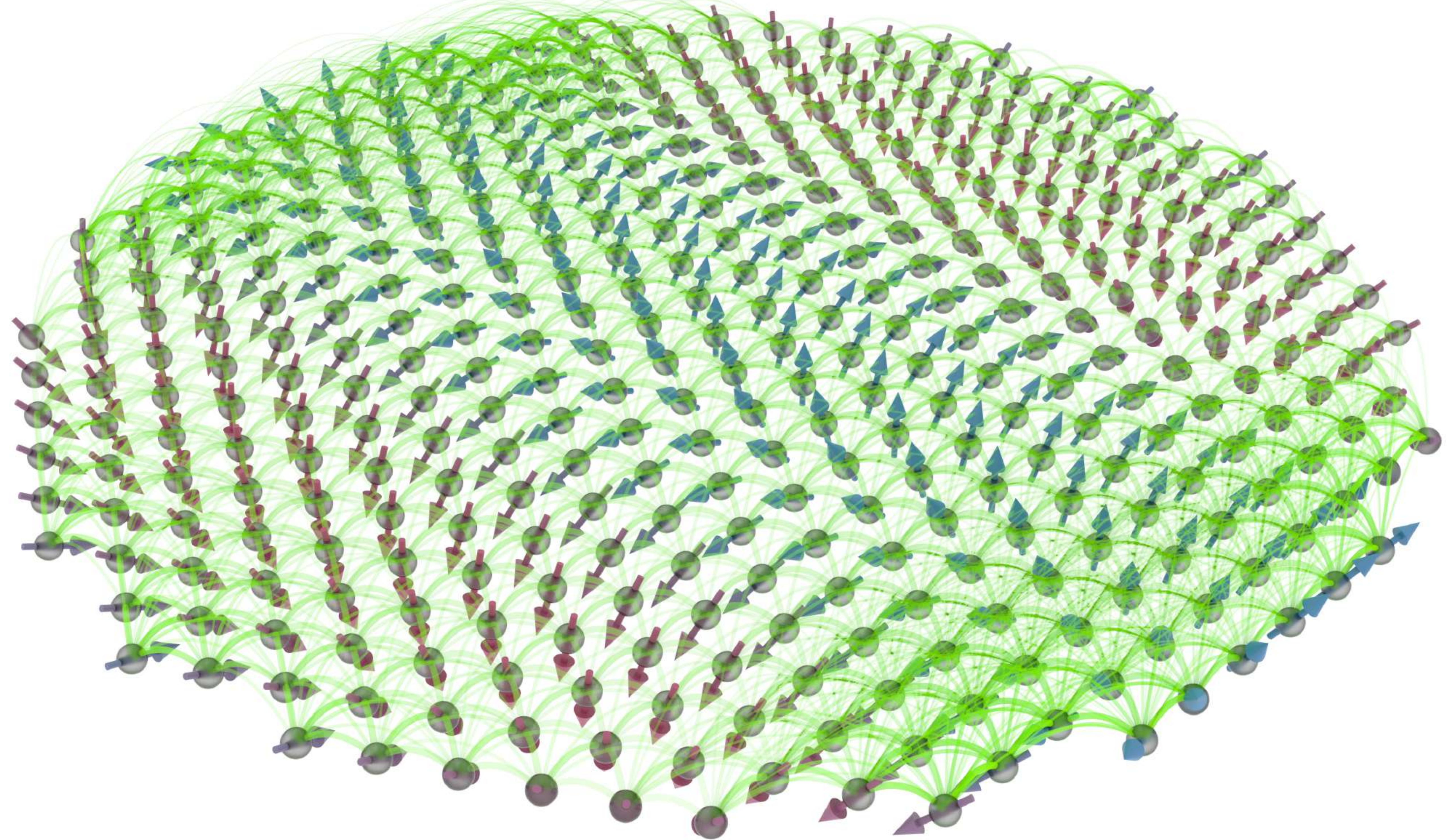

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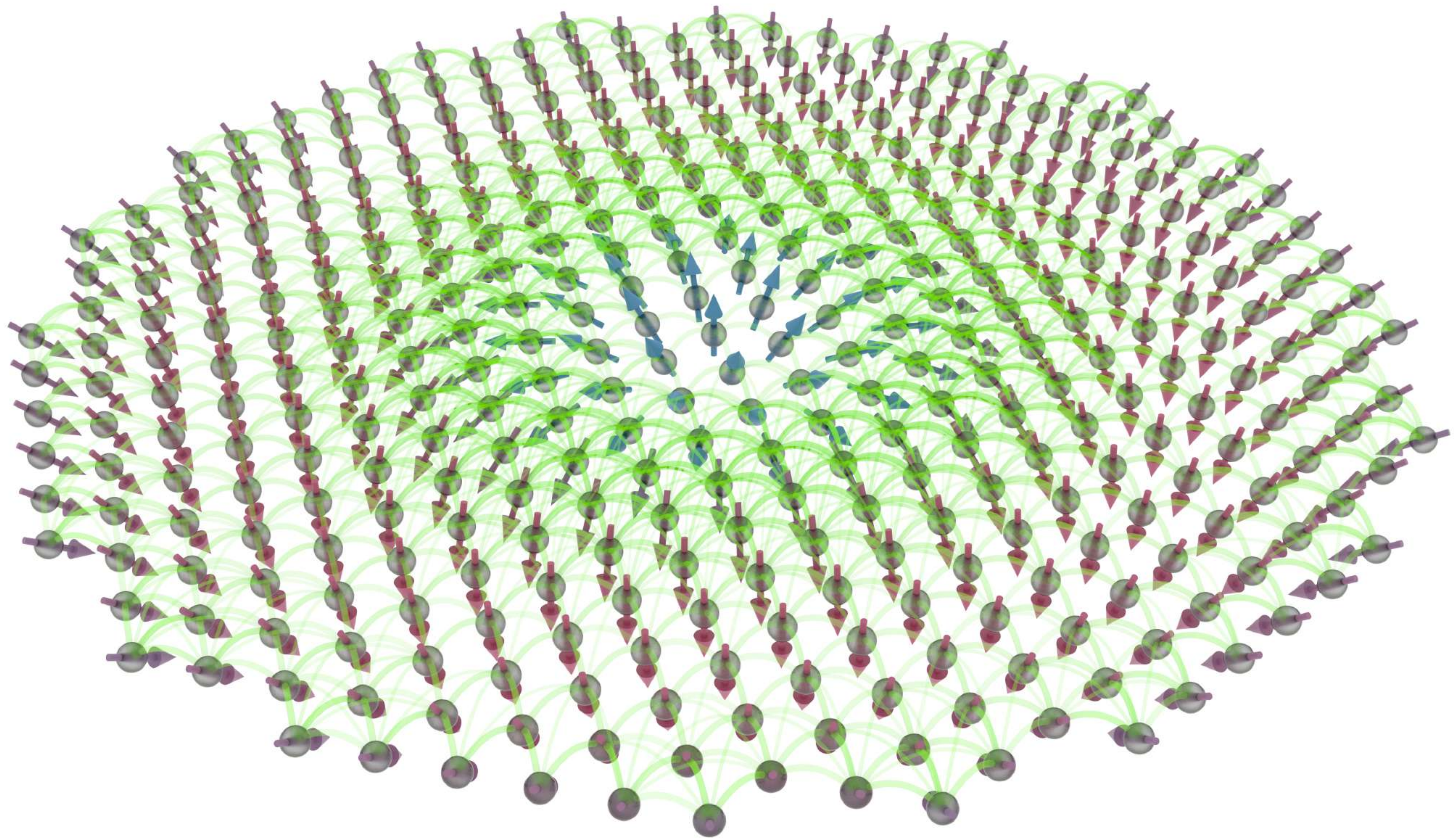
- confirm entanglement shared between two spins (Osterloh et al., Nature 416)
- quantum skyrmion ground state



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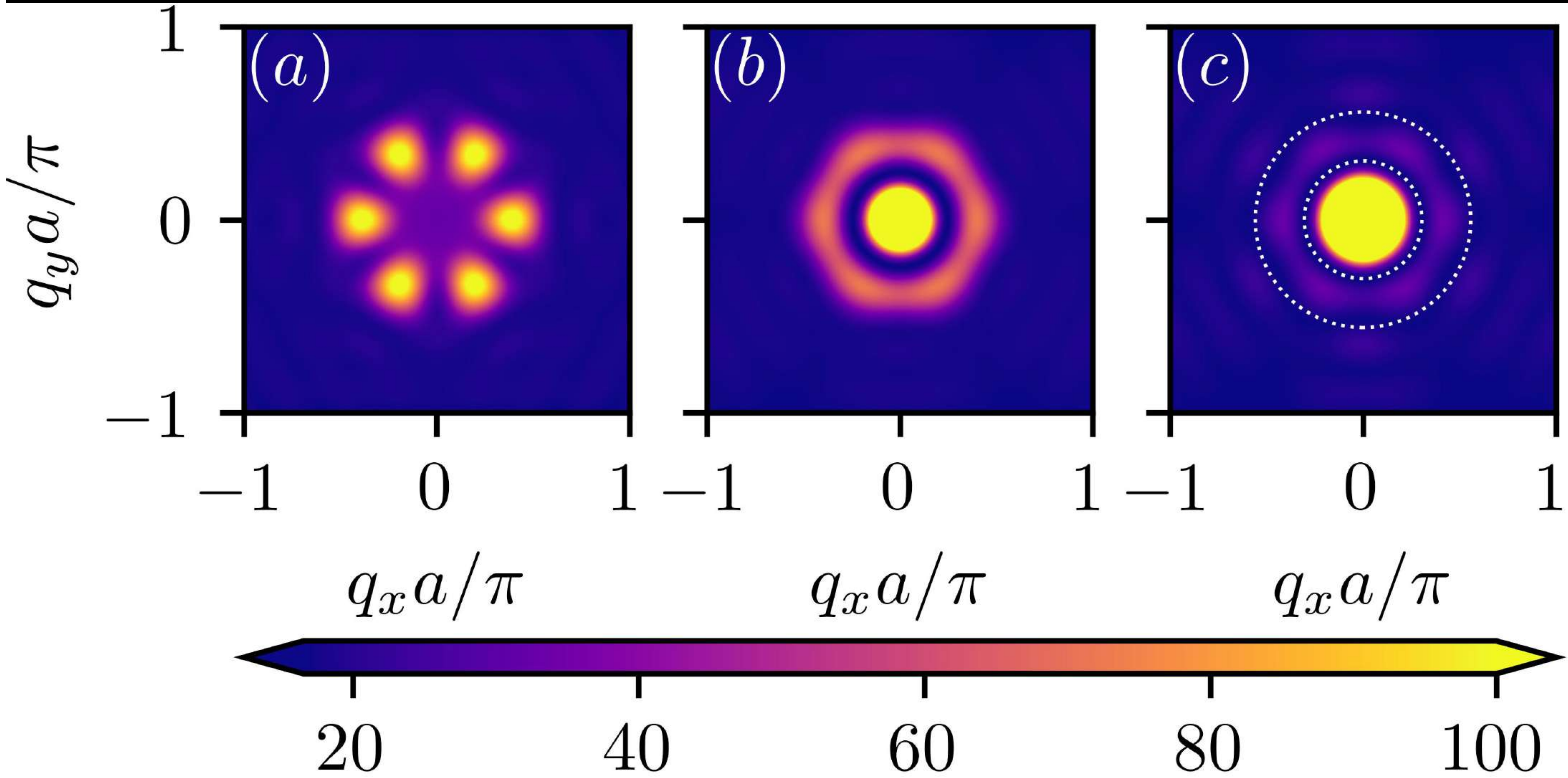
- Elastic magnetic differential neutron scattering cross section (extracted from spin structure factors)



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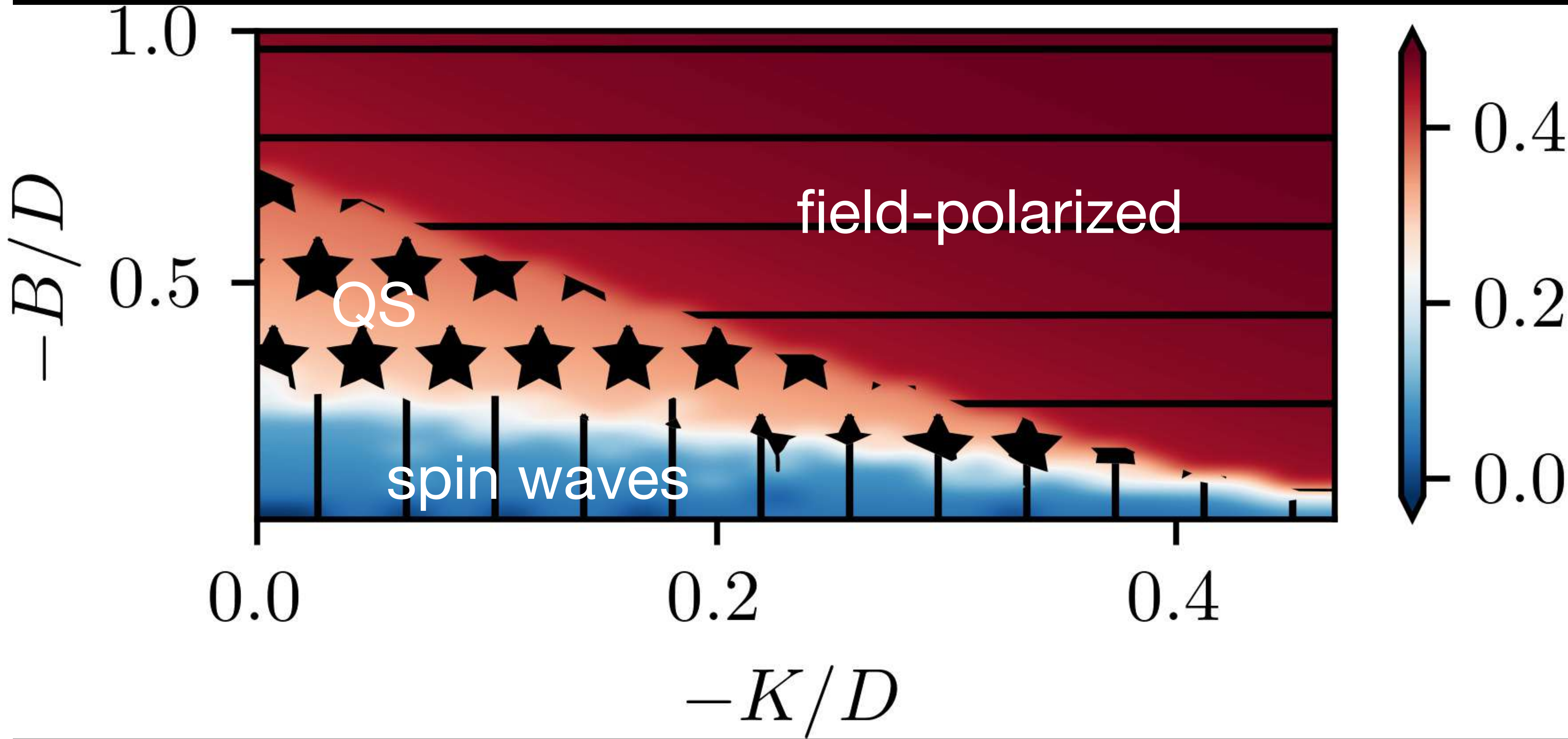
- Add anisotropy to study meltdown of the skyrmion phase
  - $\hat{H}_{\text{pert.}} = -K \sum_{\langle \boldsymbol{r}, \boldsymbol{r}' \rangle} \hat{S}^z_{\boldsymbol{r}} \hat{S}^z_{\boldsymbol{r}'}$



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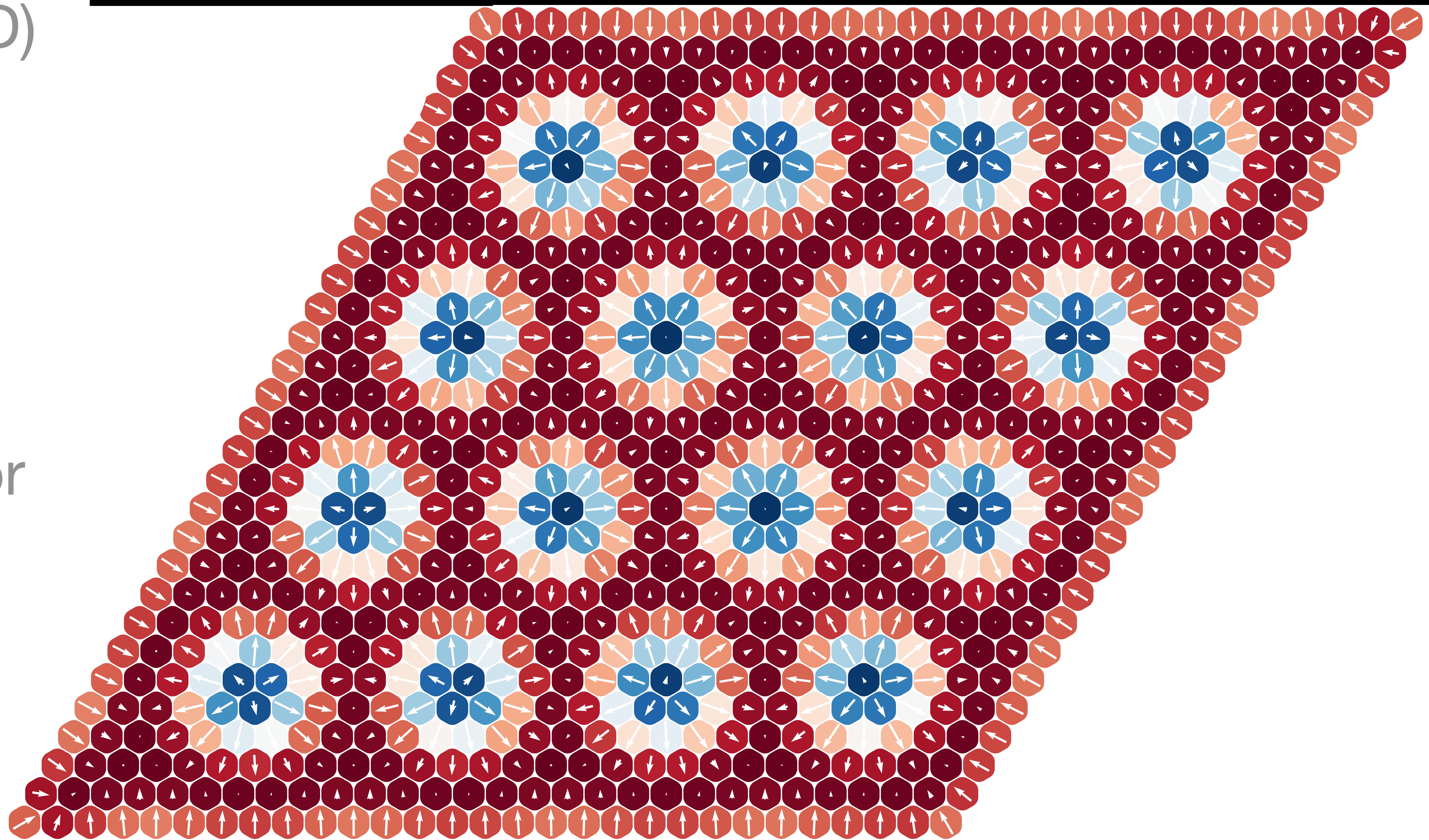
- locality of entanglement allows to access large system sizes — skyrmion lattices!



Quantum Spintronics

Future directions

- analytic description of the quantum skyrmion lattice phase
- find and utilize unique dynamics of quantum skyrmion lattices
  - tensor networks
    - state tomography
    - time evolution with tensor networks
  - finite temperature simulations (using auxiliary field QMC?)

$$\hat{H} = J \sum_{\langle \boldsymbol{r}, \boldsymbol{r}' \rangle} \left(\hat{\boldsymbol{S}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}'} + \boldsymbol{D}_{\boldsymbol{r}-\boldsymbol{r}'} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}} \times \hat{\boldsymbol{S}}_{\boldsymbol{r}'} + \boldsymbol{B} \cdot \hat{\boldsymbol{S}}_{r} \boldsymbol{\delta}_{\boldsymbol{r},\boldsymbol{r}'} \right)$$

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- To use Hubbard-Stratonovich trafo’s, we must write interactions as perfect squares

$$\epsilon_{\alpha\beta\gamma} \hat{S}^{\beta}_{\boldsymbol{r}} \hat{S}^{\gamma}_{\boldsymbol{r}'} = \frac{|\epsilon_{\alpha\beta\gamma}|}{2} \left(\hat{S}^{\beta}_{\boldsymbol{r}} + \mathrm{sign}(\epsilon_{\alpha\beta\gamma}) \hat{S}^{\gamma}_{\boldsymbol{r}'} \right)^2 + \ldots$$

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- Sign problem? If yes, how bad is it…

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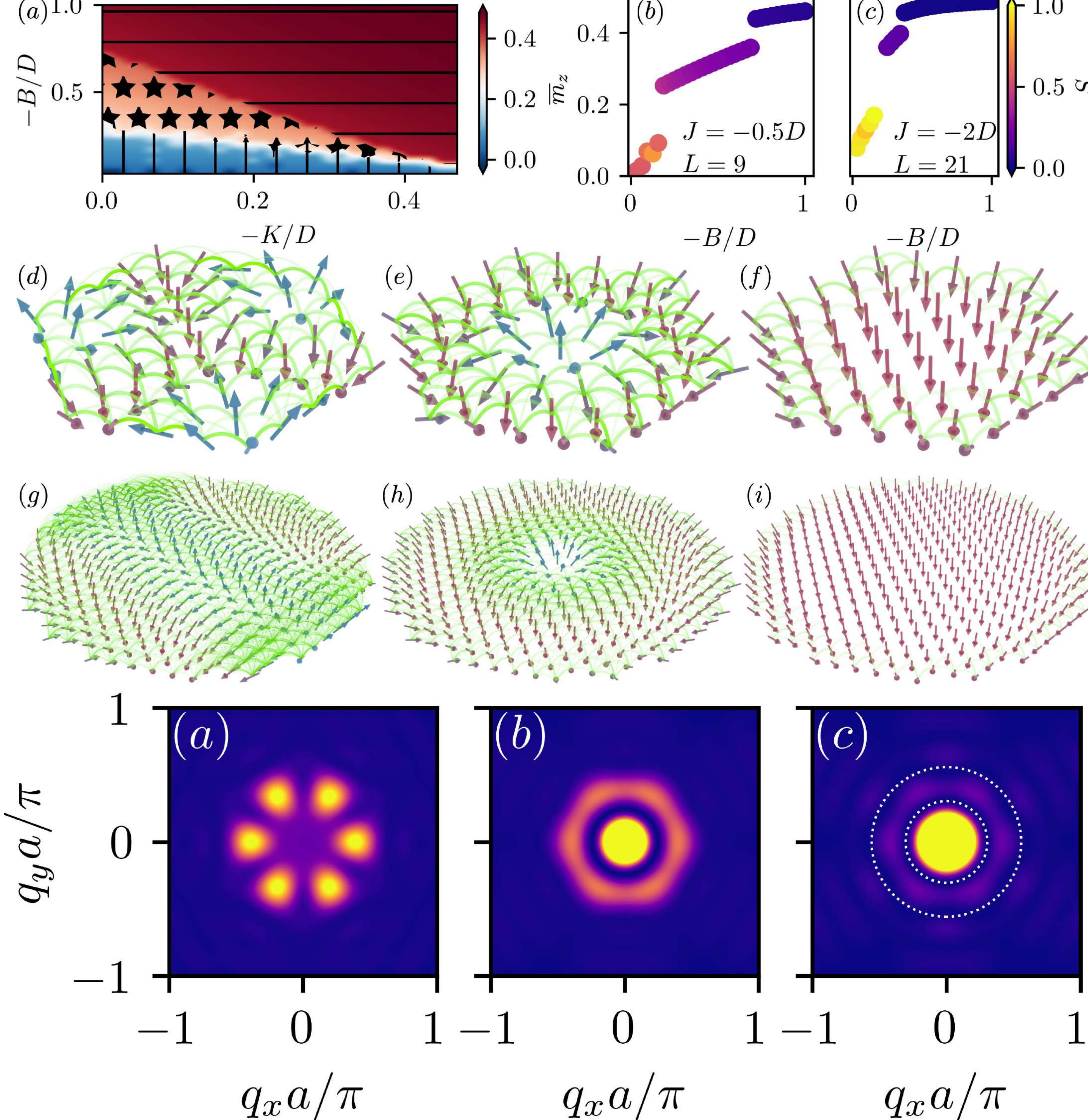

Acknowledgements

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Quantum skyrmion lattices in Heisenberg Ferromagnets

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- Prof. Thomas Schmidt