Mixed bubbles in Bose-Bose mixtures

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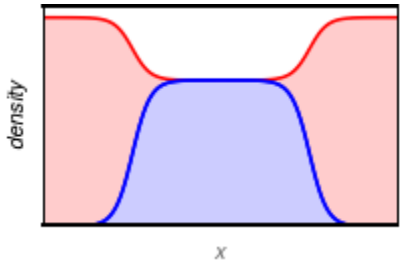
arXiv:2008.05870

Bose-Bose mixture, mean field

Mean-field energy density: $\frac{E_{MF}}{\text{Volume}} = \frac{g_{11} n_1^2 + g_{22} n_2^2 + 2 g_{12} n_1 n_2}{2}$

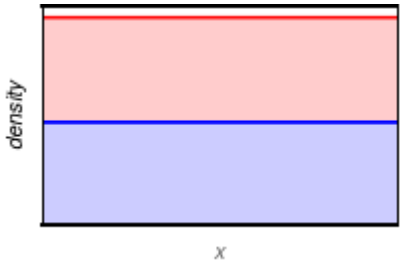
g_{12}

$g_{12} > \sqrt{g_{11} g_{22}}$ phase separation

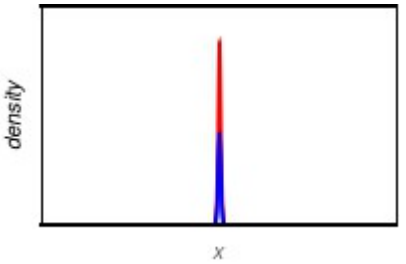


mean-field stability

$g_{11} > 0, \quad g_{22} > 0, \quad \text{and} \quad g_{12}^2 < g_{11} g_{22}$



$g_{12} < -\sqrt{g_{11} g_{22}}$ collapse



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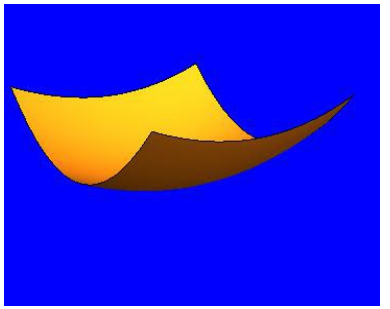

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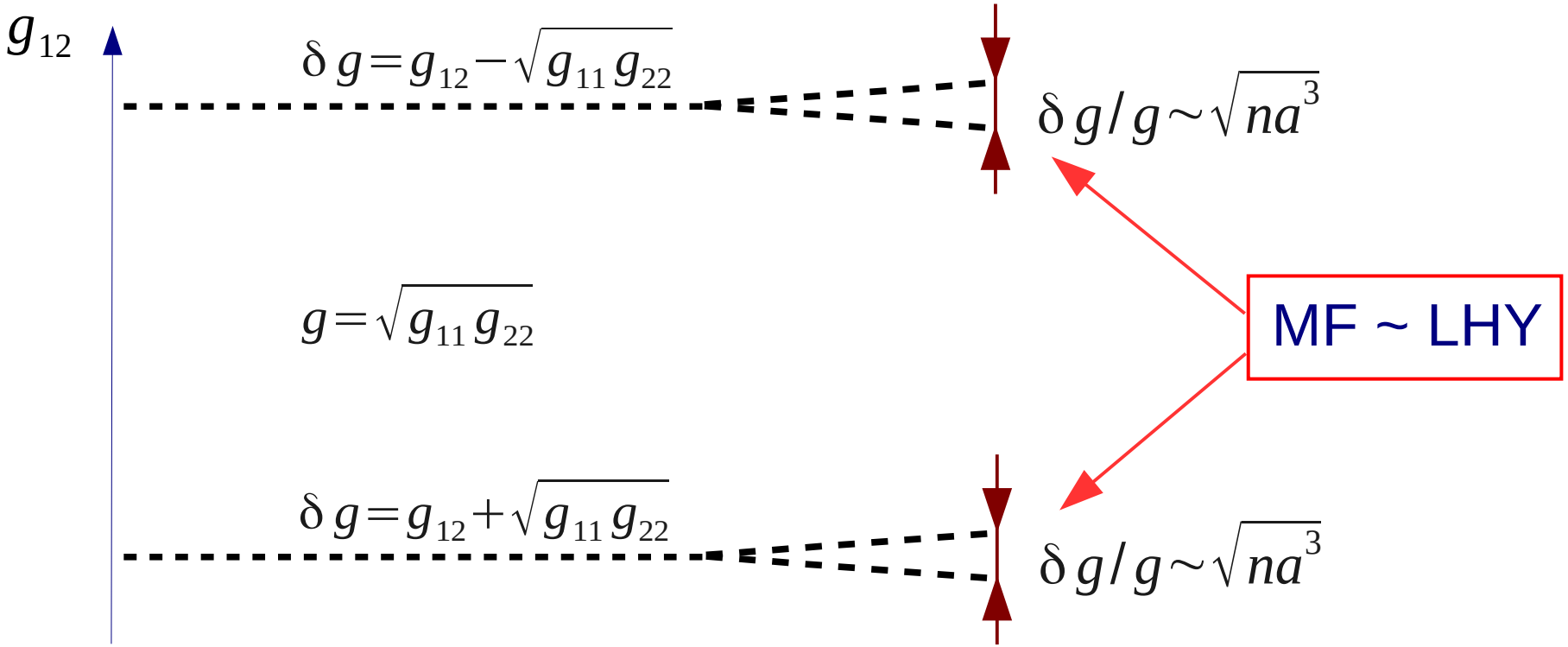
MF+LHY (or Bogoliubov or BMF or Larsen?)

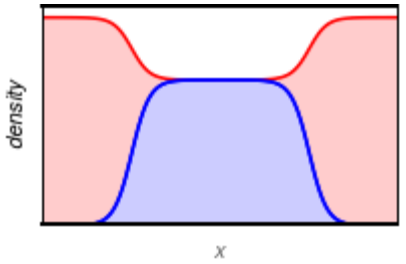
$$E_{MF}+E_{LHY}=\frac{1}{2}(n_1\ n_2)\begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}+E_{LHY}\sim gn_+^2\pm\delta g\, n_-^2+\underbrace{\frac{8}{15\pi^2}(g_{11}n_1+g_{22}n_2)^{5/2}}$$

3D, mass balanced



MF+LHY (or Bogoliubov or BMF or Larsen?)

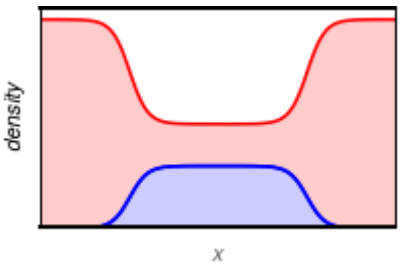
phase separation
- pure bubbles



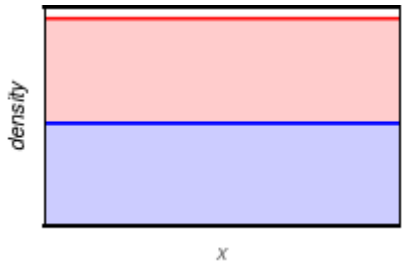
g_{12}

$\sqrt{g_{11}g_{22}}$

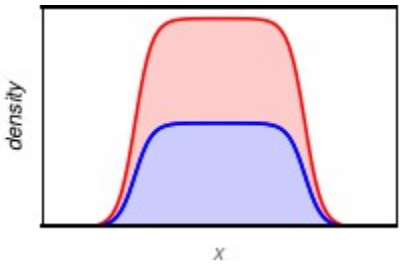
phase separation
- mixed bubbles



miscible phase

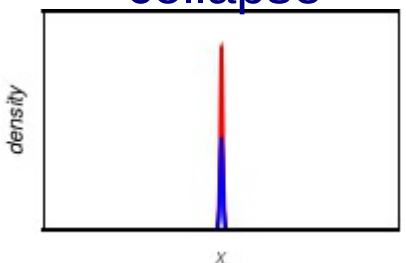


quantum droplets



$-\sqrt{g_{11}g_{22}}$

collapse



LHY correction (3D, mass balanced)

Bogoliubov theory

$$E_{\pm}(k)=\sqrt{c_{\pm}^2 k^2+k^4/4}\,; \quad c_{\pm}^2=\frac{g_{11}n_1+g_{22}n_2\pm\sqrt{(g_{11}n_1-g_{22}n_2)^2+4g_{12}^2 n_1 n_2}}{2}$$

$$\frac{E}{\text{Volume}}=\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}+\frac{1}{2}\sum_{\pm}\sum_{k}\left[E_{\pm}(k)-k^2/2-c_{\pm}^2\right]=$$

$$=\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}+\frac{8}{15\pi^2}\left(c_+^5+c_-^5\right)+\ldots$$

$$gn_+^2\pm\delta g n_-^2 \qquad (g_{11}n_1+g_{22}n_2)^{5/2}(1+\ldots\delta g/g) \qquad (\delta g n)^{5/2}$$

Power counting:

MF $\propto g n^2$, leading BMF $\propto g n^2\sqrt{ng^3}$, higher order terms $\propto g n^2(ng^3), g n^2(ng^3)^{7/4}$

n_+

$$\pm\delta g n_-^2+\left[g(An_++Bn_-)\right]^{5/2}-\mu_- n_- \to min \text{ at fixed } n_+$$

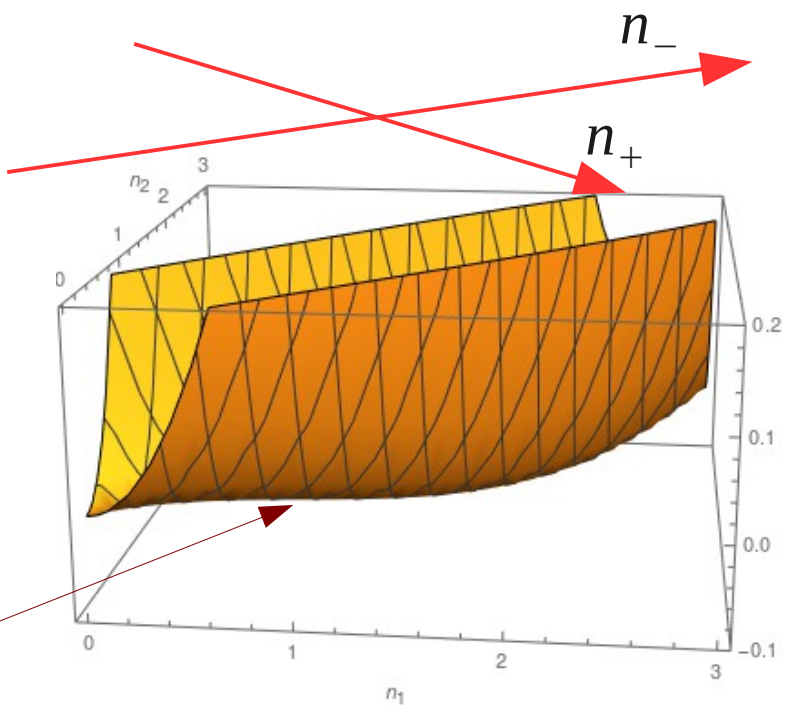
Quantum droplet

$$\delta g = g_{12} + \sqrt{g_{11} g_{22}} \ll \sqrt{g_{11} g_{22}} = g$$

$$n_+ \propto \sqrt{g_{11}}\, n_1 - \sqrt{g_{22}}\, n_2 \approx 0$$

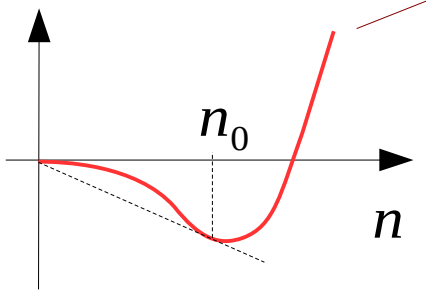
The mean-field term “locks” the ratio $\dfrac{n_2}{n_1} = \sqrt{\dfrac{g_{11}}{g_{22}}}$

$$n_- \propto \sqrt{g_{22}}\, n_1 + \sqrt{g_{11}}\, n_2 \propto n$$



$$\frac{E}{\text{volume}} = \delta g\, n^2 + \ldots (gn)^{5/2}$$

$\delta g < 0$ ⇒



⇒ Equilibrium with vacuum. Saturation density

$$n_0 \propto \frac{1}{a^3}\left(\frac{\delta g}{g}\right)^2$$

Note: $\delta g / g \sim \sqrt{n a^3}$

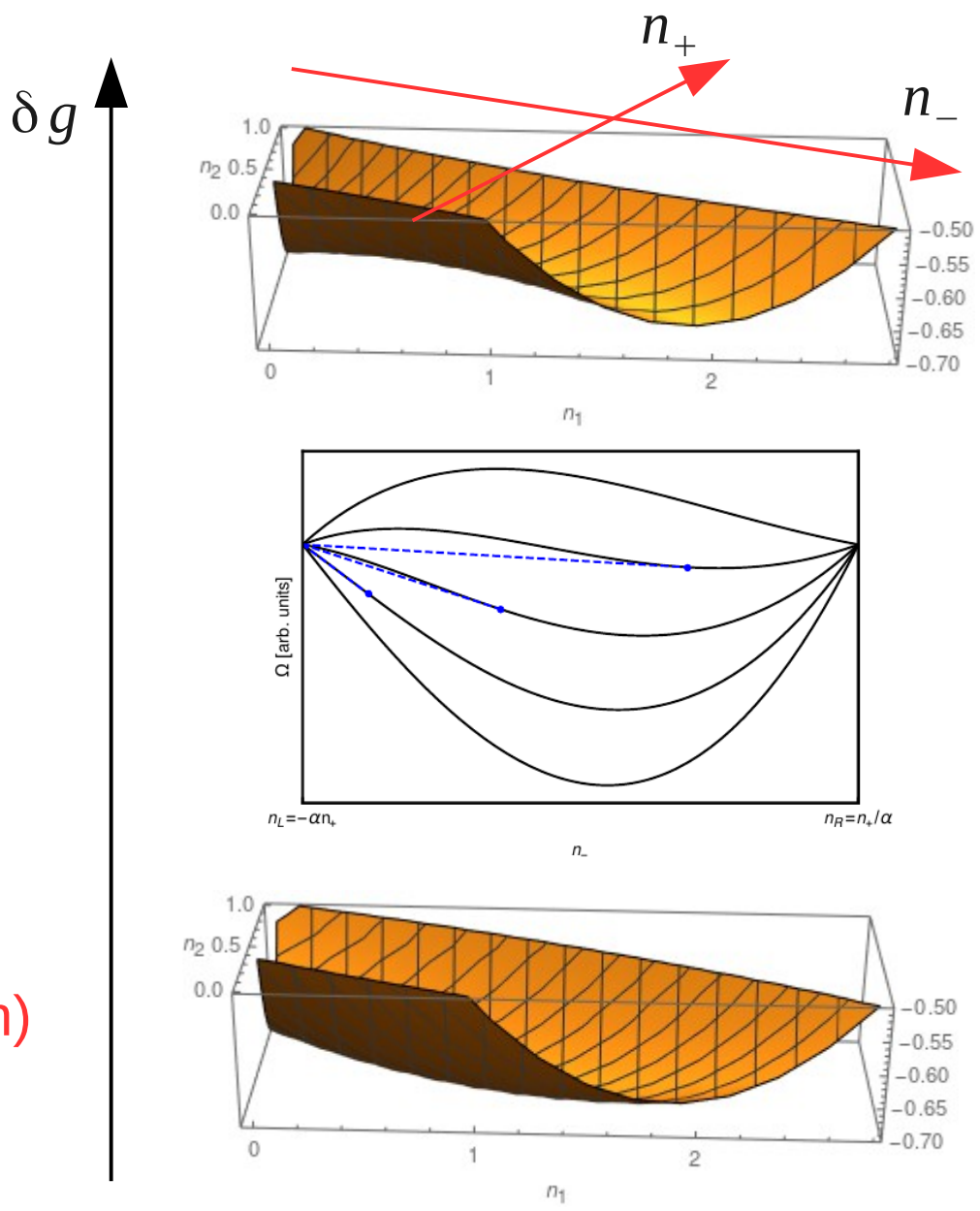
Mixed bubble

$$\delta g = g_{12} - \sqrt{g_{11} g_{22}} \ll \sqrt{g_{11} g_{22}} = g$$

$$n_+ \propto \sqrt{g_{11}} n_1 + \sqrt{g_{22}} n_2 \approx const \quad \text{``density''}$$

$$n_- \propto -\sqrt{g_{22}} n_1 + \sqrt{g_{11}} n_2 \qquad \text{``spin''}$$

$$\Omega = -\delta g n_-^2 + \left[g \left(A n_+ + B n_- \right) \right]^{5/2} - \mu_- n_-$$



First-order transition (phase separation) between pure and mixed phases for

$$g\sqrt{na^3} \sim \delta g_{min} < \delta g < \delta g_{max} \sim g\sqrt{na^3}$$

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$$\wp$$

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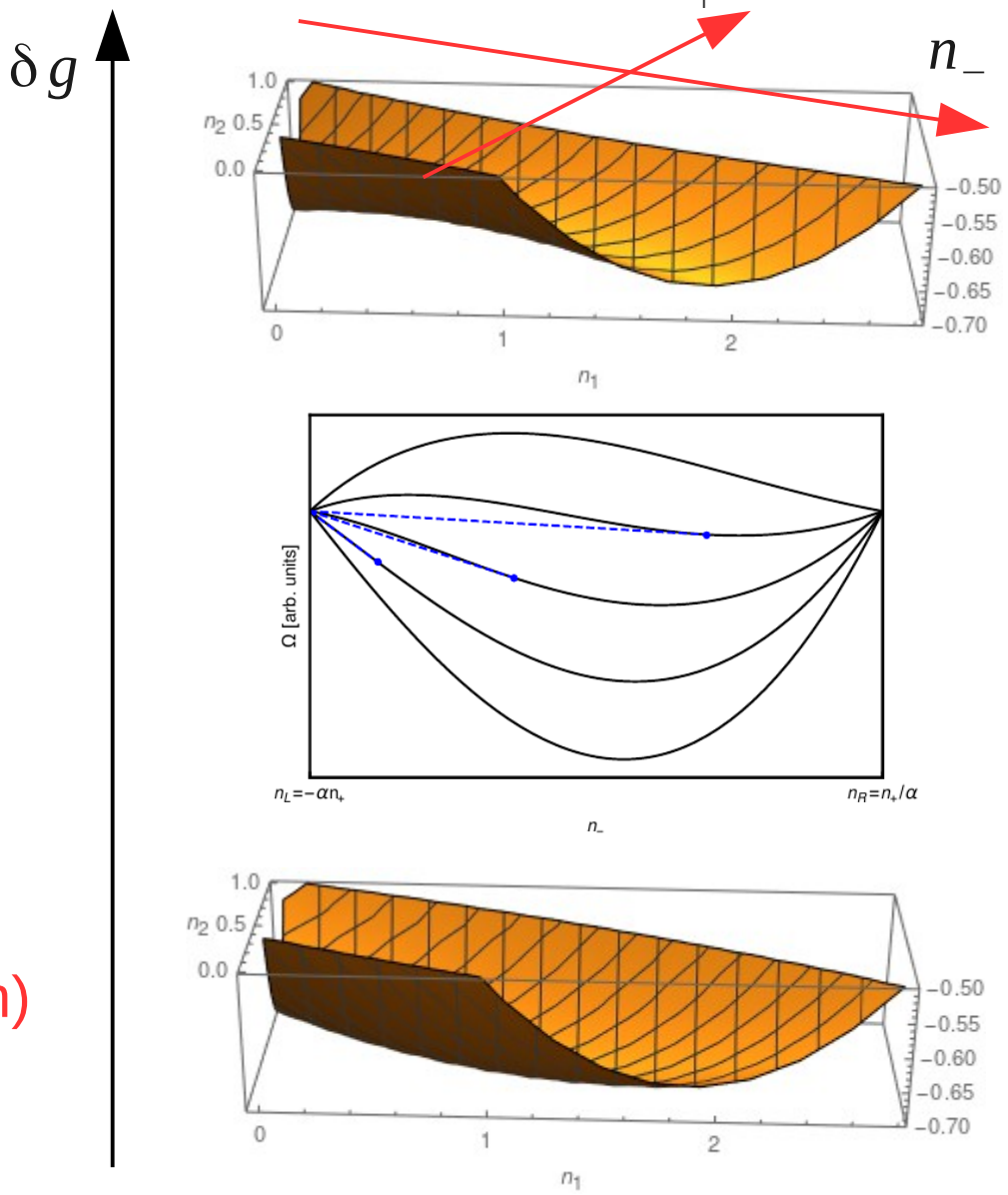

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$$g_{11} n_1 + g_{22} n_2$$

For $g_{22} > g_{11}$ concave-convex shape

For $g_{11} > g_{22}$ convex-concave shape

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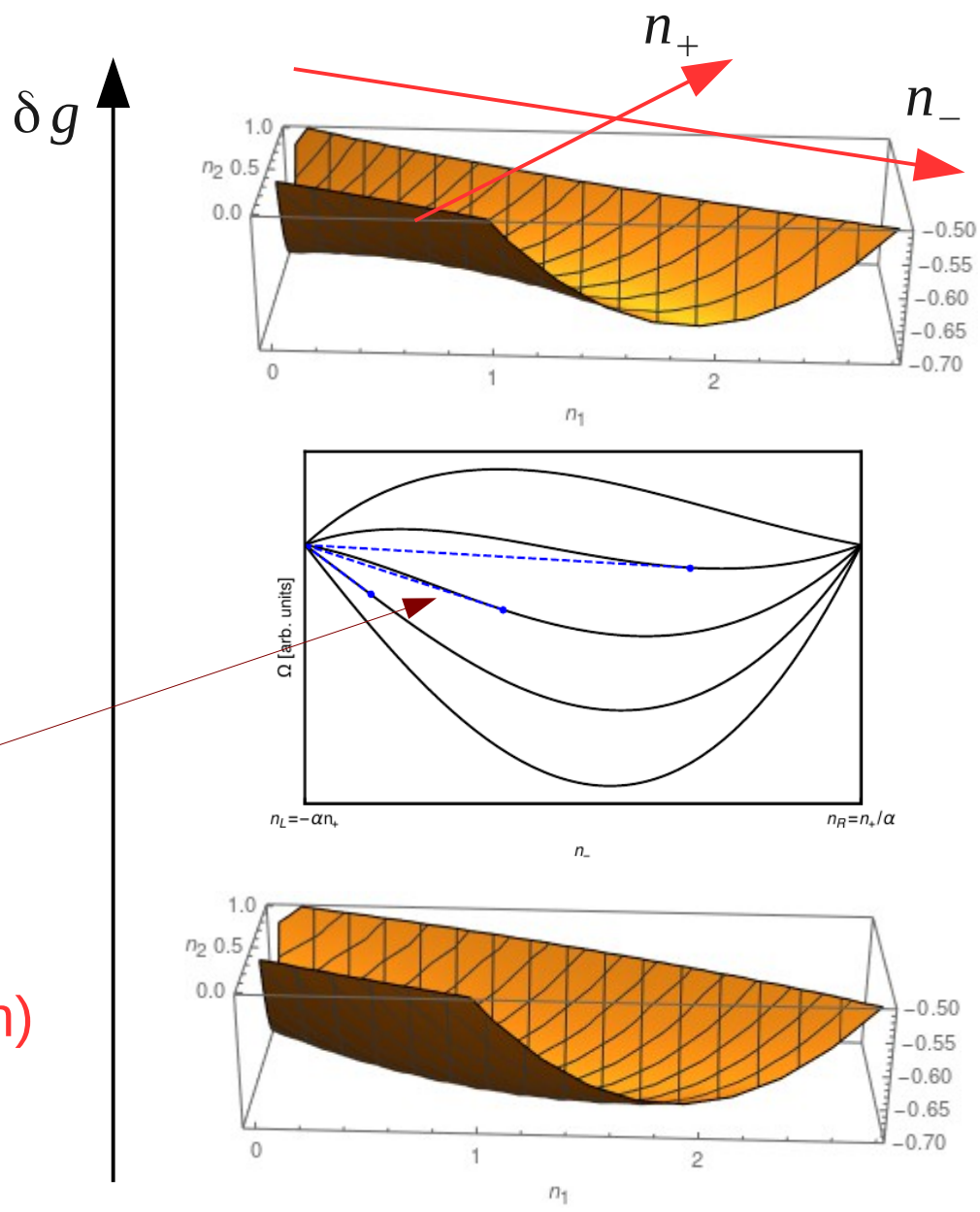
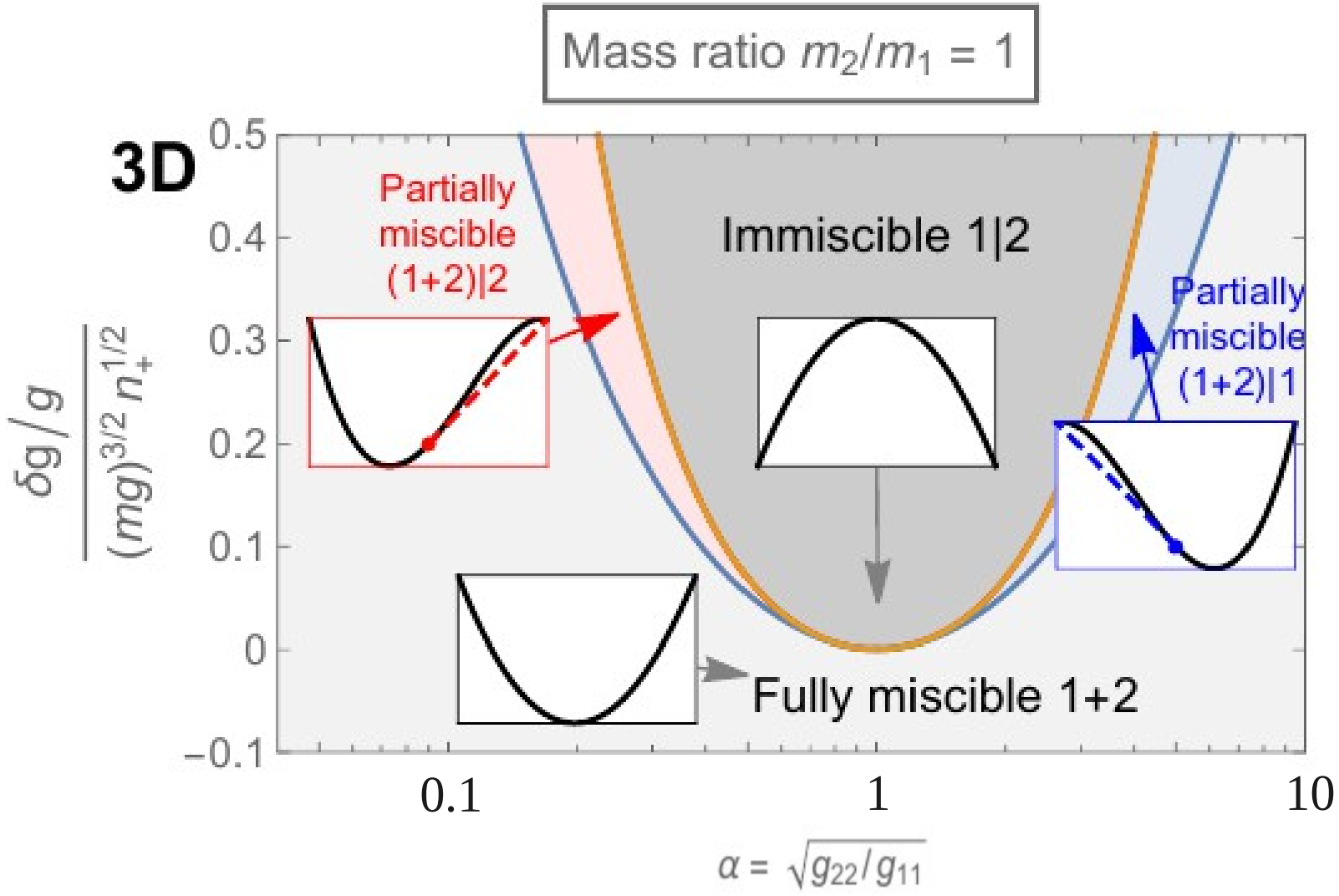
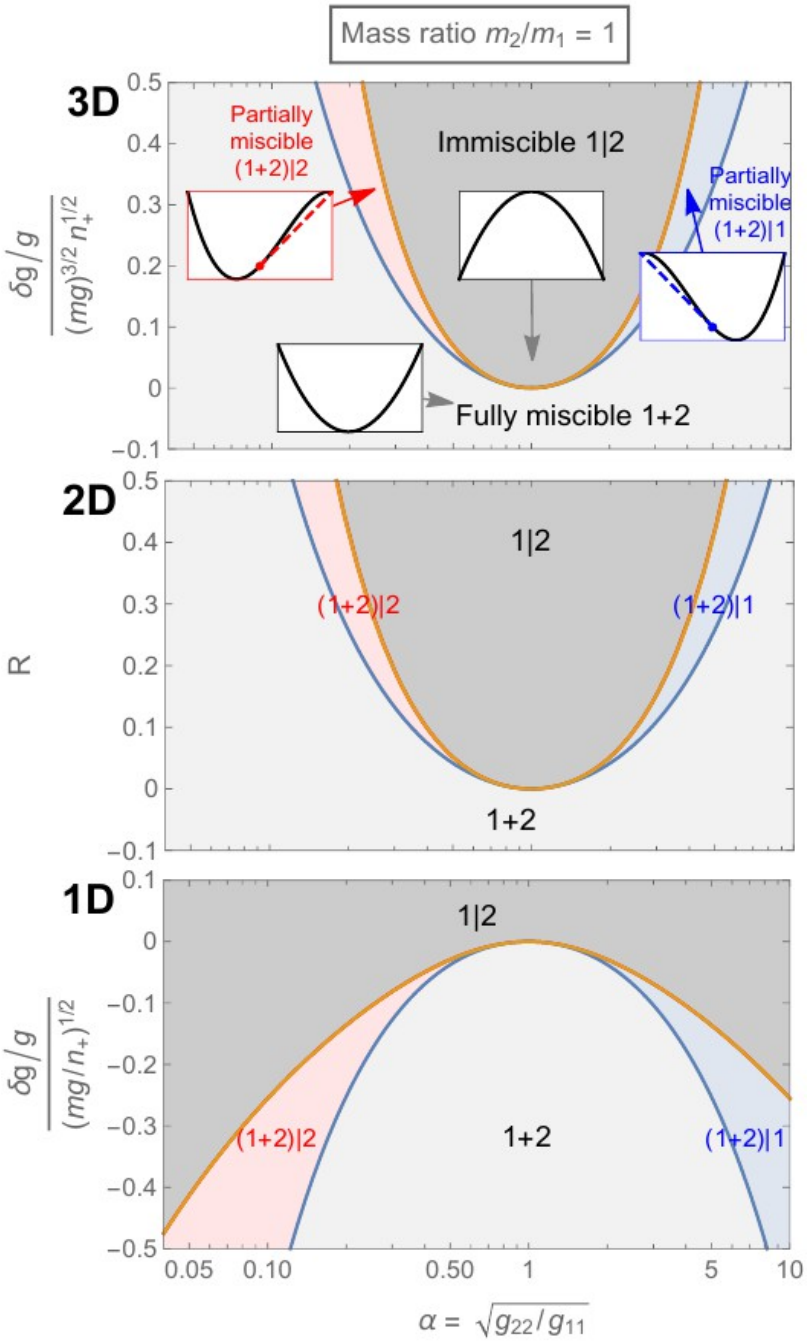



Mass ratio $m_2/m_1 = 1$
3D
Partially miscible (1+2)|2
Immiscible 1|2
Partially miscible (1+2)|1
Fully miscible 1+2
$\frac{\delta g/g}{(mg)^{3/2} n_+^{1/2}}$
0.5
0.4
0.3
0.2
0.1
0
−0.1
0.1
1
10
$\alpha = \sqrt{g_{22}/g_{11}}$


Mass ratio m_2/m_1 = 1
3D
Partially miscible (1+2)|2
Immiscible 1|2
Partially miscible (1+2)|1
Fully miscible 1+2
$\frac{\delta g/g}{(mg)^{3/2} n_+^{1/2}}$
2D
1|2
(1+2)|2
(1+2)|1
1+2
R
1D
1|2
(1+2)|2
1+2
(1+2)|1
$\frac{\delta g/g}{(mg/n_+)^{1/2}}$
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
0.05
0.10
0.50
1
5
10
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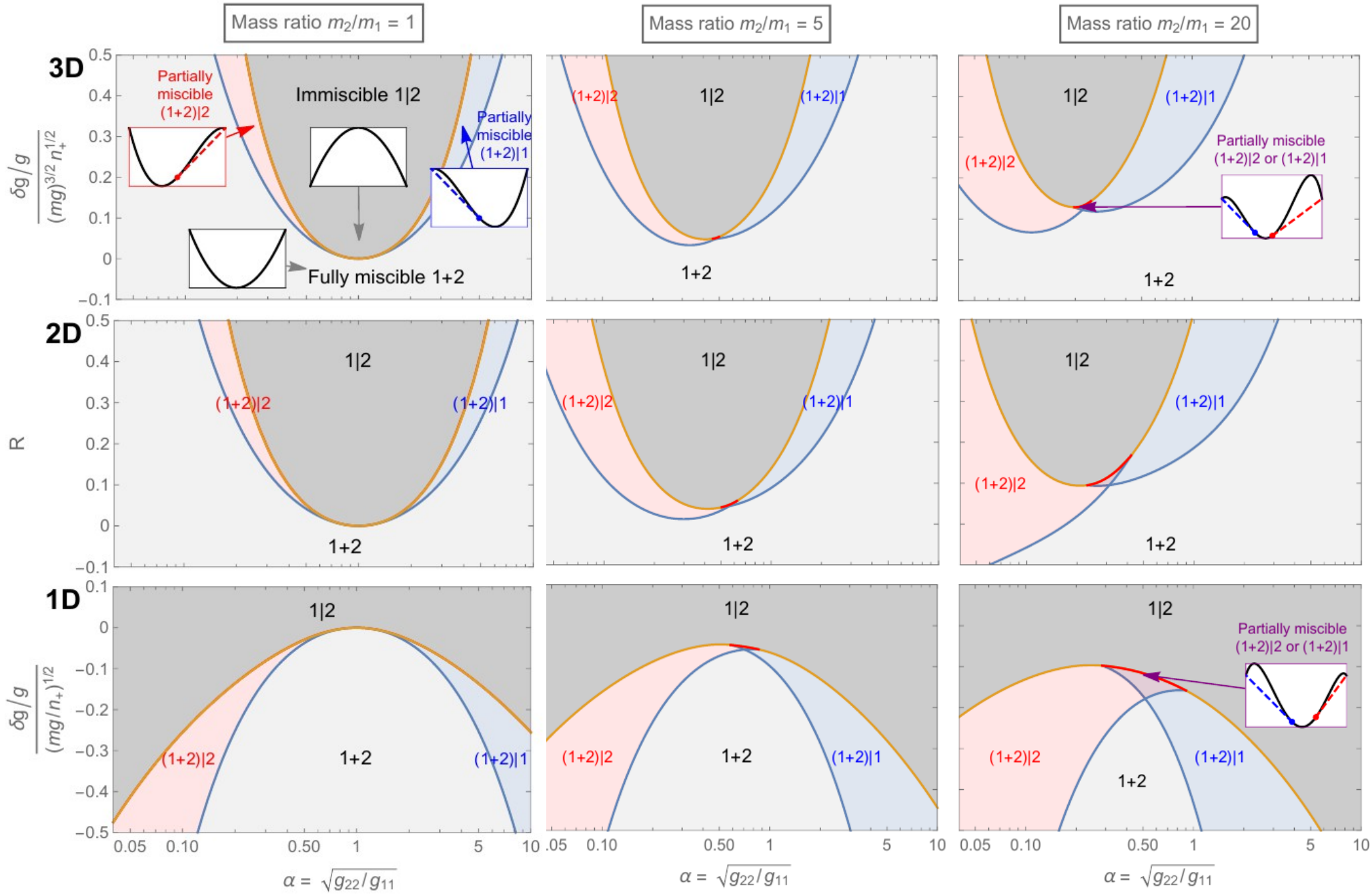
Mass ratio m_2/m_1 = 1
Mass ratio m_2/m_1 = 5
Mass ratio m_2/m_1 = 20
3D
2D
1D
R
$\frac{\delta g/g}{(mg)^{3/2} n_+^{1/2}}$
$\frac{\delta g/g}{(mg/n_+)^{1/2}}$
Partially miscible (1+2)|2
Immiscible 1|2
Partially miscible (1+2)|1
Fully miscible 1+2
1|2
(1+2)|2
(1+2)|1
1+2
Partially miscible (1+2)|2 or (1+2)|1
$\alpha = \sqrt{g_{22}/g_{11}}$

Summary

- Bubbles of mixed phase immersed in a pure component – yet another manifestation of BMF

- Bubble region widens with the interaction, with the mass imbalance, and with decreasing dimensionality

- Plenty of candidates among available ultracold mixtures, but need a flat trapping potential

Outlook

- Finite-size effects, quantum pressure, surface tension, external confinement,...

- Dynamics: excitations, motion of a bubble through the host gas (impossible with pure bubble in 1D), Andreev-Bashkin effect,...

- Beyond-Bogoliubov calculation, towards the strongly interacting regime (for instance, in 1D), Tonks-Bose = Fermi-Bose mixture? Mass-imbalanced Fermi-Fermi mixtures? etc. etc...

Thank you for your attention!