Mixtures of Ultracold Atomic Gases

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Outline

1. Two component-coherently coupled Bose gas
2. Domain wall solution
3. Pair of vortices – precession
4. Decay of the domain wall
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1. Two component-coherently coupled Bose gas
2. Domain wall solution
3. Pair of vortices – precession
4. Decay of the domain wall
5. Mixtures of Bose and Fermi superfluids
6. Phase diagram at unitarity
7. Dark-bright solitons at unitarity
8. Dark-bright solitons at BEC-BCS crossover
9. Conclusions
Two component BEC's

Two coupled GPE's:

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\begin{align*}
\text{i} \hbar \partial_t \psi_1 &= (H_0 + g_{11} |\psi_1|^2 + g_{12} |\psi_2|^2) \psi_1 \\
\text{i} \hbar \partial_t \psi_2 &= (H_0 + g_{22} |\psi_2|^2 + g_{12} |\psi_1|^2) \psi_2 \\
\text{with} \quad H_0 &= -\frac{\hbar^2}{2m} \nabla^2 + V(x)
\end{align*}
\]
**Two component BEC's**

**✔ Two coupled GPE's:**

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 i\hbar \frac{\partial \psi_1}{\partial t} = \left( H_0 + g_{11} |\psi_1|^2 + g_{12} |\psi_2|^2 \right) \psi_1 \\
 i\hbar \frac{\partial \psi_2}{\partial t} = \left( H_0 + g_{22} |\psi_2|^2 + g_{12} |\psi_1|^2 \right) \psi_2
\]

with \( H_0 = -\frac{\hbar^2}{2m} \nabla^2 + V(x) \)

**✔ Phase separation:**

\[
\frac{g_{11} g_{22}}{g_{12}^2} > 1 \quad - \text{miscible regime}
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\frac{g_{11} g_{22}}{g_{12}^2} < 1 \quad - \text{immiscible regime}
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S. Beattie et al., PRL 110 (2013)
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✔ dark-bright soliton:

From: A. Sartori and A. Recati, EPJD 2013

S. Beattie et al., PRL 110 (2013)
Coherently coupled BEC's

Two GPE's with coherent \((Rabi)\) coupling:

\[
\begin{align*}
\frac{i\hbar}{\hbar} \partial_t \psi_1 &= \left( H_0 + g_{11} |\psi_1|^2 + g_{12} |\psi_2|^2 \right) \psi_1 - \frac{1}{2} \hbar \Omega_R \psi_2 \\
\frac{i\hbar}{\hbar} \partial_t \psi_2 &= \left( H_0 + g_{22} |\psi_2|^2 + g_{12} |\psi_1|^2 \right) \psi_2 - \frac{1}{2} \hbar \Omega_R \psi_1
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with \(H_0 = -\frac{\hbar^2}{2m} \nabla^2 + V(x)\)
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Two GPE's with coherent (Rabi) coupling:

\[ i\hbar \partial_t \psi_1 = (H_0 + g_{11} |\psi_1|^2 + g_{12} |\psi_2|^2)\psi_1 - \frac{1}{2} \hbar \Omega_R \psi_2 \]

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Energy density \( e(n_\uparrow, n_\downarrow) = \frac{1}{2} g_{\uparrow\uparrow} n_\uparrow^2 + \frac{1}{2} g_{\downarrow\downarrow} n_\downarrow^2 + g_{\uparrow\downarrow} n_\uparrow n_\downarrow - \Omega_R \cos(\phi_\uparrow - \phi_\downarrow) \sqrt{n_\uparrow n_\downarrow} \)
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✔ Two possible ground states: miscible and immiscible
✔ Second-order phase transition due to \( \Omega \)
✔ Phases tend to be equal to each other
✔ Symmetry breaking (conservation of the total N only)
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hybridisation

Coherently coupled BEC's

\[ \Omega_R = 0 \]

\[ \Omega_R \neq 0, g_{11} = g_{22} \]

Second order phase transition:

Gap opens

\[ g_{12} = g_{11} + \frac{2\Omega}{n} \]

(for \( g_{11} = g_{22} \))

Domain wall

The densities $n_1$ and $n_2$ are frozen degrees of freedom.

$$E(\phi_1, \phi_2) = \frac{\hbar^2}{2m} [n_1 (\nabla \phi_1)^2 + n_2 (\nabla \phi_2)^2] - \Omega_R \cos(\phi_1 - \phi_2) \sqrt{n_1 n_2}$$
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Domain wall solution (local minimum):

$$\phi_1 = 4 \frac{n_2}{n} \arctan e^{kz} \quad \phi_2 = -4 \frac{n_1}{n} \arctan e^{kz}$$

and $$k^2 = mn\Omega/\hbar \sqrt{n_1 n_2}$$

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Surface tension $E_{\text{wall}} - E_{\text{GS}}$:

$$\sigma = 8 \frac{\hbar^{3/2} \sqrt{\Omega n}}{\sqrt{m}} \left( \frac{n_1 n_2}{n^2} \right)^{3/4}$$

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MT et al., in preparation
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Total energy (Thomas-Fermi):

\[ E(d) = 2E_v + E_{dw} + E_{int} \]
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if \( E_v \ll E_{\text{wall}} \)

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MT et al., in preparation
Decay of the domain wall

- Critical value of $\Omega_R$
- $\hbar\Omega_c = \frac{1}{3} n\delta g$
- Decay due to density fluctuations for $\Omega_R > \Omega_C$
- Smaller pieces of domain wall

*MT et al., in preparation*
Mixtures of Bosons and Fermions

- $^6$Li fermions
- $^7$Li bosons ($a_s > 0$)
- In optical dipole trap
- $N_f \gg N_b$
- Initial imbalance

From: Igor Ferrier-Barbut et al., Science 2014

Also: M. Delehaye et al. 2015
Mixtures of Bosons and Fermions

- $^6$Li fermions
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- Plato at the centre, where SF pairing occurs
- Dynamics of the dipole modes
- $\omega_f/\omega_b = \sqrt{7/6}$
- Long-lived oscillations below critical velocity $v_c$
- Beats – energy transfer between the clouds
- agrees across the BCS-BEC crossover

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Mixtures of Bosons and Fermions

Phase Diagram from the Equation of State

\[ E = \frac{1}{2} g_{bb} n_b^2 + g_{bf} n_b n_f + \frac{3}{5} E_F n_f \]

Ideal gas of fermions
Mixtures of Bosons and Fermions

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Linear stability condition:

\[ \frac{\partial \mu_i}{\partial n_j} \geq 0 \]

Viverit \textit{at al.}, PRA 2000
Mixtures of Bosons and Fermions

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\[ n_f^{1/3} \leq \frac{(6\pi^2)^{2/3}}{12\pi} \frac{m_{bf}^2 a_{bb}}{m_b m_f a_{bf}^2} \]

Viverit et al., PRA 2000
Mixtures of Bosons and Fermions

Bose-Fermi mixture at unitarity

MT et al., in preparation
Mixtures of Bosons and Fermions

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Mixtures of Bosons and Fermions

Bose-Fermi mixture at unitarity

\[ n_f^{1/3} \leq \eta \left( \frac{6\pi^2}{12\pi} \right)^{2/3} \frac{m_{bf}^2 a_{bb}}{m_b m_f a_{bf}^2} \]

MT et al., in preparation
Mixtures of Bosons and Fermions

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MT et al., in preparation
Mixtures of Bosons and Fermions

Bose-Fermi mixture at BEC and BCS

Phase separation
Soliton
Order parameter - soliton

MT et al., in preparation
Mixtures of Bosons and Fermions

Bose-Fermi mixture at BEC and BCS

Phase separation

Soliton

Order parameter - soliton

Partial separation

complete separation

coexistence

MT et al., in preparation
Mixtures of Bosons and Fermions

Bose-Fermi mixture at BEC and BCS

MT et al., in preparation

Solution of two coupled GPE's with mean-field parameters
Mixtures of Bosons and Fermions

For $g_{bf} n_0 = E_F$, the Fermi gas is coherent on both sides of the solitonic depletion (solid line).

For $g_{bf} n_0 = 1.7 E_F$, there is no coherence and therefore it is a phase separation (dashed line).

Density and $\Delta$ after a sign change for wherever $\Delta < 0$
Conclusions

1. The coherently coupled BEC's admit a domain wall solution between two vortex lines,

2. The tension of the domain wall causes the vortex pair to precess,

3. Increasing the Rabi coupling above the threshold causes the domain wall to decay,

4. The mean-field equations restore the three phases predicted by the LDA,

5. The soliton's coherence survives at the partial separation and is destroyed at a complete separation.
Thank you for your attention