Propagation of quantum correlations after a quench in the Mott-insulator regime of the Bose-Hubbard model

Konstantin Krutitsky

Patrick Navez

Friedemann Queisser

Ralf Schützhold

UNIVERSITÄT DUISBURG ESSEN

Bose-Hubbard model

$$\hat{H} = -\frac{J}{Z}\sum_{\mu\nu} T_{\mu\nu}\,\hat{b}_\mu^\dagger \hat{b}_\nu + \frac{U}{2}\sum_\mu \hat{n}_\mu(\hat{n}_\mu - 1)$$

$$i\partial_t\hat{\rho} = \left[\hat{H},\hat{\rho}\right]$$

Reduced density matrices

$$\hat{\rho}_\mu = \mathrm{tr}_{\not{\mu}}\{\hat{\rho}\} \qquad \hat{\rho}_{\mu\nu} = \mathrm{tr}_{\not{\mu}\not{\nu}}\{\hat{\rho}\} \qquad \text{etc}$$

e.g. $\langle \hat{n}_\mu^2 \rangle = \mathrm{tr}\left(\hat{\rho}_\mu \hat{n}_\mu^2\right) \qquad \langle \hat{b}_\mu^\dagger \hat{b}_\nu \rangle = \mathrm{tr}\left(\hat{\rho}_{\mu\nu}\hat{b}_\mu^\dagger \hat{b}_\nu\right)$ etc

$$\hat{\rho}_{\mu\nu}^{\text{corr}} = \hat{\rho}_{\mu\nu} - \hat{\rho}_\mu\hat{\rho}_\nu$$

$$\hat{\rho}_{\mu\nu\lambda}^{\text{corr}} = \hat{\rho}_{\mu\nu\lambda} - \hat{\rho}_{\mu\nu}^{\text{corr}}\hat{\rho}_\lambda - \hat{\rho}_{\mu\lambda}^{\text{corr}}\hat{\rho}_\nu - \hat{\rho}_{\nu\lambda}^{\text{corr}}\hat{\rho}_\mu - \hat{\rho}_\mu\hat{\rho}_\nu\hat{\rho}_\lambda$$

Equations of motion for the correlations

$$i\partial_t\hat{\rho}_\mu = \left[\hat{H}_\mu, \hat{\rho}_\mu\right] + \frac{1}{Z}\sum_{\kappa\neq\mu} tr_\kappa \left[\hat{H}_{\mu\kappa} + \hat{H}_{\kappa\mu}, \hat{\rho}^{\text{corr}}_{\mu\kappa} + \hat{\rho}_\mu\hat{\rho}_\kappa\right]$$

$$\begin{aligned}
i\partial_t\hat{\rho}^{\text{corr}}_{\mu\nu} &= \left[\hat{H}_\mu, \hat{\rho}^{\text{corr}}_{\mu\nu}\right] + \frac{1}{Z}\left[\hat{H}_{\mu\nu}, \hat{\rho}^{\text{corr}}_{\mu\nu} + \hat{\rho}_\mu\hat{\rho}_\nu\right] \\
&-\frac{\hat{\rho}_\mu}{Z}\, tr_\mu \left[\hat{H}_{\mu\nu} + \hat{H}_{\nu\mu}, \hat{\rho}^{\text{corr}}_{\mu\nu} + \hat{\rho}_\mu\hat{\rho}_\nu\right] \\
&+\frac{1}{Z}\sum_{\kappa\neq\mu,\nu} tr_\kappa \left[\hat{H}_{\mu\kappa} + \hat{H}_{\kappa\mu}, \hat{\rho}^{\text{corr}}_{\mu\nu}\hat{\rho}_\kappa + \hat{\rho}^{\text{corr}}_{\nu\kappa}\hat{\rho}_\mu + \hat{\rho}^{\text{corr}}_{\mu\nu\kappa}\right] + (\mu \leftrightarrow \nu)
\end{aligned}$$

etc

$$\hat{H}_\mu = \frac{U}{2}\hat{n}_\mu\left(\hat{n}_\mu - 1\right) \qquad\qquad \hat{H}_{\mu\nu} = -J\,T_{\mu\nu}\,\hat{b}^\dagger_\mu\hat{b}_\nu$$

Strategies to solve

- Expansion in $1/Z$ [for details see PRA **89**, 033616 (2014)]
- Truncate eqs. [EPJ Quantum Technology **1**:12 (2014)]

Results

Dynamics after quench $J/U = 0 \to 0.1$ (Mott insulator regime)

$$|\psi_{\text{initial}}\rangle = \bigotimes_{\mu} |1\rangle_{\mu}$$

1D chain

2D square lattice

periodic boundary conditions

- Particle-number distribution

  $p_{\mu}(n)$

- One-body density matrix

  $\langle \hat{b}_{\mu}^{\dagger} \hat{b}_{\nu} \rangle$

- comparison with exact diagonalization (small systems)

2-point vs 3-point correlations in 1D and 2D

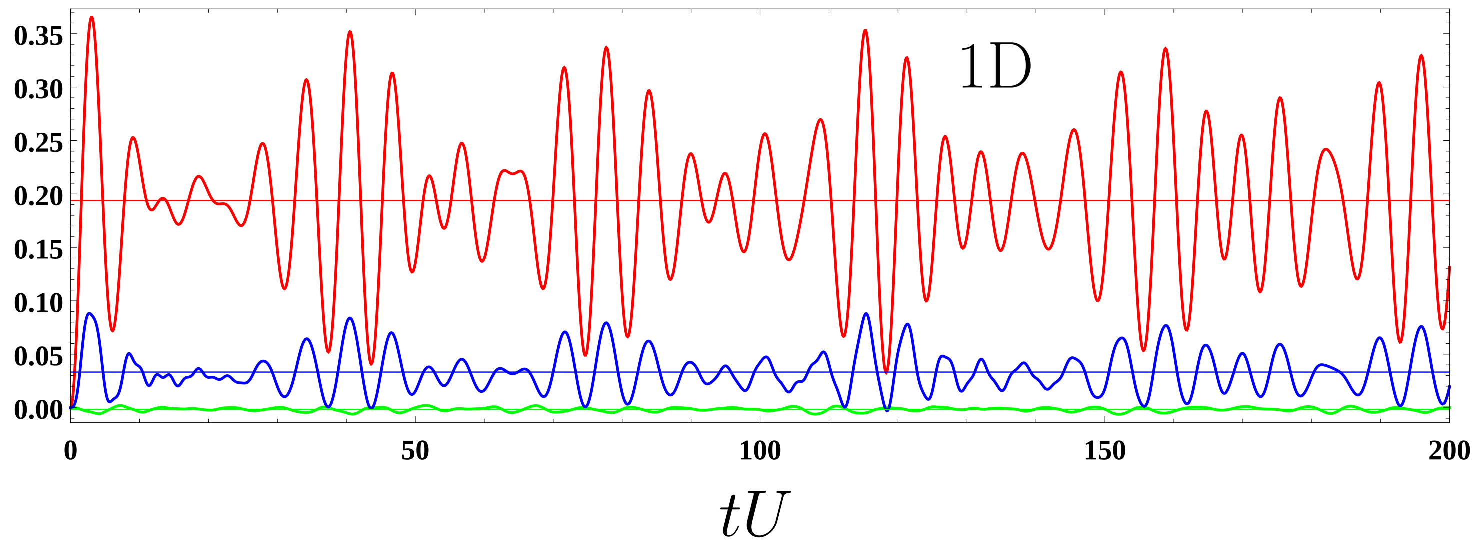
$J/U = 0 \to 0.1$ exact diagonalization

$N = L = 11$

$\langle \hat{a}_1^\dagger \hat{a}_2 \rangle$

$\langle \hat{a}_1 (\hat{a}_2^\dagger)^2 \hat{a}_3 \rangle$

$\langle \hat{n}_1 \hat{a}_2^\dagger \hat{a}_3 \rangle^c$

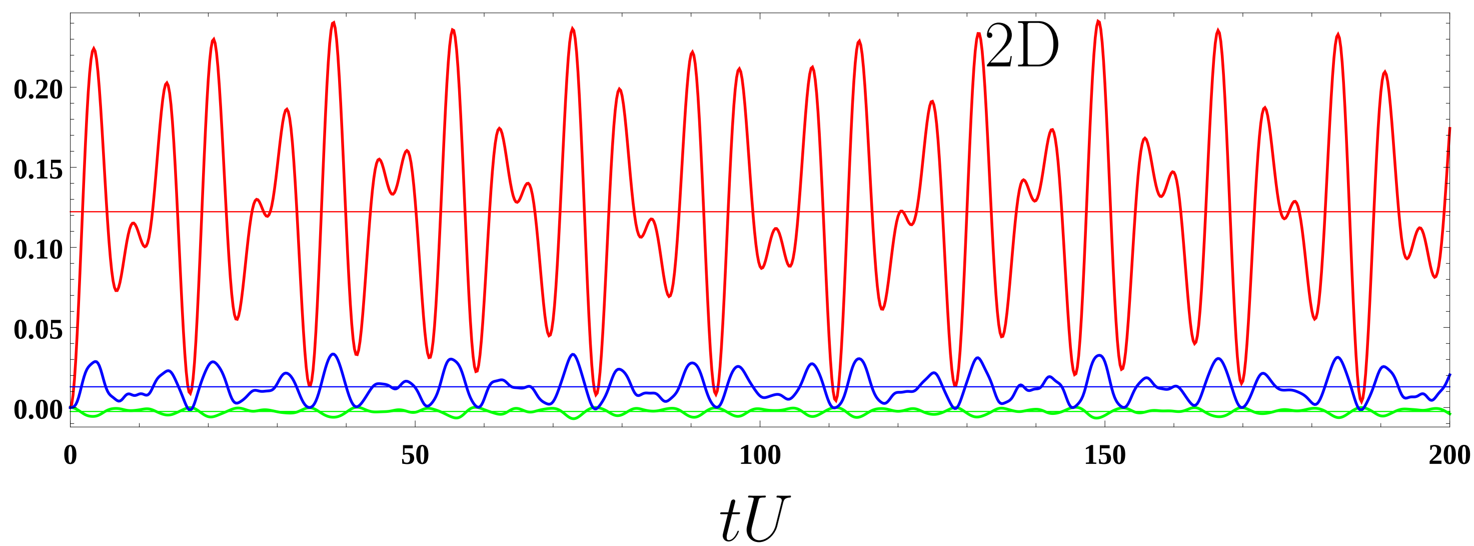


2D: 3×3

$\langle \hat{a}_1^\dagger \hat{a}_2 \rangle$

$\langle \hat{a}_1 (\hat{a}_2^\dagger)^2 \hat{a}_3 \rangle$

$\langle \hat{n}_1 \hat{a}_2^\dagger \hat{a}_3 \rangle^c$



Expansion in $1/Z$

PRA **89**, 033616 (2014)

$$\hat{\rho}_\mu = \mathcal{O}\,(1) \qquad \hat{\rho}^{\text{corr}}_{\mu\nu} = \mathcal{O}\left(\frac{1}{Z}\right) \qquad \hat{\rho}^{\text{corr}}_{\mu\nu\lambda} = \mathcal{O}\left(\frac{1}{Z^2}\right) \qquad \text{etc}$$

$\mathcal{O}(1)$: Gutzwiller mean-field equations $\rightarrow$ no time dynamics

$$\mathcal{O}\left(\frac{1}{Z}\right): \qquad p_\mu(0) = p_\mu(2) = \frac{4J^2}{N}\sum_{\mathbf{k}} T^2_{\mathbf{k}} \frac{1-\cos(\omega_{\mathbf{k}} t)}{\omega^2_{\mathbf{k}}}$$

$$\omega_{\mathbf{k}} = \sqrt{U^2 - 6JUT_{\mathbf{k}} + J^2 T^2_{\mathbf{k}}} \qquad T_{\mathbf{k}} = \frac{1}{NZ}\sum_{\mu_1\mu_2} T_{\mu_1\mu_2} e^{i\mathbf{k}\cdot(\mathbf{x}_{\mu_1}-\mathbf{x}_{\mu_2})}$$

$$\langle \hat{b}^\dagger_{\mu_1}\hat{b}_{\mu_2}\rangle = \frac{4JU}{N}\sum_{\mathbf{k}} T_{\mathbf{k}} \frac{1-\cos(\omega_{\mathbf{k}} t)}{\omega^2_{\mathbf{k}}}\, e^{i\mathbf{k}\cdot(\mathbf{x}_{\mu_1}-\mathbf{x}_{\mu_2})}$$

maximum group velocity $\mathbf{v}_{\mathrm{max}} = \max \nabla_{\mathbf{k}} \omega_{\mathbf{k}}$

hypercubic lattice in D dimensions with small J/U:

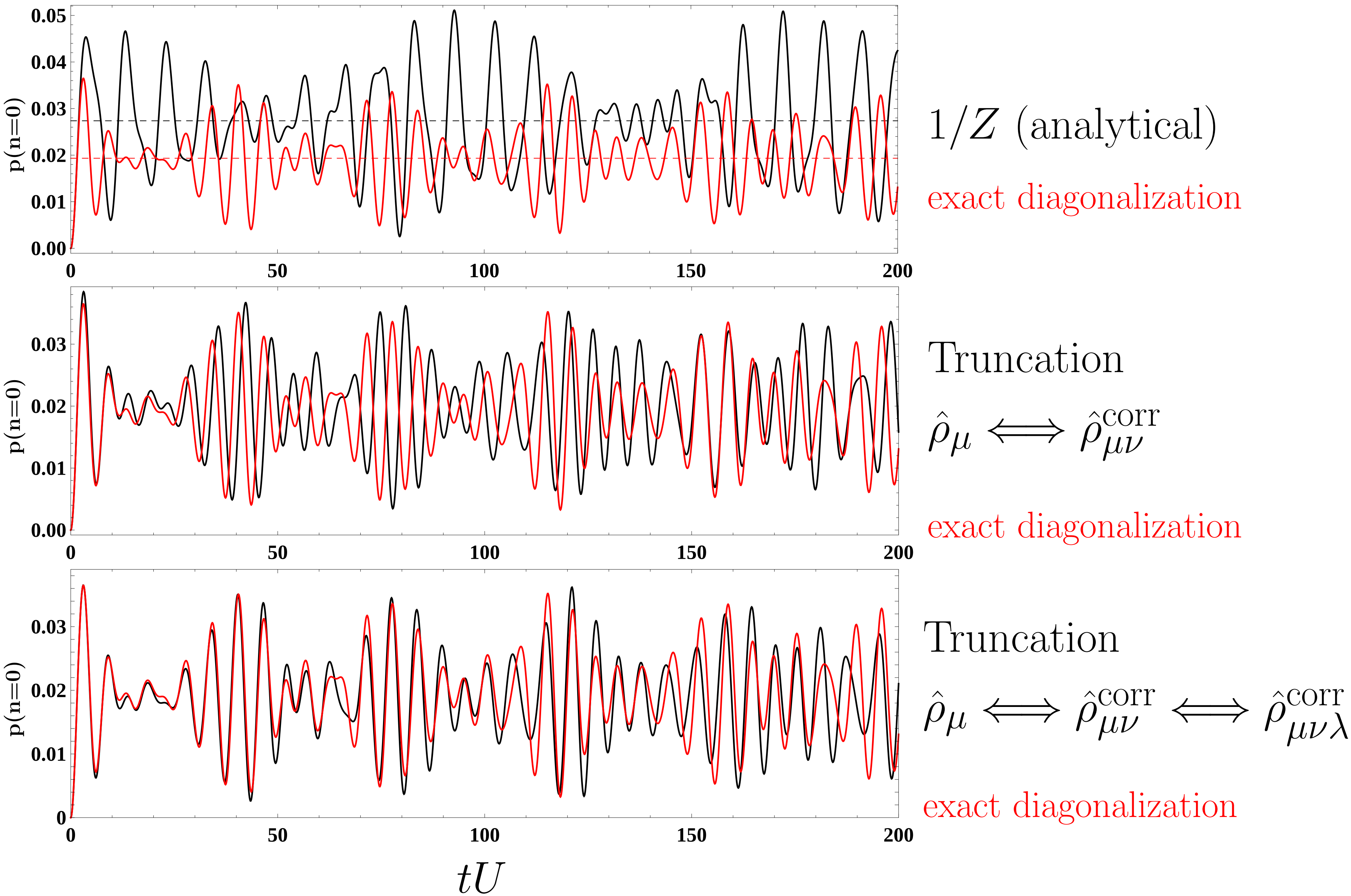
$$v_{\mathrm{max}} \approx 3J/D \text{ along the lattice axes}$$
$$v_{\mathrm{max}} \approx 3J/\sqrt{D} \text{ along the diagonal}$$

similar result in one dimension: P. Barmettler et al, PRA **85**, 053625 (2012)

experimental realization: M. Cheneau et al, Nature **481**, 484 (2012)

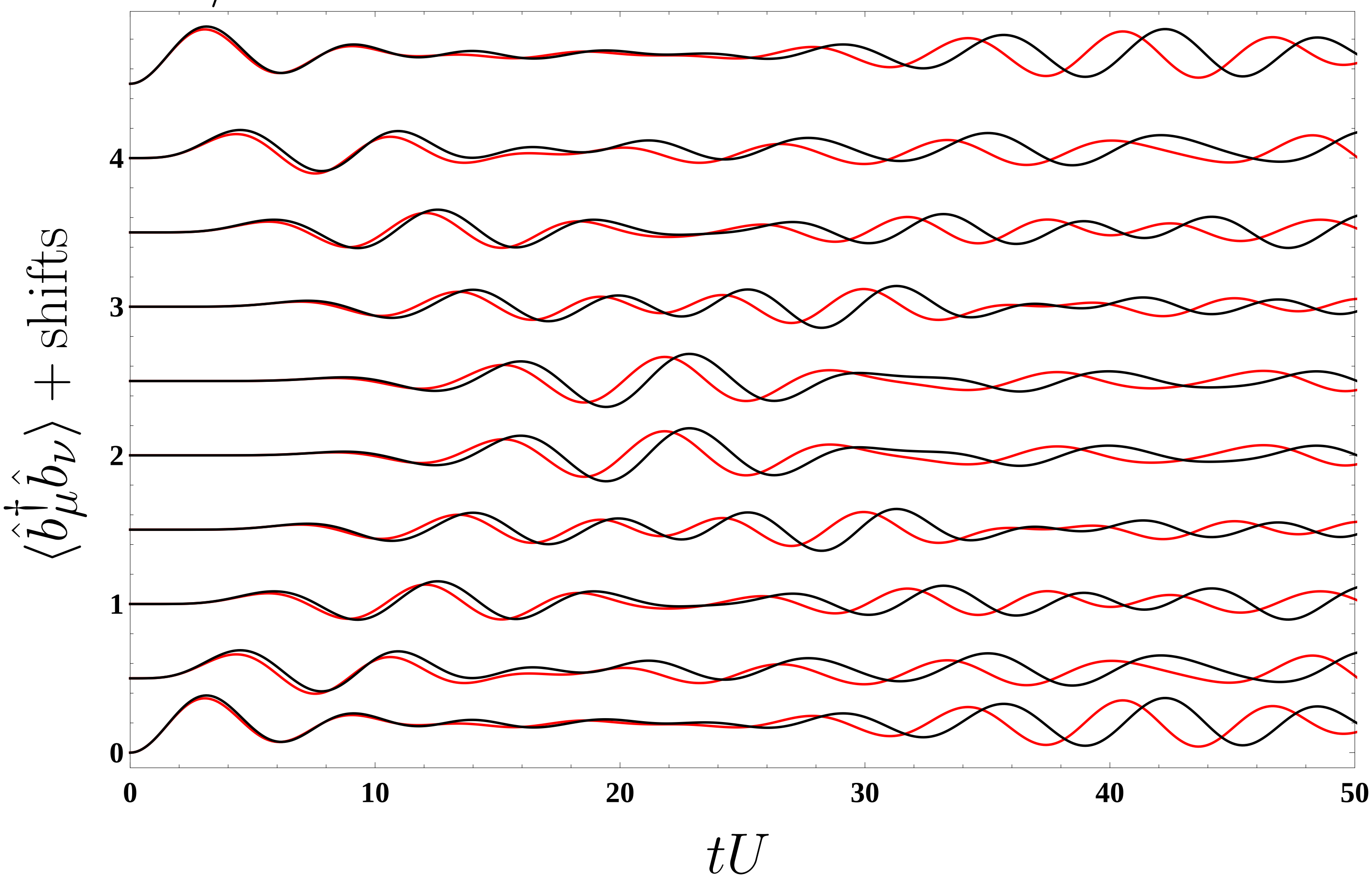
Dynamics after quench in 1D

$J/U = 0 \rightarrow 0.1 \qquad N = L = 11$



Dynamics after quench in 1D

$J/U = 0 \to 0.1$ $N = L = 11$

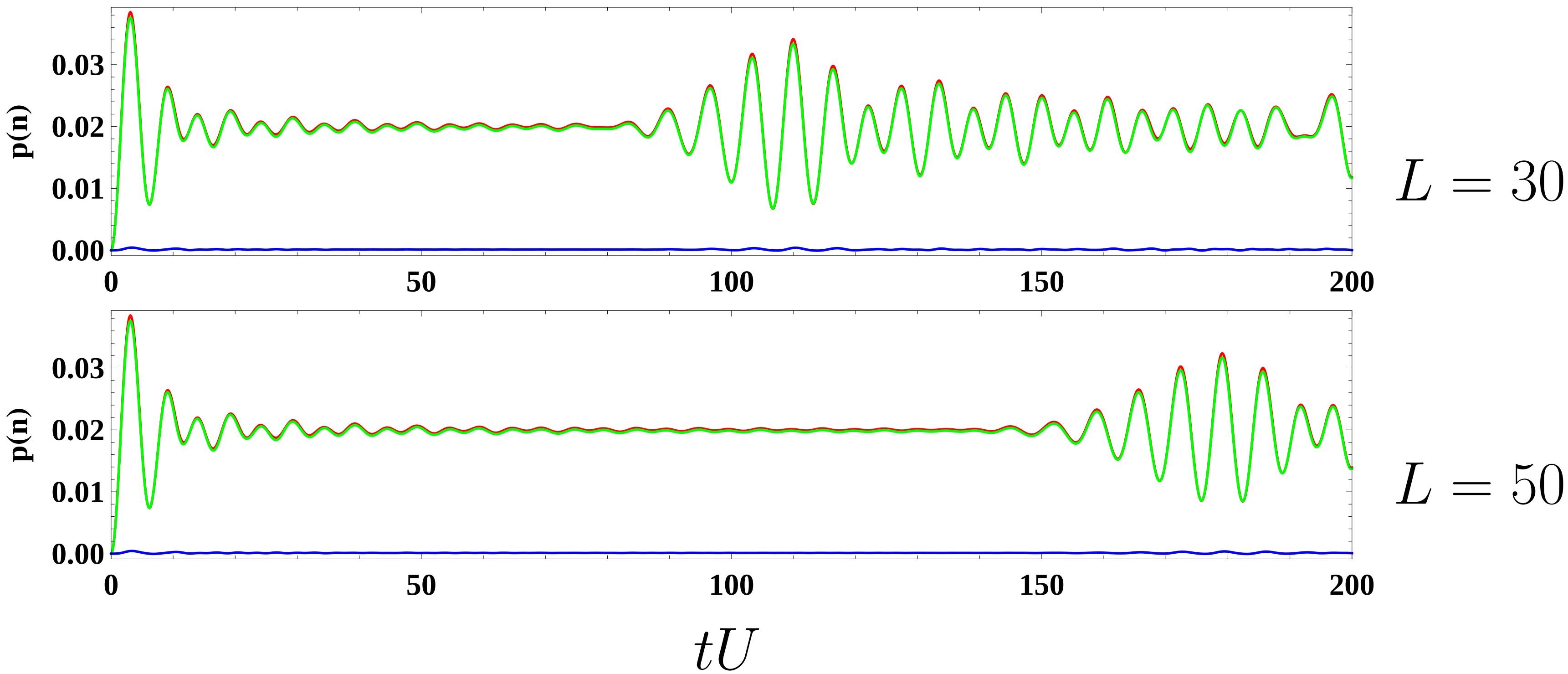


Truncation

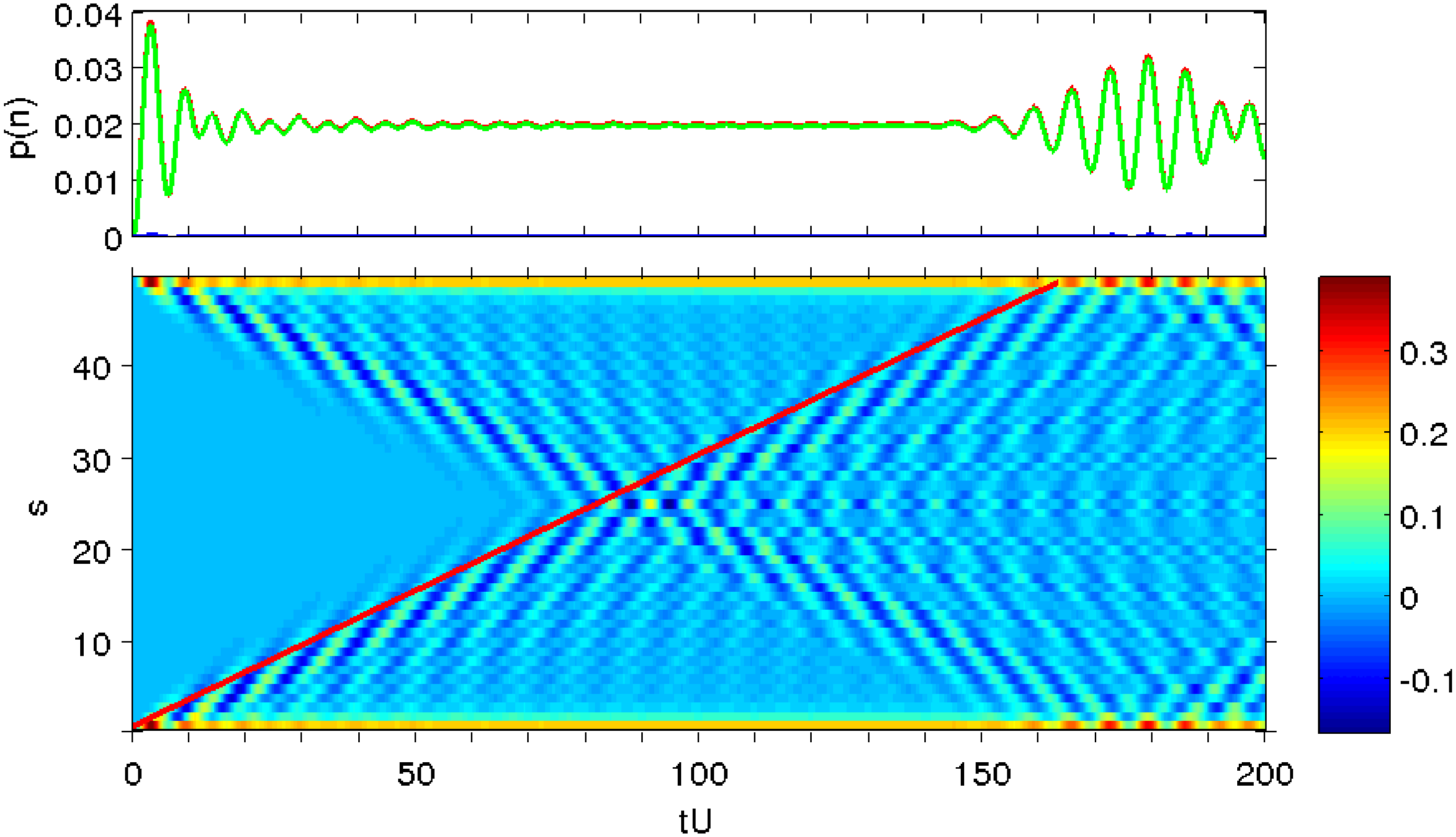
$\hat{\rho}_\mu \Longleftrightarrow \hat{\rho}^{\mathrm{corr}}_{\mu\nu}$

exact diagonalization

Dynamics after quench in 1D



Dynamics after quench in 1D



$L = 50$

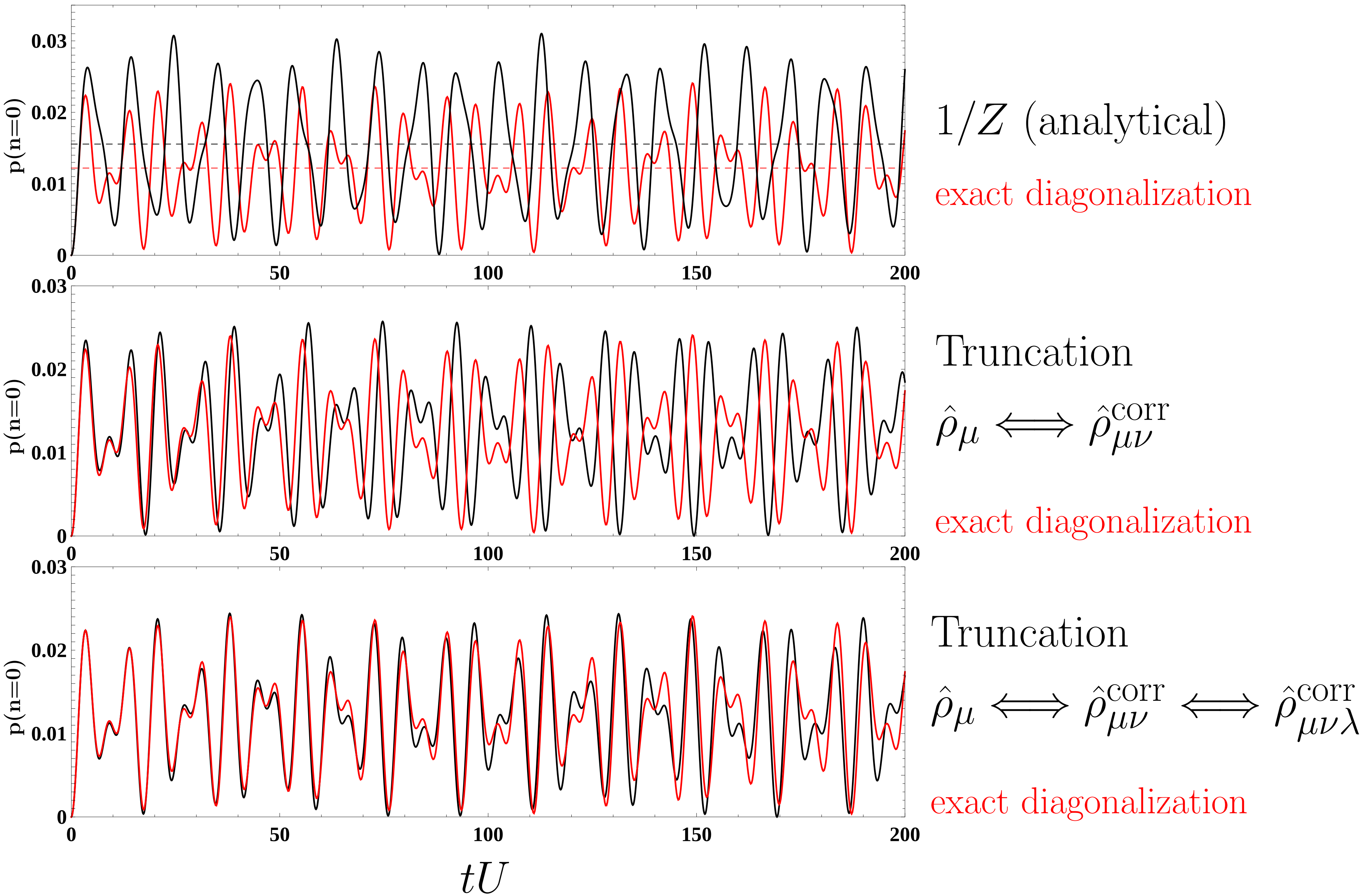
$\langle \hat{b}_\mu^\dagger \hat{b}_{\mu+s} \rangle$

$s = v_{\max} t$

$v_{\max} \approx 3J$

Dynamics after quench in 2D

$J/U = 0 \rightarrow 0.1$ $\qquad 3 \times 3$

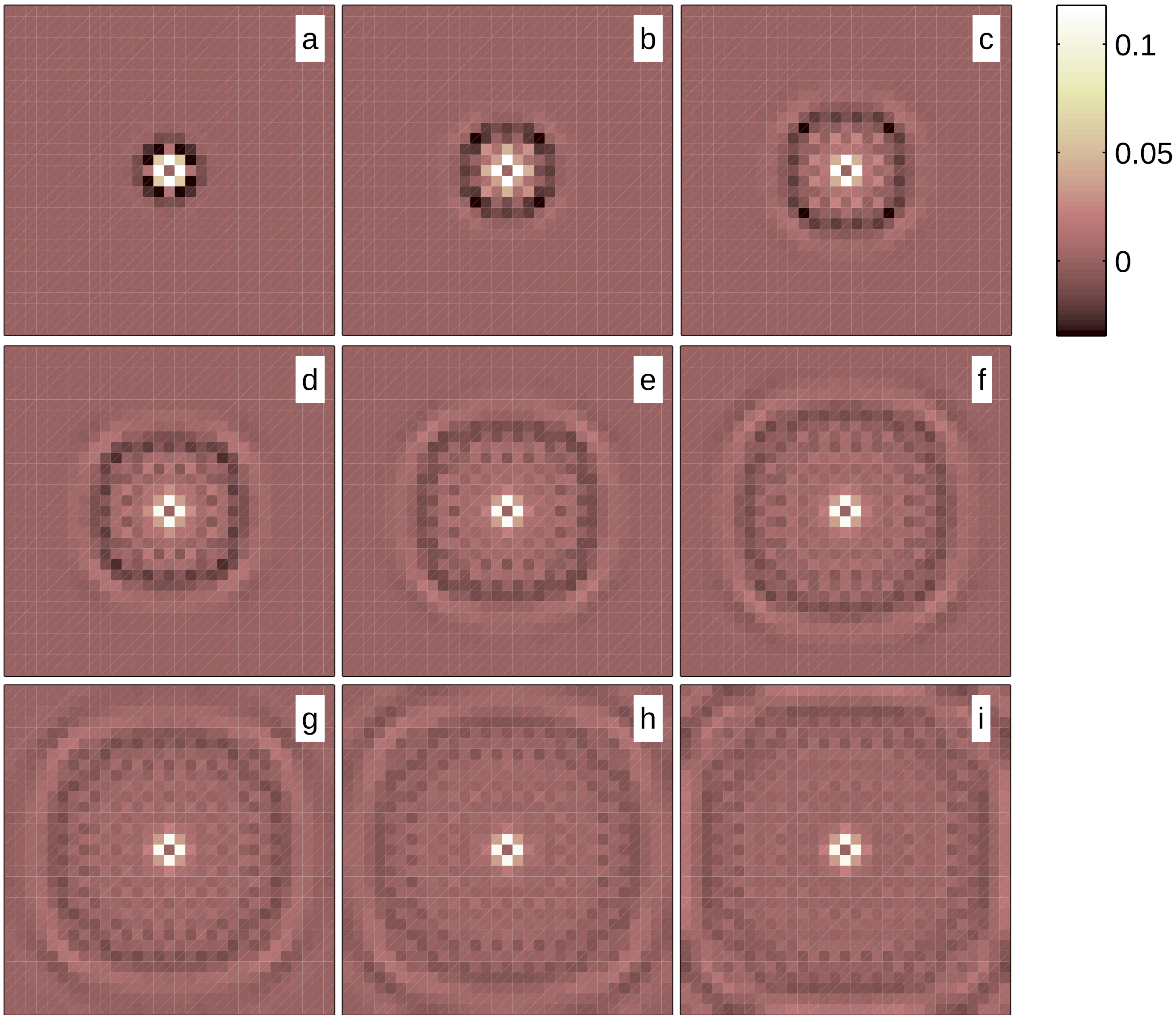


Dynamics after quench in 2D

$J/U = 0 \to 0.1$

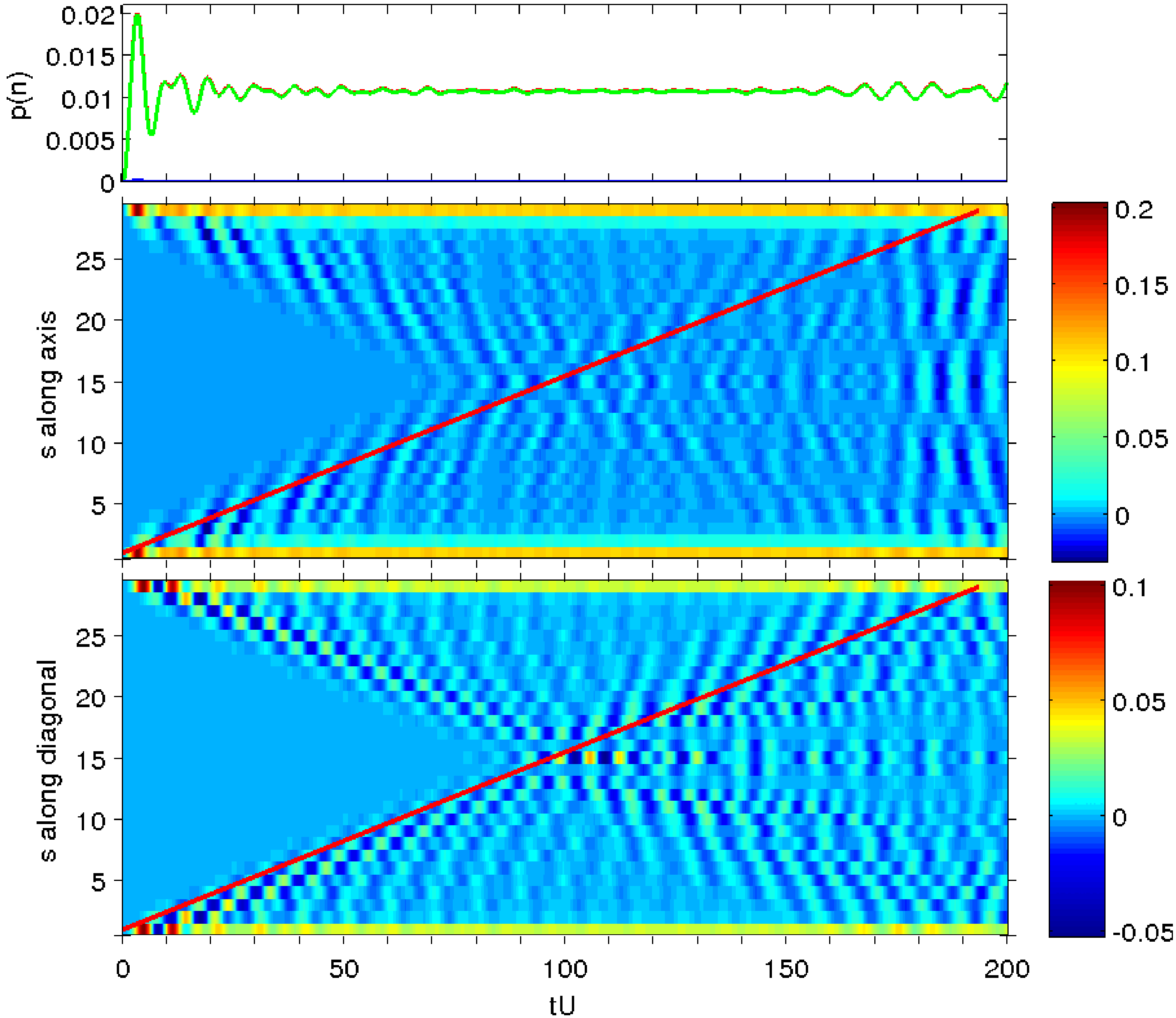
30×30 sites

$\langle \hat{b}_\mu^\dagger \hat{b}_\nu \rangle$



$tU = 10, 20, \ldots, 90$

Dynamics after quench in 2D



$\langle \hat{b}_\mu^\dagger \hat{b}_{\mu+\mathbf{e}_1 s} \rangle$

$v_{\max} \approx 3J/D$

$\langle \hat{b}_\mu^\dagger \hat{b}_{\mu+(\mathbf{e}_1+\mathbf{e}_2)s} \rangle$

$v_{\max} \approx 3J/\sqrt{D}$

Conclusions

- New method for dynamics of quantum correlations:
  higher dimensions, large systems, long time
- Results for Bose-Hubbard model in 1D and 2D
- Perturbative expansion in $1/Z$ gives qualitatively correct predictions
- Truncation schemes lead to higher accuracy
- Study of inhomogeneous systems is possible (traps, disorder)

Expansion in $1/Z$ $\rightarrow$ Phys. Rev. A **82**, 063603 (2010)
Phys. Rev. A **85**, 033625 (2012)
Phys. Rev. A **89**, 033616 (2014)
Truncated equations $\rightarrow$ EPJ Quantum Technology **1**:12 (2014)